臺大管理論叢 2015/6
第 25 卷 第 2 期 53-82
DOI: 10.6226/NTUMR.2015.Oct.GE.32

國際間接投資與風險性資本的探討：理論模型與亞洲銀行的實証分析

Risk Capital and Foreign Indirect Investment: Theory and Evidence from Banks in Asia

陳芬英 / 世新大學財務金融系副教授；國立政治大學風險管理與保險學系博士候選人
Fen-Ying Chen, Associate Professor, Department of Finance, Shih Hsin University; Ph.D. Candidate, Department of Risk Management and Insurance, National Chengchi University

Received 2012/6, Final Received 2013/10

摘 要

本文提出國際投資組合的 VaR 公式解，以利金融機構在國際間接投資中管理市場風險。本模型可視為 Kupiec (1999) 和 Chen and Liao (2009) 模型的一般式，其比 Kupiec (1999) 和 Chen and Liao (2009) 模型更適合目前國際投資的狀況。本文以日本、台灣、韓國的銀行為例，實證發現當銀行中的外國資產的投資權重越大，則以 VaR 模型衡量的風險性資產更接近實際的風險性資本，而且次級房貸發生後，本模型更能準確衡量三國金融機構的風險性資本。此外，我們也發現外國資產的投資權重與相對 VaR 存在先下降後上升的趨勢，因此本文續而最小化 VaR 求得國內外資產的最適配置比例。實證發現，在金融危機期間，銀行業者會減少國外資產的投資比例以最小化 VaR，但在經濟狀況正常時，反而增加國外資產的比重。

【關鍵字】風險性資本、國際投資組合、匯率風險

Abstract

This article presents an analytical value at risk (VaR) for financial institutions to manage the market risk of international portfolios in foreign indirect investment. The model incorporates the Kupiec's (1999) model and the Chen and Liao (2009) model, but it more appropriately fits the real world. Taking Japan, Taiwan, and Korea as examples, the empirical results showed that the increase in the weight of a financial institution's foreign assets strongly enabled its VaR capital allocation to be close to the actual capital. Additionally, the VaR capital bias and default probability bias trended to 1, implying that the VaR model accurately captured the actual risk capital for international portfolios over the subprime mortgage crisis. Alternatively, the relationship between the weights of foreign assets and the VaRs first decreased and then increased. Thus, the VaRs were minimized to obtain the optimal foreign and domestic weights. The empirical results illustrated that bank managers decreased the optimal proportions of foreign assets in Japan, Taiwan or Korea during the subprime crisis, while they increase the weights in a normal economy.

【Keywords】risk capital, international portfolios, exchange rate risk

臺灣大學學術期刊資料庫

1. Introduction

Foreign indirect investment (also referred to as foreign portfolio investment) is the investment made indirectly by foreign private investors through intermediary financial rulings or the financial market. With the liberalization and globalization of capital markets, foreign portfolio investment rose rapidly around the world. In Taiwan, official monthly statistics illustrate that the average of foreign assets relative to domestic assets has been around 46% at domestic commercial banks over the past ten years. In Japan, the ratio is at least 5% and around 9% in Korea. On average, the weight of foreign assets is around 20% at Asian banks and growing. Thus, controlling the market risk from portfolios composed of domestic and foreign assets is an increasing concern for financial institutions.

The VaR approach is widely viewed as a measure of a portfolio's market risk. The VaR amount is generally regarded as a risk capital measure by banking regulators in the world. Various methods can be used to calculate the VaR amount. Basically, approaches to VaR can be usefully classified into two broad groups, the parametric approach[1] and the nonparametric approach.[2] However, one of the well-known shortcomings of the parametric approach is it underestimates the frequency of extreme events. Thus, while not making any distributional assumptions about asset returns, the nonparametric method overcomes the disadvantages of the parametric approach.

Despite the benefits of both approaches, the focus of previous research has been on either the risk management of domestic portfolios (Kupiec, 1999), or foreign portfolios (Chen and Liao, 2009), but not both in the same context. Hence, this article's aim is to develop a VaR model for international portfolios in a period of capital market liberalization and globalization based on the parametric approach.

Kupiec (1999) showed there is a bias in the way VaR-based capital allocation schemes measure risk capital. VaR capital measures do not accurately account for the necessity to maintain equity to cover unexpected losses because VaR capital does not measure the interest

1 In the case of parametric techniques, returns are modeled using normal distribution. This approach is the most common because it is the easiest and fastest method to implement. Research related to these ideas was introduced by Jorion (1996, 2007), Longerstaey and Zangari (1996), Simons (1997), Duffie and Pan (1997), Kupiec (1995, 1999), Brooks and Persand (2002), and Chen and Liao (2009).

2 Studies in the framework of nonparametric approaches follow the research of Hendrick (1996), Jamshidian and Zhu (1997), Hull and White (1998a, 1998b), Barone-Adesi, Giannopoulos, and Vosper (1999), Barone-Adesi and Giannopoulos (2001), Linsmeier and Pearson (2000), Brooks and Persand (2003), and Huang and Lin (2004).

臺灣大學財務金融研究所

that must be paid to debt holders. Also, he demonstrated the bias is a function of instantaneous drift rates and asset return volatility. As to international portfolios, this article disclosed how much bias exists and other elements that affect the bias.

In this paper, the optimal proportions of domestic assets and foreign assets were determined by minimizing VaRs with no constraints. The finding was that investors decreased the optimal proportions of foreign assets in Japan, Taiwan or Korea during the subprime crisis period, and increased the normal economy weights.

The next section outlines the model and a closed-form solution derived to calculate VaRs for international portfolios. In the third section, the analytical formula of the VaR bias was derived. Section 4 provides a comparative static analysis on the risk capital measured by the VaR approach and the VaR bias. Balance sheets of domestic, regional banks and commercial banks in Taiwan, Japan, and Korea were used as samples to inspect the accuracy of the model in terms of the VaR bias criterion, before the subprime mortgage crisis and after. The last section is a conclusion.

2. Model Formulation

In this section, some assumptions on model formulation are presented. The framework by Kupiec (1999) was extended to derive a closed-form solution for an international VaR model in continuous time, a general international VaR model that incorporates both the Kupiec (1999) and Chen and Liao (2009) models.

This paper assumed: (i) a firm's value was composed of the value of a kind of domestic assets and n sorts of foreign assets valued in n classes of currencies, (ii) the capital market is a complete market with no transaction fee or tax, (iii) there exists a theoretical riskless interest rate for lenders and borrowers, (iv) the asset returns were normally distributed over the interest interval, (v) exchange rates were quoted at the price of one unit of the foreign currency in domestic dollars, (vi) investment strategies were constant over the investment horizon, and (vii) the dynamic processes of asset price and exchange rates follow the Geometric Brownian Motion (GBM) as shown below, respectively:

$$\frac{dA_{d,t}}{A_{d,t}} = \mu_{d,t}dt + \sigma_{d,t}\,dW_t, \tag{1}$$

$$\frac{dA_{f_i,t}}{A_{f_i,t}} = \mu_{f_i,t}dt + \sigma_{f_i,t}\,dP_t, \tag{2}$$

臺灣大學學術期刊資料庫

$$\frac{de_{i,t}}{e_{i,t}} = \mu_{e_i,t}dt + \sigma_{e_i,t}\,dQ_t, \tag{3}$$

where $\mu_{d,t}$ and $\sigma_{d,t}$ are the drift rate and volatility of domestic asset returns at time t, respectively; the $\mu_{f_i,t}$ and $\sigma_{f_i,t}$ are the drift rate and volatility for the *i*th kind of foreign asset returns for $i = 1,2,.....,n$ at time t, respectively; $\mu_{e_i,t}$ and $\sigma_{e_i,t}$ are the drift rate and volatility for the *i*th kind of exchange rate at time t, respectively. W_t, P_t and Q_t are one dimensional Brownian motions defined in a filtered probability space (Ω, F, p) under the original probability measure, p. Moreover, the correlation coefficients among the three Brownian motions are defined as $corr(dW_t, dP_t) = \rho_{d,f}$, $corr(dP_t, dQ_t) = \rho_{f,e}$, and $corr(dW_t, dQ_t) = \rho_{d,e}$. Thus, the covariance among domestic assets, foreign assets, and the exchange rate is respectively denoted by

$$\text{cov}\left(\frac{dA_{d,t}}{A_{d,t}}, \frac{dA_{f_i,t}}{A_{f_i,t}}\right) = \rho_{d,f_i}\sigma_{d,t}\sigma_{f_i,t}\,dt, \qquad \text{cov}\left(\frac{dA_{d,t}}{A_{d,t}}, \frac{de_{i_i,t}}{e_{i_i,t}}\right) = \rho_{d,e_i}\sigma_{d,t}\sigma_{e_i,t}\,dt,$$

and

$$\text{cov}\left(\frac{dA_{f_i,t}}{A_{f_i,t}}, \frac{de_{i,t}}{e_{i_i,t}}\right) = \rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t}\,dt, \quad \text{in which } \rho_{j,k} \text{ is a correlation coefficient}$$

between j and k, $j \neq k$.

Now, consider the potential daily loss exposure to long trading positions. Typically, the VaR is a specific left-hand critical value for a potential loss distribution. Given conventions, one can define the daily losses valued in domestic dollars relative to the end-of-period expected asset value, called relative VaR and denoted by $VaR(mean)$, as follows:

$$-VaR(mean) \equiv V_\alpha - E_t(V_T), \tag{4}$$

in which VaR(mean) > 0, and the $E_t(.)$ is the expected value conditional on information at time t. The V_α is the firm value denominated in domestic dollars given a confidence level of α. V_t is the firm value at time T (the investment horizon), which consists of a type of domestic asset and n sorts of foreign assets, also denoted by

$$V_T = A_{d,T} + \sum_{i=1}^{n} e_{i,T}A_{f_i,T}\ .$$

Alternatively, the VaR(0) can represent the VaR in domestic dollars relative to the initial asset value, namely absolute VaR:

$$-VaR(0) \equiv V_\alpha - V_T, \tag{5}$$

With VaR(0) > 0.

Before deriving the VaR analytical formula for an international portfolio, Proposition 1

臺灣大學學術期刊資料庫

must be used.

Proposition 1: Given the dynamic processes for asset prices and exchange rates following the GBM, the dynamic process of V_t can be expressed

$$\frac{dV_t}{V_t}=\left[\gamma_t\mu_{d,t}+\sum_{i=1}^{n}\beta_{i,t}(\mu_{e_i,t}+\mu_{f_i,t}+\rho_{f_i,e_i}\sigma_{e_i,t}\sigma_{f_i,t})\right]dt+\gamma_t\sigma_{d,t}\,dW_t+\left[\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\right]dQ_t+\left[\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}\right]dP_t$$

with $V_t=A_{d,t}+\sum_{i=1}^{n}e_{i,t}A_{f_i,t}$, $\gamma_t=\frac{A_{d,t}}{V_t}$, and $\beta_{i,t}=\frac{e_{i,t}A_{f_i,t}}{V_t}$. The γ_t and $\beta_{i,t}$ are also named the weights of a variety of domestic assets and the ith kind of foreign asset, respectively.

Appendix A provides a detailed proof of Proposition 1. Note that the weights do not vary over the investment horizon based on assumption (vi).

Using Proposition 1 and equations (4) and (5), the analytical formulas for the relative VaR and absolute VaR of international portfolios can be derived as

$$VaR(mean)=\left|Min\left\{V_t\left\{\begin{array}{l}\exp\left[(\gamma_t\mu_{d,t}+\sum_{i=1}^{n}\beta_{i,t}(\mu_{e_i,t}+\mu_{f_i,t}+\rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t})-\frac{1}{2}\sigma_t^2)(T-t)+Z_\alpha\sigma_t\sqrt{T-t}\right]\\ -\exp\left[(\gamma_t\mu_{d,t}+\sum_{i=1}^{n}\beta_{i,t}(\mu_{e_i,t}+\mu_{f_i,t}+\rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t}))(T-t)\right]\end{array}\right\},0\right\}\right|$$

and

$$VaR(0)=\left|Min\left\{V_t\left\{\exp\left[(\gamma_t\mu_{d,t}+\sum_{i=1}^{n}\beta_{i,t}(\mu_{e_i,t}+\mu_{f_i,t}+\rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t})-\frac{1}{2}\sigma_t^2)(T-t)+Z_\alpha\sigma_t\sqrt{T-t}\right]-1\right\},0\right\}\right|, \tag{6}$$

in which the Z_α stands for a critical value with a given probability α, and

$$\sigma_t=\sqrt{\gamma_t^2\sigma_{d,t}^2+\left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\right)^2+\left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}\right)^2+2\rho_{d,e_i}\gamma_t\sigma_{d,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}+2\rho_{d,f_i}\gamma_t\sigma_{d,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}+2\rho_{f_i,e_i}\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}}.$$

Equation (6) is provided in **Appendix B**.

The analytical formula in equation (6) stands for the VaR capital allocation measured for an international portfolio, and it includes some important elements, such as the volatility of underlying assets, exchange rate volatility, the correlation coefficient among domestic assets, foreign assets and exchange rate, and the weight of domestic and foreign assets. Equation (6) can also be reduced to Kupiec's (1999)[3] solution as $\gamma_t = 1$, $\beta_{i,t} = 0$, which shows

3 Kupiec (1999) shows the relative VaR and absolute VaR as follows, respectively:

$$VaR_k(mean)=\left|Min\left\{A_{d,t}\left[e^{(\mu_{d,t}-\frac{1}{2}\sigma_{d,t}^2)(T-t)+Z_\alpha\sqrt{\sigma_{d,t}^2(T-t)}}-e^{\mu_{d,t}(T-t)}\right],0\right\}\right|, \quad \text{and } VaR_k(0)=\left|Min\left\{A_{d,t}\left[e^{(\mu_{d,t}-\frac{1}{2}\sigma_{d,t}^2)(T-t)+Z_\alpha\sqrt{\sigma_{d,t}^2(T-t)}}-1\right],0\right\}\right|.$$

臺灣大學學術期刊資料庫

that a firm value is only composed of domestic assets. Alternatively, equation (6) goes to the closed-form solution of Chen and Liao (2009)[4] as $\gamma_t = 0$, $\beta_{i,t} = 1$ indicating a firm's value is only made up of foreign assets. Clearly, equation (6) is a general form. It can be incorporated into the analytic solutions in the Kupiec (1999) and Chen and Liao (2009) models. In other words, Kupiec (1999) presented a VaR model to manage market risk to assets valued in domestic currency. Chen and Liao (2009) developed a VaR model for risk management of foreign assets in continuous time. The focus of the previous research has only been on either domestic or foreign assets, but not both in the same context. In reality, investors especially banks generally buy not only domestic assets, but also foreign assets in highly integrated global financial markets. Therefore, this paper examined stochastic assets (domestic assets and foreign assets) to provide a general VaR to manage portfolio market risks. The model is more appropriate than the Kupiec (1999) model and the Chen and Liao (2009) model for real world situations.

Jorion (2007) illustrated that relative VaR is more conservative if the mean value is positive. The more conservative measure is more appropriate. Hence, a relative VaR measure was adopted throughout this article.

3. VaR Bias for International Portfolios

The VaR bias reported by Kupiec (1999) comprises a VaR capital bias and default probability bias. It will be explained for international portfolios.

3.1 VaR Capital Bias

VaR capital bias is measured by the ratio of VaR equity capital allocation to the actual equity capital allocation[5] that is required to achieve the target default rate (in percent). Briefly,

4 Chen and Liao (2009) derived the relative VaR and absolute VaR of foreign-issued assets below, respectively:

$$VaR_c\ (mean) = \left| Min\left\{ A_{f_i,t}\, e_{i,t} \left\{ \begin{array}{l} \exp\left[(\mu_{f_i,t} + \mu_{e_i,t} - \frac{1}{2}\sigma_{f_i,t}^2 - \frac{1}{2}\sigma_{e_i,t}^2)(T-t) + Z_\alpha \sqrt{(\sigma_{f_i,t}^2 + \sigma_{e_i,t}^2 + 2\rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t})(T-t)} \right] \\ -\exp\left[(\mu_{f,t} + \mu_{Q,t} + \rho_{f,Q}\sigma_{f,t}\sigma_{Q,t})(T-t) \right] \end{array} \right\}, 0 \right\} \right|,$$

and

$$VaR_c\ (0) = \left| Min\left\{ A_{f_i,t}\, e_{i,t} \left\{ \exp\left[(\mu_{f_i,t} + \mu_{e_i,t} - \frac{1}{2}\sigma_{f_i,t}^2 - \frac{1}{2}\sigma_{e_i,t}^2)(T-t) + Z_\alpha \sqrt{(\sigma_{f_i,t}^2 + \sigma_{e_i,t}^2 + 2\rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t})(T-t)} \right] - 1 \right\}, 0 \right\} \right|.$$

5 The actual equity capital allocation is briefly named the actual capital in Kupiec (1999).

臺灣大學學術期刊資料庫

$$\text{VaR capital bias}\,(VB) = \frac{\text{VaR capital}}{\text{Actual capital}}. \tag{7}$$

As equation (7) moves closer to 1, the VaR capital is nearing the actual capital. When equation (7) is greater than 1, a firm's actual capital is less than its VaR capital. Also, it implies the firm is undercapitalized and exposed to potentially wider fluctuations than firms with sufficient capitalization.

Kupiec (1999) used the Black and Scholes (1973) and Merton (1974) models (the BSM model) to estimate actual capital. Under the BSM model, the firm's equity is a call option on the firm's underlying value, with a strike price equal to the face value of the firm's debt, and debt maturity equal to the option's time to maturity. A firm's equity value at time T can be defined as:

$$E_T = Max(V_T - K, 0).$$

Note that V and K stand for the market value of assets and the book value of debt, respectively. If one uses a riskless asset (B_t) as a numeraire, $\frac{E_t}{B_t}$ is a martingale under a risk neutral probability measure $\tilde{p}$ conditional on the information at time t, or $\frac{E_t}{B_t} = E^{\tilde{p}}\left[\frac{E_T}{B_T}\middle| F_t\right]$. Thus, the value of a firm's equity at time t can be written as

$$E_t = e^{-r_t(T-t)}\, E_t^{\tilde{P}}[Max(V_T - K, 0)], \tag{8}$$

where $E_t^{\tilde{P}}[.]$ is the expected value conditional on the information at time t under the risk neutral probability measure, $\tilde{p}$. Traditionally, the value of equity today (E_t) is viewed as actual capital.

Originally, the dynamic process of a firm's value with domestic and foreign assets is described by Proposition 1. Assuming

$$\sigma_t d Y_t = \gamma_t \sigma_{d,t} d W_t + \left[\sum_{i=1}^{n} \beta_{i,t} \sigma_{e_i,t}\right] d Q_t + \left[\sum_{i=1}^{n} \beta_{i,t} \sigma_{f_i,t}\right] d P_t$$

and $\mu_t = \gamma_t \mu_{d,t} + \sum_{i=1}^{n} \beta_{i,t} (\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i, e_i} \sigma_{e_i,t} \sigma_{f_i,t})$,

Proposition 1 can be rewritten

$$\frac{dV_t}{V_t} = \mu_t dt + \sigma_t dY \tag{9}$$

Where Y_t is a one dimensional Brownian motion under the original probability measure, and

臺灣大學學術期刊資料庫

$$\sigma_t = \sqrt{\gamma_t^2\sigma_{d,t}^2 + \left[\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\right]^2 + \left[\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}\right]^2 + 2\rho_{d,e_i}\gamma_t\sigma_{d,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t} + 2\rho_{d,f_i}\gamma_t\sigma_{d,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t} + 2\rho_{f_i,e_i}\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}}.$$

By means of Girsanov's theorem, the relationship between the original probability measure and the risk neutral probability measure is $dY_t = d\tilde{Y}_t + \sigma_t dt$. Then, the expected value of $\frac{dV_t}{V_t}$ is equal to the riskless rate per unit of time under the risk neutral probability measure, $E^{\tilde{P}}[\frac{dV_t}{V_t}|F_t] = r_{d,t}dt$. Thus, equation (9) becomes

$$\frac{dV_t}{V_t} = r_{d,t}dt + \sigma_t d\tilde{Y}_t,$$

$$\text{or } \frac{dV_t}{V_t} = r_{d,t}dt + \gamma_t\sigma_{d,t}d\tilde{W}_t + \left[\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\right]d\tilde{Q}_t + \left[\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}\right]d\tilde{P}. \tag{10}$$

Using Ito's lemma, one can obtain $V_T = V_t e^{(r_{d,t}-\frac{1}{2}\sigma_t^2)(T-t)+\sigma_t\tilde{Y}_{T-t}}$. Equation (11) can be derived:

$$E^{\tilde{P}}[\,V_T 1_{\{V_T>\mathrm{K}\}}|\,F_t\,] = V_t e^{r_{d,t}(T-t)}\Phi(d_1), \text{ and}$$
$$E^{\tilde{P}}[\,1_{\{V_T>\mathrm{K}\}}|\,F_t\,] = \Phi(d_2), \tag{11}$$

with $d_1 = \frac{\ln\frac{V_t}{K} + (r_{d,t} + \frac{1}{2}\sigma_t^2)(T-t)}{\sigma_t\sqrt{T-t}}$, $d_2 = \frac{\ln\frac{V_t}{K} + (r_{d,t} - \frac{1}{2}\sigma_t^2)(T-t)}{\sigma_t\sqrt{T-t}}$, and

$$\sigma_t = \sqrt{\gamma_t^2\sigma_{d,t}^2 + \left[\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\right]^2 + \left[\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}\right]^2 + 2\rho_{d,e_i}\gamma_t\sigma_{d,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t} + 2\rho_{d,f_i}\gamma_t\sigma_{d,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t} + 2\rho_{f_i,e_i}\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}}.$$

Therefore, the closed form solution of the value of today's equity is

$$E_t = V_t\Phi(d_1) - Ke^{-r_{d,t}(T-t)}\Phi(d_2). \tag{12}$$

Equation (12) can be reduced to the BSM model formula as $\gamma_t = 1$, $\beta_{i,t} = 0$. The actual capital is also $e^{-r_{d,t}(T-t)}\{VaR(mean)\}$.[6]

The par value of a firm's debt, *K*, can be derived by the default probability desired to achieve the VaR significance level, where a firm has an end-of-period value below K. Therefore, $K = V_t e^{(\mu_t - \frac{1}{2}\sigma_t^2)(T-t)+Z_\alpha\sigma_t\sqrt{T-t}}$. Again, K equals the expected value of the firm's end-of-period value minus *VaR*(*mean*) or $E_t[V_T]$–V*aR*(*mean*) (see, for example, Jorion (2007)).

6 See Jorion (2007).

臺灣大學學術期刊資料庫

Hence, the VaR capital bias is measured by the ratio of equation (12) to equation (6)

$$\text{VB} = \frac{VaR(mean)}{V_t\Phi(d_1) - Ke^{-r_{d,t}(T-t)}\Phi(d_2)}, \text{ or}$$

$$\frac{VaR(0)}{V_t\Phi(d_1) - Ke^{-r_{d,t}(T-t)}\Phi(d_2)}, \tag{13}$$

$$\text{with } d_1 = \frac{\ln\frac{V_t}{K} + (r_{d,t} + \frac{1}{2}\sigma_t^2)(T-t)}{\sigma_t\sqrt{T-t}}, \quad d_2 = \frac{\ln\frac{V_t}{K} + (r_{d,t} - \frac{1}{2}\sigma_t^2)(T-t)}{\sigma_t\sqrt{T-t}},$$

$$\sigma_t = \sqrt{\gamma_t^2\sigma_{d,t}^2 + \left[\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\right]^2 + \left[\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}\right]^2 + 2\rho_{d,e_i}\gamma_t\sigma_{d,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t} + 2\rho_{d,f_i}\gamma_t\sigma_{d,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t} + 2\rho_{f_i,e_j}\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}},$$

$$VaR(mean) = \left| Min\left\{ V_t\left\{ \begin{array}{l} \exp\left[(\gamma_t\mu_{d,t} + \sum_{i=1}^{n}\beta_{i,t}(\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t}) - \frac{1}{2}\sigma_t^2)(T-t) + Z_\alpha\sigma_t\sqrt{T-t}\right] \\ -\exp\left[(\gamma_t\mu_{d,t} + \sum_{i=1}^{n}\beta_{i,t}(\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t})(T-t)\right] \end{array} \right\}, 0 \right\} \right|,$$

and

$$VaR(0) = \left| Min\left\{ V_t\left\{ \exp\left[(\gamma_t\mu_{d,t} + \sum_{i=1}^{n}\beta_{i,t}(\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t}) - \frac{1}{2}\sigma_t^2)(T-t) + Z_\alpha\sigma_t\sqrt{T-t}\right] - 1\right\}, 0 \right\} \right|.$$

3.2 Default Probability Bias

The default probability bias is defined as the ratio between the actual default probability and the nominal default probability measured by the VaR.

$$\text{Default probability bias}\,(DB) = \frac{\text{Actual default probability}}{\text{Default probability measured by VaR measure}}.$$

Actual default probability can be measured by Merton (1974) formula, when a firm's value is less than the book value of the firm's debt. The actual default probability is $\Phi(d_3)$. Hence, the default probability bias has a confidence level of α, denoted

$$\text{DB} = \frac{\Phi(d_3)}{\alpha} \tag{14}$$

$$\text{with } d_3 = \frac{\ln\frac{K}{V_t} - (\mu_t - \frac{1}{2}\sigma_t^2)(T-t)}{\sigma_t\sqrt{T-t}}, \quad \mu_t = \gamma_t\mu_{d,t} + \sum_{i=1}^{n}\beta_{i,t}(\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i,e_i}\sigma_{e_i,t}\sigma_{f_i,t}),$$

臺灣大學學術期刊資料庫

and

$$\sigma_t = \sqrt{\gamma_t^2\sigma_{d,t}^2 + \left[\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\right]^2 + \left[\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}\right]^2 + 2\rho_{d,e_i}\gamma_t\sigma_{d,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t} + 2\rho_{d,f_i}\gamma_t\sigma_{d,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t} + 2\rho_{f_i,e_i}\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}}.$$

4. Sensitivity Analysis

In this section, a comparative static analysis was performed on the VaR capital and VaR bias. This article assumed: (i) the firm's value consisted of a variety of domestic assets and a class of foreign assets, and the exchange rate was the ratio of domestic currency to foreign currency; (ii) the initial firm's value was $100, and book value of the firm's debt was $90; (iii) the critical value was –2.33 at a given α of 0.01, and the investment horizon was set at one year (T–t = 1).

Monthly domestic and foreign asset-log returns data from domestic, regional, and commercial banks in Taiwan, Japan, and Korea were used as samples, respectively. In May, 2013, the Central Bank of Taiwan reported the number of domestic bank head offices at 41, with 3,433 branches in Taiwan. The ratio of their deposits relative to all bank deposits was higher than 70%. In Japan, the Bank of Japan reported that domestically licensed banks included city banks (6), regional banks (64), regional banks II (41), and trust banks (31) in May, 2013. Deposits at regional banks were more substantial than in other banks in Japan. In Korea, banks are divided into commercial banks and specialized banks. Commercial banks consist of nationwide, local banks and branches of foreign banks. Special banks refer to financial institutions established under a special act, rather than the Banking Act, and their main enterprise is the banking businesses. Specialized banks include the Korea Development Bank, the Export-Import Bank of Korea, the Industrial Bank of Korea, the National Agricultural Cooperative Federation, the National Federation of Fisheries Cooperatives, and others. Official statistics report the number of commercial banks was 52, far more than the five specialized banks in December, 2011. Thus, Korean commercial banks were used as samples in this paper.

Generally speaking, all bank assets can be split into domestic and foreign-currency assets for each country. Foreign-currency assets were composed of stocks, derivatives, bonds, and other securities, valued in foreign currencies in Taiwan, Japan, and Korea. The three countries' banks have similar capital requirements. The minimum capital requirement rule for the Bank of Japan, Bank of Taiwan, and Bank of South Korean are at least 8% of a bank's international settlement ratio (BIS ratio).

臺灣大學博碩士論文期刊資料庫

The monthly asset's log returns were computed using the following formula:

$$R_t = \log\left(\frac{P_t}{P_{t-1}}\right),$$

where P_t is the value of investments in domestic assets or foreign assets, from the domestic, regional, and commercial bank's balance sheets as offered by the Central Bank of Taiwan, Bank of Japan, and Bank of Korea. Investments in domestic stocks and securities were used as the value of domestic assets, and investments in foreign stocks and securities as the value of foreign assets. The time window spanned from January, 2000 to December, 2012, and was divided into two periods. Period I was before the subprime mortgage crisis, from January, 2000 to July, 2007, when monthly asset returns totaled 91. Period II was from August, 2007 to December, 2012, when monthly asset returns were 65. The time break at August, 2007 reflects the subprime mortgage crisis that hit America that summer.

As shown in Table 1, a set of model parameters were estimated for various samples. Note that the estimated results of $\mu_{d,t}$, $\mu_{f_i,t}$, and $\mu_{e_i,t}$ were the expected value of equation (15) plus their half variance. That is $E\left[d(\ln A_{j,t})\right]+\frac{1}{2}\sigma^2_{j,t}$, for $j = d, f_{i,t}, e_{i,t}, j \neq i$. The riskless rate was estimated based on the interest rate of three-month certificate deposits.

$$\begin{aligned}
d(\ln A_{d,t}) &= (\mu_{d,t} - \frac{1}{2}\sigma^2_{d,t})dt + \sigma_{d,t}\, dW_t, \\
d(\ln A_{f_i,t}) &= (\mu_{f_i,t} - \frac{1}{2}\sigma^2_{f_i,t})dt + \sigma_{f_i,t}\, dP, \\
d(\ln e_{i,t}) &= (\mu_{e_i,t} - \frac{1}{2}\sigma^2_{e_i,t})dt + \sigma_{e_i,t}\, dQ.
\end{aligned} \tag{15}$$

Through Table 1, the respective average values of model parameters can be obtained across three countries. The initial value of these parameters for sensitivity analysis, which means the initial value of the riskless interest rates was 0.02579,[7] and the domestic weight was calculated to be 0.8674, while the foreign weight was 0.1326, domestic mean returns was 0.0062, foreign mean returns were 0.0359, the exchange rate's means returns were 0.0101, domestic asset volatility was 0.0678, foreign asset volatility was 0.0033, exchange rate volatility was 0.0270. Correlation coefficients between domestic assets and foreign

7 $\frac{0.0018+0.0084333}{2} = 0.00511665$; $\frac{0.0217978+0.0163286}{2} = 0.0190632$; $\frac{0.0501+0.0563}{2} = 0.0532$.
$(0.00511665 + 0.0190632 + 0.0532) \div 3 = 0.02579$.

臺灣大學學術期刊資料庫

assets, domestic assets and exchange rates, and foreign assets and exchange rates were 0.0410, -0.2053, and 0.3154, respectively.

Table 1 Estimation of Model Parameters in Various Periods

Parameter	Japan		Taiwan		Korea	
	Period I	Period II	Period I	Period II	Period I	Period II
r_t	0.0018000	0.0084333	0.0217978	0.0163286	0.0501000	0.0563000
γ_t	0.9974884	0.9982780	0.7197946	0.6550665	0.9420988	0.8915431
β_t	0.0025116	0.0017220	0.2802054	0.3449335	0.0579012	0.1084569
$\mu_{d,t}$	0.0012295	0.0053171	0.0059130	0.0034456	0.0112260	0.0102740
$\mu_{f,t}$	0.1596620	-0.0107720	0.0146820	0.0110090	0.0150230	0.0258340
$\sigma_{d,t}$	0.0103497	0.0110762	0.0091230	0.0055611	0.0140850	0.0102500
$\sigma_{f,t}$	0.1487740	0.0876910	0.0150700	0.0157040	0.0514070	0.0879890
$\mu_{e,t}$	0.0030430	-0.0056520	0.0029226	0.0017070	-0.0019000	0.0197880
$\sigma_{e,t}$	0.0226410	0.0345310	0.0141429	0.0207910	0.0227260	0.0470550
$\rho_{d,f}$	0.1463370	0.3566610	-0.0682020	0.0746110	-0.1846370	-0.0789420
$\rho_{d,e}$	0.0757240	-0.5819920	-0.3478620	-0.1232430	-0.0636670	-0.1907420
$\rho_{f,e}$	0.2605640	0.0156460	0.4844560	0.5692030	0.2647890	0.2974470

*Note: Supposing the value of the banks in Japan, Taiwan, and Korea was made up of a variety of domestic assets and foreign assets (valued in US. dollars), and the exchange rate was the ratio of domestic dollars to US. dollars.

4.1 Sensitivity Analysis of VaR Capital

Using equation (6), the impact of volatility and correlation coefficients on the relative VaR capital allocation is shown in Figures 1A and 1B, respectively. There was one consistent phenomenon, that the relative VaR increased monotonically as volatilities and correlation coefficients grew. The reason is that increasing volatility and correlation coefficients made the potential losses rise. Additionally, the sensitivity of foreign asset volatility was higher than other asset's volatilities. The impact of correlation coefficients between foreign assets and exchange rates on VaR capital was greater than other correlation coefficients. Figure 1C also illustrates the relationship between foreign asset weights and the VaRs first decreased and then increased, implying that the percentage of foreign assets decreases an international portfolio's market risk at the beginning, then increases market risk in the end.

In Figures 1A and 1B, VaR capital sensitivity for correlation coefficients are weak, while Figure 1C was more sensitive than Figures 1A and 1B.

臺灣大學學術期刊資料庫

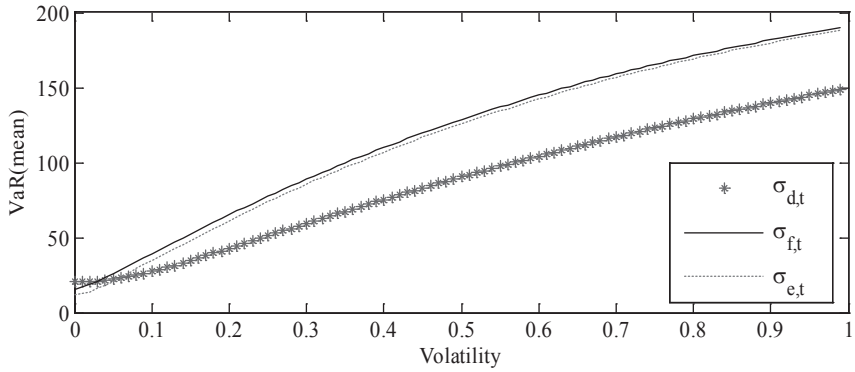


Figure 1A The Impact of Volatility on VaR

*Note that $\sigma_{d,t}$, $\sigma_{f,t}$, and $\sigma_{e,t}$ represent the volatility Y of domestic asset returns, foreign asset returns and foreign exchange rate returns, respectively.

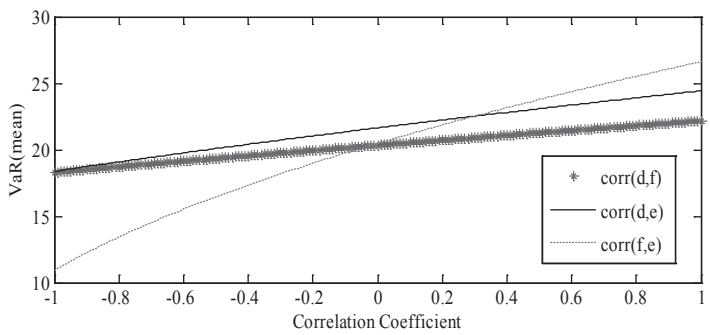


Figure 1B The Impact of the Correlation Coefficient on VaR

*Note that corr(d,f), corr(d,e) and corr(f,e) denote the correlation coeffcients between domestic asset returns and foreign asset returns, domestic asset returns and foreign exchange rate returns, foreign asset returns and foreign exchange rate returns, respectively.

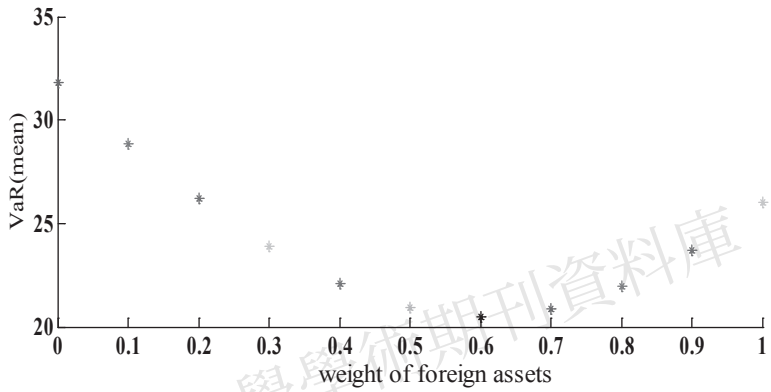


Figure 1C The Impact of Weights of Foreign Assets on VaR

臺灣大學學術期刊資料庫

4.2 Sensitivity Analysis of the VaR Bias

Figures 2A, 2B, and 2C showed that the effect of volatility, correlation coefficients, and the weights of foreign assets on the VaR capital bias generally increased based on equations (13) and (14), respectively. Figure 2B demonstrated that the sensitivity of correlation coefficients between foreign assets and domestic assets was greater than the other correlation coefficients. Also, the VaR capital bias was less than 1, and increased as the correlation coefficients increased implying that the VaR capital allocation was moving closer to the actual capital. As illustrated in Figure 2C, as the weight of foreign assets rose, the VaR capital bias also rose implying that increasing foreign assets enabled the VaR capital allocation to closer to the actual capital.

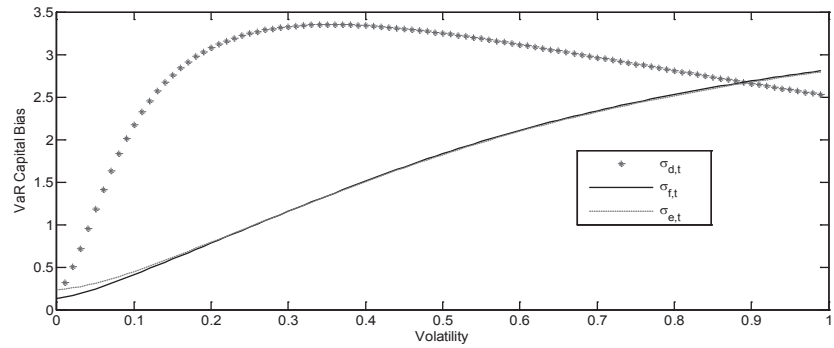


Figure 2A The Impact of Volatility on the VaR Capital Bias

*Note that $\sigma_{d,t}$, $\sigma_{f,t}$, and $\sigma_{e,t}$ represent the volatility of domestic asset returns, foreign asset returns, and foreign exchange rate returns, respectively.

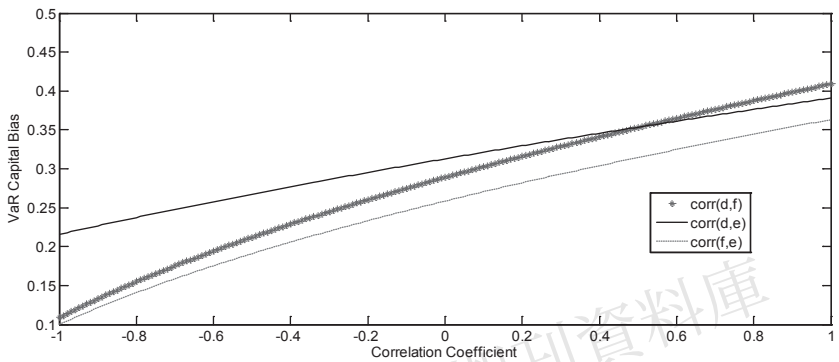


Figure 2B The Impact of the Correlation Coefficient on the VaR Capital Bias

*Note that corr(d,f), corr(d,e), and corr(f,e) denote the correlation coeffcients between domestic asset returns and foreign asset returns, domestic asset returns and foreign exchange rate returns, foreign asset returns, and foreign exchange rate returns, respectively.

臺灣大學商學期刊資料庫

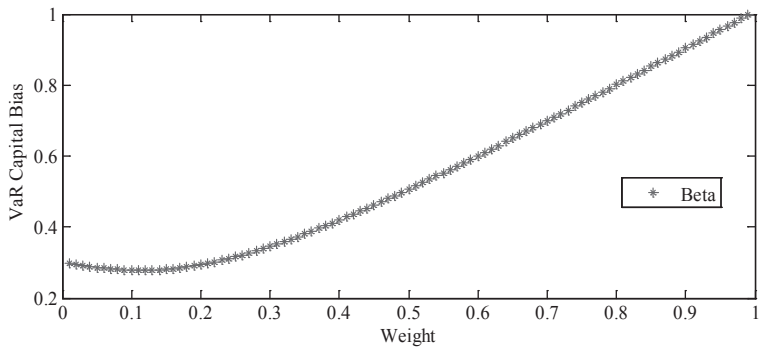


Figure 2C The Impact of Weights of Foreign Assets on the VaR Capital Bias
*Note that Beta stands for the weights of foreign asset.

Figures 3A, 3B, and 3C indicate that volatility, correlation coefficients, and the weights of foreign assets positively affect the default probability bias. Notice that the true default probability is always in excess of the target default probability implied by VaR since the default probability bias was greater than 1, and the bias increased at high levels of volatility, correlation coefficients, and weights of foreign assets. That is, the target VaR default rate was downward-biased as volatility, correlation coefficients, and foreign asset weights increased. Intuitively, these figures show that as volatility, correlation coefficients, and foreign asset weights shrink, the potential losses decline, and the actual default probability decreases.

Additionally, the debt's par value greatly affected the VaR capital bias and default probability bias shown in Figure 4. As the book value of debt increased, the actual default probability estimated in the BSM model rose. Thus the relation of the debt's par value and default probability bias was positive. In general, the magnitude of VaR capital bias was lower than 1 indicating that the actual capital was greater than the VaR capital allocation. Alternatively, Figure 4 illustrates that the default probability bias was always higher than VaR capital bias when the book value of debt increased. This implies that the impact of the debt's book value on default probability was greater than on VaR capital allocation.

Throughout Figures 2A, 2B, 2C, 3A, 3B, and 3C, the sensitivities of VaR bias and default probability bias for the correlation coefficients was the weakest.

臺灣大學學術期刊資料庫

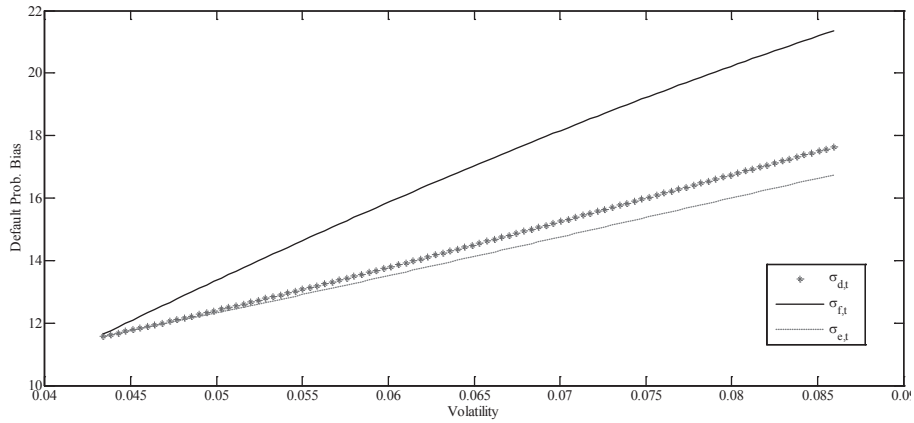


Figure 3A The Impact of Volatility on the Default Probability Bias

*Note that $\sigma_{d,t}$, $\sigma_{f,t}$, and $\sigma_{e,t}$ represent the volatility of domestic asset returns, foreign asset returns, and foreign exchange rate returns, respectively.

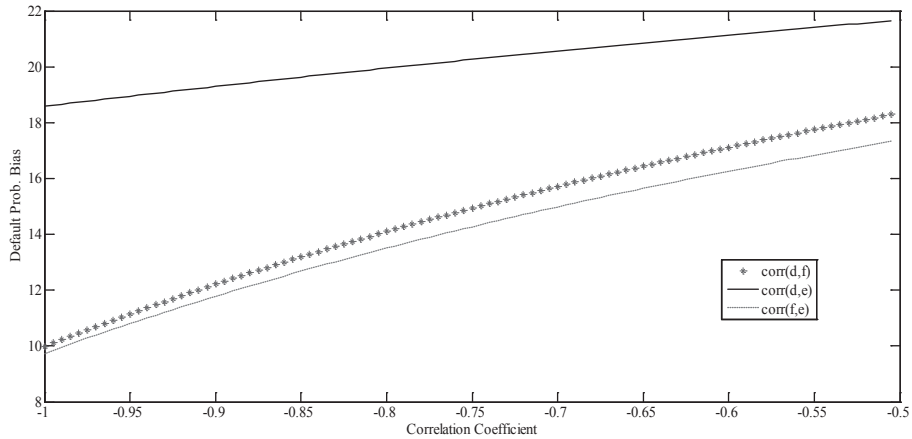


Figure 3B The Impact of the Correlation Coefficient on the Default Probability Bias

*Note that corr(d,f), corr(d,e), and corr(f,e) denote the correlation coefficients between domestic asset returns and foreign asset returns, domestic asset returns and foreign exchange rate returns, foreign asset returns and foreign exchange rate returns, respectively.

臺灣大學學術期刊資料庫

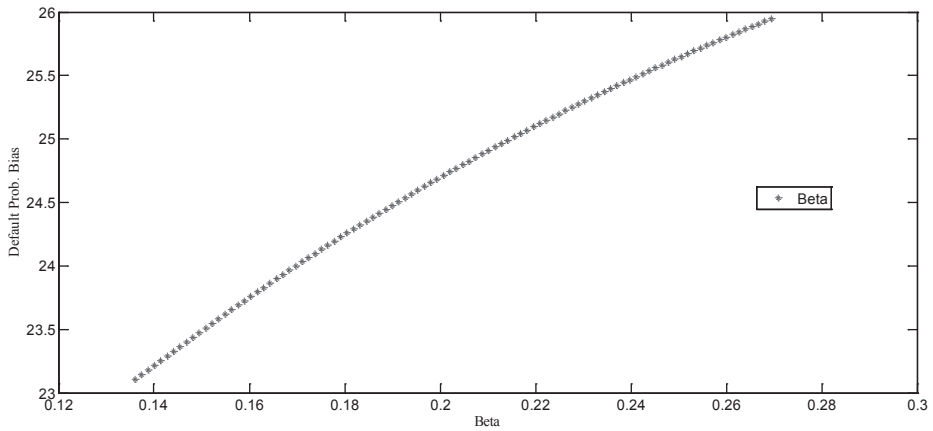


Figure 3C The Impact of Foreign Asset Weights on the Default Probability Bias

*Note that Beta stands for the weights of foreign asset.

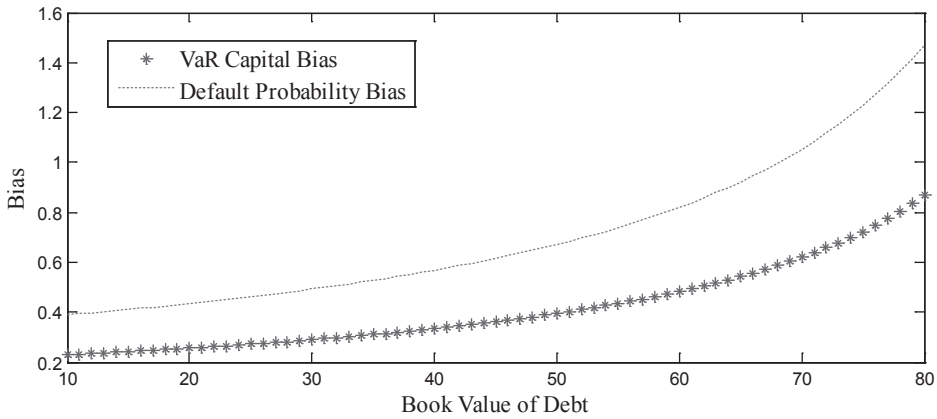


Figure 4 The Impact of the Book Value of Debt on the Bias

5. Model Evaluation

In this section, the capability of the VaR measure as related to international portfolios will be examined in terms of the VaR capital bias and default probability bias between the two periods (Period I and Period II).

5.1 Default Probability Bias Criterion

After estimation of model parameters illustrated in Table 1 for various samples, Table 2 shows the values of the relative VaR and absolute VaR measures in Period I were greater

臺灣大學學術期刊資料庫

than the values in Period II in Japan, while the values in Period I were less than in Period II in both Taiwan and Korea. In addition, the values of the relative VaR were higher than those of the absolute VaR for each country in every period.

In this section, the empirical analysis was performed to explain the default probabilities of the banks in Japan, Taiwan, and Korea and the accuracy of the model from the perspective of default probability bias. Tables 3, 4, and 5 indicate the default probability bias for Taiwan was less than 1 in Period I; whereas it was greater than 1 in Period II, as correlation coefficients, asset weights of foreign assets and volatility increased. Thus, in Taiwan, the nominal VaR default rate was downward-biased in Period II and upward-biased in Period I; equally the actual default probability was upward-biased in Period II and downward-biased in Period I. As for Japan, the default probability, based on the VaR measure, was mostly downward-biased in both Period I and II as correlation coefficients and volatility increased. In Taiwan and Japan, there existed a common situation that default probability bias was always higher in Period II than in Period I in the various scenarios. This denotes that the true default probability was greater in Period II than in Period I. Additionally, the VaR default probabilities in Taiwan and Japan were downward-based during Period II. Conversely, in Korea, the default rate bias was almost smaller than 1 during Period I and II. In general, Korea's situation was different from Taiwan and Japan.

Obviously, foreign asset proportions strongly moved the default probability bias close to 1 in Korea during both periods. Overall, there was a trend where the bias slowly moved toward 1, and it described that the nominal probability of default implied by the VaR for international portfolios came near to the actual probability of default in any case. Thus, the results show the performance of the model presented by the article under VaR capital bias criterion was accurate.

Table 2 VaR Measurement in Two Measures for Various Periods

Country	VaR(mean)		VaR(0)	
	Period I	Period II	Period I	Period II
Taiwan	1.610645	2.492830	0.986723	1.980870
Japan	2.156145	2.080330	2.087321	1.771370
Korea	1.965845	2.477730	1.342921	1.449970

臺灣大學學術期刊資料庫

Table 3 The VaR Bias for Alternative Correlation Coefficients in Various Periods

Correlation Coefficient		Taiwan				Japan				Korea			
		Period I		Period II		Period I		Period II		Period I		Period II	
		DB	VB	DB	VB	DB	VB	DB	VB	DB	VB	DB	VB
$\rho_{d,e}$	$\rho_{f,e}$	**Panel A:** $\rho_{f,e}$ change as correlation coefficient of domestic assets and foreign assets and $\rho_{d,e}$ are fixed.											
-0.5	-0.75	0.6576	1.1160	1.2041	0.4652	1.2161	1.0184	1.2978	1.0309	0.6768	0.8015	0.8052	0.8990
-0.5	-0.5	0.6762	1.1081	1.1494	0.5828	1.1148	1.0082	1.2444	1.0249	0.7776	0.9016	0.9147	0.9101
-0.5	-0.25	0.6846	1.0938	1.0588	0.6426	1.0135	1.0069	1.1091	1.0105	0.8781	0.9317	0.9201	0.9596
-0.5	0	0.6932	1.0881	1.0486	0.6820	1.0061	1.0014	1.0597	1.0086	0.9415	0.9916	0.9537	0.9956
-0.25	-0.75	0.7359	1.0999	1.2283	0.5442	1.1350	1.0259	1.4060	1.0376	0.6874	0.9016	0.8571	0.9155
-0.25	-0.5	0.7432	1.0842	1.1524	0.6202	1.2111	1.0158	1.2637	1.0275	0.7878	0.9281	0.8683	0.9341
-0.25	-0.25	0.7475	1.0771	1.1286	0.6665	1.1025	1.0845	1.1416	1.0179	0.8882	0.9412	0.9415	0.9529
-0.25	0	0.7549	1.0621	1.0477	0.6992	1.0462	1.0117	1.0381	1.0135	0.9315	0.9769	0.9893	0.9866
$\rho_{d,f}$	$\rho_{f,e}$	**Panel B:** $\rho_{f,e}$ change as correlation coefficient of domestic assets and exchange rates and $\rho_{d,f}$ are fixed.											
-0.5	-0.75	0.6779	1.1921	1.3399	0.3877	1.3328	1.0269	1.4277	1.0432	0.7483	0.9475	0.7968	0.8964
-0.5	-0.5	0.6819	1.1612	1.2912	0.5195	1.2314	1.0265	1.2837	1.0279	0.7696	0.9477	0.8372	0.9105
-0.5	-0.25	0.6831	1.1119	1.1713	0.5829	1.1301	1.0261	1.2467	1.0172	0.8608	0.9478	0.9483	0.9716
-0.5	0	0.6875	1.1098	1.0985	0.6239	1.0287	1.0258	1.0479	1.0101	0.9831	0.9480	0.9999	0.9967
-0.25	-0.75	0.6831	1.1012	1.3716	0.4594	1.3252	1.0185	1.4375	1.0154	0.7681	0.8005	0.8409	0.8970
-0.25	-0.5	0.6857	1.0914	1.2918	0.5501	1.2238	1.0184	1.2577	1.0087	0.8627	0.9006	0.9105	0.9214
-0.25	-0.25	0.6939	1.0891	1.1981	0.6018	1.1224	1.0181	1.1556	1.0034	0.9691	0.9437	0.9812	0.9533
-0.25	0	0.6942	1.0425	1.0988	0.6373	1.0211	1.0182	1.0521	1.0026	0.9812	0.9688	0.9973	0.9997
$\rho_{f,e}$	$\rho_{d,e}$	**Panel C:** $\rho_{d,e}$ change as correlation coefficient of domestic assets and exchange rates and $\rho_{f,e}$ are fixed.											
-0.5	-0.75	0.7017	1.1169	1.3385	0.4746	1.3158	1.021	1.4059	1.0392	0.7833	0.8015	0.8884	0.8333
-0.5	-0.5	0.7034	1.1089	1.2981	0.5284	1.2148	1.0182	1.2584	1.0339	0.8506	0.9015	0.8941	0.9522
-0.5	-0.25	0.7089	1.0951	1.1511	0.5658	1.1138	1.0158	1.1482	1.032	0.8626	0.9496	0.9683	0.9744
-0.5	0	0.7135	1.0895	1.0981	0.5941	1.0128	1.0136	1.0483	1.0313	0.9569	0.9797	0.9998	0.9999
-0.25	-0.75	0.7217	1.1282	1.4805	0.5580	1.3145	1.0208	1.4464	1.0316	0.811	0.8016	0.9000	0.8207
-0.25	-0.5	0.7227	1.1125	1.3802	0.5882	1.2135	1.0181	1.291	1.0304	0.8483	0.8918	0.9050	0.9223
-0.25	-0.25	0.7255	1.1094	1.2065	0.6121	1.1125	1.0158	1.2416	1.0289	0.8882	0.9498	0.9510	0.9747
-0.25	0	0.7335	1.0914	1.1008	0.6316	1.0115	1.0136	1.094	1.0278	0.9839	0.9799	0.9845	0.9802

*Note that DB and VB denote the default probability bias, and the VaR capital bias, respectively.

臺灣大學管理學術期刊資料庫

Table 4 The VaR Bias for Alternative Weights in Various Periods

Weight		Taiwan				Japan				Korea			
		Period I		Period II		Period I		Period II		Period I		Period II	
γ_t	β_t	DB	VB	DB	VB	DB	VB	DB	VB	DB	VB	DB	VB
1	0	0.5596	1.0847	1.8035	0.7269	1.8794	1.0587	1.9104	1.1536	0.6945	0.9224	0.8220	0.9815
0.9	0.1	0.7298	1.0709	1.7377	0.8680	1.8284	1.0696	1.8391	1.1498	0.6519	0.9608	0.8965	0.9902
0.8	0.2	0.7719	1.0517	1.6056	0.9210	1.6992	1.0599	1.7462	1.1023	0.6890	0.9779	0.9309	0.9925
0.7	0.3	0.7958	1.0466	1.5316	0.9247	1.5998	1.0404	1.6468	1.0906	0.7319	0.9851	0.9335	0.9939
0.6	0.4	0.8835	1.0349	1.4769	0.9321	1.4977	1.0378	1.5316	1.0878	0.7619	0.9890	0.9497	0.9941
0.5	0.5	0.8032	1.0307	1.3960	0.9449	1.3665	1.0347	1.3805	1.0758	0.7821	0.9915	0.9576	0.9953
0.4	0.6	0.7139	1.0131	1.3271	0.9587	1.2484	1.0325	1.2707	1.0615	0.7961	0.9932	0.9820	0.9967
0.3	0.7	0.7113	1.0112	1.2277	0.9662	1.2094	1.0308	1.2069	1.0594	0.9060	0.9944	0.9887	0.9971
0.2	0.8	0.6520	1.0088	1.1178	0.9788	1.0568	1.0295	1.0706	1.0426	0.9133	0.9954	0.9899	0.9975
0.1	0.9	0.6405	1.0078	1.0349	0.9878	1.0446	1.0285	1.0324	1.0349	0.9687	0.9961	0.9911	0.9988
0	1	0.5467	1.0072	1.0168	0.9967	1.0023	1.0110	1.0133	1.0265	0.9928	0.9967	0.9985	1.0003

*Note that DB and VB denote the default probability bias and the VaR capital bias, respectively.

臺灣大學學術期刊資料庫

Table 5 The VaR Bias for Alternative Volatilities in Various Samples

Volatility (%)	Taiwan				Japan				Korea			
	Period I		Period II		Period I		Period II		Period I		Period II	
	DB	VB	DB	VB	DB	VB	DB	VB	DB	VB	DB	VB
$\sigma_{d,t}$	**Panel A:** Changes in volatility to domestic asset returns											
20	0.8671	0.8682	1.0177	1.1949	0.5230	1.8697	1.0058	1.8344	1.0258	0.6646	0.9174	0.7786
25	0.8791	0.8802	1.0151	1.1406	0.6331	1.6436	1.0053	1.7176	1.0221	0.7496	0.9282	0.8606
30	0.8862	0.8873	1.0134	1.1046	0.7361	1.4997	1.0051	1.6056	1.0197	0.8397	0.9388	0.9488
35	0.8902	0.8913	1.0123	1.0787	0.8321	1.4304	1.0039	1.5961	1.0180	0.9327	0.9492	0.9551
40	0.8921	0.8932	1.0114	1.0588	0.8599	1.1486	1.0033	1.2880	1.0168	0.9915	0.9595	0.9992
$\sigma_{f,t}$	**Panel B:** Changes in volatility to foreign asset returns											
20	0.8824	1.0394	1.0398	0.6514	1.7569	1.0424	1.8724	1.0802	0.7905	0.9359	0.8119	0.9559
25	0.8976	1.0329	1.0247	0.7916	1.6542	1.0364	1.7553	1.0432	0.8996	0.9431	0.9029	0.9624
30	0.9051	1.0286	1.0146	0.9271	1.5514	1.0321	1.7079	1.0192	0.9123	0.9602	0.9368	0.9798
35	0.9431	1.0256	1.0073	0.9572	1.2485	1.0283	1.4602	1.0125	0.9603	0.9615	0.9860	0.9826
40	0.9813	1.0233	1.0018	0.9872	1.0455	1.0255	1.1124	1.0082	0.9798	0.9715	0.9913	0.9984
$\sigma_{e,t}$	**Panel C:** Changes in volatility to exchange rates											
20	0.8980	1.0476	1.1040	0.6294	1.7423	1.0379	1.9761	1.0615	0.7968	0.9248	0.8808	0.9521
25	0.9118	1.0308	1.0956	0.7675	1.5378	1.0346	1.7927	1.0349	0.8518	0.9433	0.9179	0.9602
30	0.9596	1.0264	1.0272	0.9294	1.3332	1.0307	1.4346	1.0161	0.9019	0.9449	0.9581	0.9646
35	0.9867	1.0233	1.0770	0.9655	1.1285	1.0277	1.2008	1.0123	0.9349	0.9605	0.9846	0.9745
40	0.9929	1.0209	1.0483	0.9765	1.0236	1.0253	1.0666	1.0017	0.9567	0.9722	1.0033	0.9949

*Note that DB and VB denote the default probability bias and the VaR capital bias, respectively.

臺灣大學管理學術期刊審查中

5.2 VaR Capital Bias Criterion

The VaR model's performance can be examined using the VaR capital bias criterion. Conditional on the levels of correlation coefficients and the weights of foreign assets as shown in Tables 3 and 4, the VaR capital bias in Japan declined and was greater than 1 in both Period I and II. However, the bias generally increased and was less than 1 in Korea during the two periods. In Taiwan, the bias was negatively related to the correlation coefficients and its magnitude was over unity in Period I; the relationship was positive and its magnitude was under unity in Period II. In Taiwan, the VaR capital allocation exceeded the actual capital in Period I, and was under the actual capital in Period II. On the other hand, there was a negative relationship between the weight of foreign assets and Taiwan's bias in Period I, the bias was greater than 1. However, there was a positive relationship between the weights of foreign assets and the bias for Taiwan in Period II, the bias was less than 1.

As for the relation between volatility and the VaR capital bias, Table 5 shows that it was a negative relationship and the magnitude of the bias was larger than 1 in Japan during the two periods. For Korea, the relationship was positive and the VaR capital allocation was smaller than the actual capital in both periods. As for Taiwan, the bias decreased as volatilities of foreign assets and exchange rate increased in Period I, and its value was over 1. However, the bias rose and almost dropped below 1 as the volatilities of foreign assets and exchange rate increased in Period II.

Throughout Tables 3, 4, and 5, there are two interesting phenomena. First, banks in Japan should increase actual risk capital, since measurements greater than 1 meaning the actual capital was below the VaR capital, while Korean banks keep sufficient capital on hand since a magnitude lower than 1 denotes the actual capital exceeded the VaR capital. In other words, all of Japan's banks should hold more risk capital than Korea's banks in order to prevent bankruptcy. As for Taiwan, banks generally need to increase their risk capital in normal situations, whereas they reduced risk capital to prevent a default probability during the subprime mortgage crisis. Second, despite the default probability bias and the VaR capital bias criterions, the risk capital estimated by the VaR model presented in this article generally trends to the actual capital, which implies the accurate performance of this model based on the VaR bias criterion.

6. Optimal Asset Allocation for International Portfolios

As shown in Figure 1C, the relationship between the proportion of foreign assets and

臺灣大學學術期刊資料庫

VaRs is a parabola. At the beginning, when the proportion of foreign assets increased, the maximum losses in the international portfolio decreased. But, the maximum possible losses to the international portfolio increased as the proportion of foreign assets increased. What is the optimal proportion of international holdings? This section will minimize the VaRs in order to obtain the optimal foreign and domestic assets' weights.

Therefore, the problem is to minimize the objective function of the relative VaRs shown in equation (6), subject to the sum of the weights of foreign assets and domestic assets being equal to 1, implying investors could long or short international portfolios. That is also described as follows:

$$\underset{\gamma_t,\ \beta_{i,t},\ i\in\{1,\ 2,\ 3,\ldots,n\}}{Min} \quad VaR(mean)$$

$$s.t. \qquad \gamma_t + \sum_{i=1}^{n} \beta_{i,t} = 1$$

Then, the Lagrange method is used to solve the optimal weights of foreign assets and domestic assets. The Lagrange function is described in equation (16).

$$L = VaR(mean) + \lambda(1 - \gamma_t - \sum_{i=1}^{n} \beta_{i,t}) \tag{16}$$

Then, the first order differentials on the weights of domestic assets and foreign assets, the Lagrange multiplier, respectively, were shown as below:

$$\frac{\partial L}{\partial \gamma_t} = V_t\left[\left(\mu_{d,t} - \frac{1}{2}\frac{\partial \sigma_t^2}{\partial \gamma_t}\right)(T-t) + Z_\alpha \frac{\partial \sigma_t}{\partial \gamma_t}\sqrt{T-t}\right] e^{\left[(\gamma_t\mu_{d,t} + \sum_{i=1}^{n}\beta_{i,t}(\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t}) - \frac{1}{2}\sigma_t^2)(T-t) + Z_\alpha\sigma_t\sqrt{T-t}\right]}$$

$$- V_t\mu_{d,t}e^{\left[(\gamma_t\mu_{d,t} + \sum_{i=1}^{n}\beta_{i,t}(\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t}))(T-t)\right]} = 0 \tag{17}$$

$$\frac{\partial L}{\partial \beta_{i,t}} = V_t\left[(\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t})(T-t) - \frac{1}{2}\frac{\partial \sigma_t^2}{\partial \beta_{i,t}}(T-t) + Z_\alpha \frac{\partial \sigma_t}{\partial \beta_{i,t}}\sqrt{T-t}\right]$$

$$\times e^{\left[(\gamma_t\mu_{d,t} + \sum_{i=1}^{n}\beta_{i,t}(\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t}) - \frac{1}{2}\sigma_t^2)(T-t) + Z_\alpha\sigma_t\sqrt{T-t}\right]} - V_t(\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t})(T-t)$$

$$\times e^{\left[(\gamma_t\mu_{d,t} + \sum_{i=1}^{n}\beta_{i,t}(\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t}))(T-t)\right]} = 0 \tag{18}$$

$$\frac{\partial L}{\partial \lambda} = 1 - \gamma_t - \sum_{i=1}^{n} \beta_{i,t} = 0 \tag{19}$$

From equations (17), (18), and (19), the equation (20) is obtained.

臺灣大學學術期刊資料庫

$$\mu_{d,t}\left[(\mu_{e_i,t}+\mu_{f_i,t}+\rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t})\sqrt{T-t}-F_t+Z_\alpha C_t^{-\frac{1}{2}}F_t\right]$$
$$=(\mu_{e_i,t}+\mu_{f_i,t}+\rho_{f_i,e_i}\sigma_{f_i,t}\sigma_{e_i,t})\left[(\mu_{d,t}-G_t)\sqrt{T-t}+Z_\alpha C_t^{-\frac{1}{2}}G_t\right], \quad (20)$$

with

$$C_t=\gamma_t^2\sigma_{d,t}^2+\left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\right)^2+\left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}\right)^2+2\rho_{d,e_i}\gamma_t\sigma_{d,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}+2\rho_{d,f_i}\gamma_t\sigma_{d,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}+2\rho_{f_i,e_i}\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t},$$

for $i\in\{1,2,3,....,n\}$;

$$F_t=\sigma_{e_i,t}\left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\right)+\sigma_{f_i,t}\left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}\right)+\rho_{d,e_i}\gamma_t\sigma_{d,t}\sigma_{e_i,t}$$
$$+\rho_{d,f_i}\gamma_t\sigma_{d,t}\sigma_{f_i,t}+\rho_{f_i,e_i}\left[\sigma_{f_i,t}\left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\right)+\sigma_{e_i,t}\left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}\right)\right],$$

for $i\in\{1,2,3,....,n\}$;

$$G_t=\gamma_t\sigma_{d,t}^2+\rho_{d,e_i}\sigma_{d,t}\left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i,t}\right)+\rho_{d,f_i}\sigma_{d,t}\left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i,t}\right), \text{ for } i\in\{1,2,3,....,n\}.$$

For simplicity, one can use the Matlab program to solve for the optimal weights of foreign assets, $\beta_{i,t}^*$, and domestic assets, γ_t^* for $i\in\{1,2,3,....,n\}$ in terms of equation (20). $\beta_{i,t}^*$ and γ_t^* are functions of model parameters $\sigma_{d,t}$, $\sigma_{f_i,t}$, $\sigma_{e_i,t}$, ρ_{d,f_i}, ρ_{d,e_i}, ρ_{f_i,e_i}.

Based on the estimated results of Tables (1) and (2), the optimal foreign weights in Japan, Taiwan and Korea were 0.06178, 0.52164, and 0.31351, respectively, during Period I, and 0.04178 in Japan, 0.46153 in Taiwan and 0.26156 in Korea during Period II. Table 1 discloses that the actual weights of foreign assets were 0.0025116, 0.2802054, and 0.0579012 in Japan, Taiwan, and Korea, respectively, during Period I; and, 0.001722, 0.3449335, 0.1084569 in Japan, Taiwan, and Korea, respectively, during Period II. Therefore, bank managers should increase the actual proportion of foreign assets during a normal economy or the subprime crisis period.

7. Conclusion

In reality, investors generally buy not only domestic assets but also foreign-currency assets in a highly integrated global financial market, especially banks. However, previous research only focused on either domestic assets or foreign-currency assets, but not both in

臺灣大學學術期刊資料庫

the same context. This paper provides a VaR model for the management of international portfolios including domestic assets and foreign assets. The model can be regarded as an incorporation of Kupiec's (1999) model and Chen and Liao (2009) model.

Kupiec (1999) showed that VaR capital estimates are biased owing to neglect of the interest that must be paid on debt. As to international portfolios, the magnitude of the bias in capital or default rates mainly depends on correlation coefficients, foreign-currency asset weights, and asset volatility. Furthermore, there is an interesting phenomenon where the foreign asset weights enable VaR capital bias to grow to unity, implying a firm's VaR capital allocation trends towards its actual capital. In other words, when the magnitude of foreign assets is larger, the model is more accurate. Alternatively, the empirical results show that Japanese banks should keep more sufficient capital than Korean banks. As for Taiwan, domestic banks generally need to increase risk capital in normal situations, whereas they should decrease their risk capital to prevent a default probability during the subprime mortgage crisis.

In addition, the VaRs were minimized to obtain the optimal foreign and domestic weights. The empirical evidence shows that the actual proportions of foreign-currency assets were less than the optimal weights, representing that bank managers should increase the actual proportions of foreign-currency assets during a normal economy or a subprime crisis period.

As the economy becomes more globalized, this contribution will give some instructive suggestions to financial corporations and agents.

臺灣大學學術期刊資料庫

References

Barone-Adesi, G., and Giannopoulos, K. 2001. Non-parametric VaR techniques: Myths and realities. ***Economic Notes***, 30 (2): 167-181.

Barone-Adesi, G., Giannopoulos, K., and Vosper, L. 1999. VaR without correlations for nonlinear portfolios. ***Journal of Futures Markets***, 19 (5): 583-602.

Black, F., and Scholes, M. 1973. The pricing of options and corporate liabilities. ***Journal of Political Economy***, 81 (3): 637-654.

Brooks, C., and Persand, G. 2002. Model choice and value-at-risk performance. ***Financial Analysts Journal***, 58 (5): 87-97.

_____. 2003. The effect of asymmetries on stock index return value at risk estimates. ***Journal of Risk Finance***, 4 (2): 29-42.

Chen, F. Y., and Liao, S. L. 2009. Modelling VaR for foreign-asset portfolios in continuous time. ***Economic Modelling***, 26 (1): 234-240.

Duffie, D., and Pan, J. 1997. An overview of value at risk. ***Journal of Derivatives***, 4 (3): 7-49.

Hendrick, D. 1996. Evaluation of value at risk models using historical data. ***Federal Reserve Bank of New York Economic Policy Review***, 2 (1): 39-70.

Huang, Y. C., and Lin, B. J. 2004. Value-at-risk analysis for Taiwan stock index futures: Fat tails and conditional asymmetries in return innovations. ***Review of Quantitative Finance and Accounting***, 22 (2): 79-95.

Hull, J., and White, A. 1998a. Incorporating volatility updating into the historical simulation method for VaR. ***Journal of Risk***, 1 (1): 5-19.

_____. 1998b. Value at risk when daily changes in market variables are not normally distributed. ***Journal of Derivatives***, 5 (3): 9-19.

Jamshidian, F., and Zhu, Y. 1997. Scenario simulation model: Theory and methodology. ***Finance and Stochastics***, 1 (1): 43-67.

Jorion, P. 1996. Measuring the risk in value at risk. ***Financial Analysts Journal***, 52 (6): 47-56.

_____. 2007. ***Value at Risk: The New Benchmark for Managing Financial Risk***. New York, NY: McGraw-Hill.

Kupiec, P. 1995. Techniques for verifying the accuracy of risk measurement models. ***Journal of Derivatives***, 3 (2): 73-84.

_____. 1999. Risk capital and VaR. ***Journal of Derivatives***, 7 (2): 41-52.

臺灣大學學術期刊資料庫

Linsmeier, T. J., and Pearson, N. D. 2000. Value at risk. ***Financial Analysts Journal***, 56 (2): 47-67.

Longerstaey, J., and Zangari, P. 1996. ***Riskmetrics- Technical Document (4th ed.)***. New York, NY: Morgan Guaranty Trust Co.

Merton, R. 1974. On the pricing of corporate debt: The risk structure of interest rates. ***Journal of Finance***, 29 (2): 449-470.

Simons, K. 1997. Model error. ***New England Economic Review***, 4: 17-28.

臺灣大學學術期刊資料庫

Appendix A

Owing to $V_t = A_{d,t} + \sum_{i=1}^{n} e_{i,t} A_{f_i,t}$, using Ito's lemma, one can obtain

$$\frac{dV_t}{V_t} = \gamma_t \frac{dA_{d,t}}{A_{d,t}} + \sum_{i=1}^{n} \beta_{i,t} \left[\frac{de_{i,t}}{e_{i,t}} + \frac{dA_{f_i,t}}{A_{f_i,t}} + \frac{de_{i,t}}{e_{i,t}} \frac{dA_{f_i,t}}{A_{f_i,t}} \right], \tag{A.1}$$

with $\gamma_t = \frac{A_{d,t}}{V_t}$, and $\beta_{i,t} = \frac{e_{i,t} A_{f_i,t}}{V_t}$. Substituting equations (1), (2), and (3) into (A.1), the consequences are

$$\frac{dV_t}{V_t} = \left[\gamma_t \mu_{d,t} + \sum_{i=1}^{n} \beta_{i,t} (\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i,e_i} \sigma_{e_i,t} \sigma_{f_i,t}) \right] dt + \gamma_t \sigma_{d,t} dW_t + \left[\sum_{i=1}^{n} \beta_{i,t} \sigma_{e_i,t} \right] dQ_t + \left[\sum_{i=1}^{n} \beta_{i,t} \sigma_{f_i,t} \right] dP_t.$$

Appendix B

The solution of relative VaR for international portfolios is proved by the definition of VaR, given a confidence level of α, expressed as

$$P_r(V_T \le Va) = \alpha. \tag{B.1}$$

Based on the relative VaR being denoted by $VaR(mean) \equiv E(V_T) - Va$, equation (B.1) is transformed into

$$P_r(V_T - E[V_T] \le -VaR(mean)) = \alpha. \tag{B.2}$$

By the application of Proposition 1 to equation (B.2), we obtained

$$Z_\alpha = \frac{\ln[-VaR(mean) + E(V_T)] - (\mu_t - \frac{1}{2}\sigma_t^2)(T-t) - \ln(V_t)}{\sigma_t \sqrt{T-t}}, \tag{B.3}$$

in which $\mu_t = \gamma_t \mu_{d,t} + \sum_{i=1}^{n} \beta_{i,t} (\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i,e_i} \sigma_{e_i,t} \sigma_{f_i,t})$, and

$$\sigma_t = \sqrt{\gamma_t^2 \sigma_{d,t}^2 + \left[\sum_{i=1}^{n} \beta_{i,t} \sigma_{e_i,t} \right]^2 + \left[\sum_{i=1}^{n} \beta_{i,t} \sigma_{f_i,t} \right]^2 + 2\rho_{d,e_i} \gamma_t \sigma_{d,t} \sum_{i=1}^{n} \beta_{i,t} \sigma_{e_i,t} + 2\rho_{d,f_i} \gamma_t \sigma_{d,t} \sum_{i=1}^{n} \beta_{i,t} \sigma_{f_i,t} + 2\rho_{f_i,e_i} \sum_{i=1}^{n} \beta_{i,t} \sigma_{e_i,t} \sum_{i=1}^{n} \beta_{i,t} \sigma_{f_i,t}}.$$

From equation (B.3), the result is

$$VaR(mean) = \left| Min \left\{ V_t \left\{ \begin{array}{l} \exp\left[(\gamma_t \mu_{d,t} + \sum_{i=1}^{n} \beta_{i,t} (\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i,e_i} \sigma_{f_i,t} \sigma_{e_i,t}) - \frac{1}{2}\sigma_t^2)(T-t) + Z_\alpha \sigma_t \sqrt{T-t} \right] \\ -\exp\left[(\gamma_t \mu_{d,t} + \sum_{i=1}^{n} \beta_{i,t} (\mu_{e_i,t} + \mu_{f_i,t} + \rho_{f_i,e_i} \sigma_{f_i,t} \sigma_{e_i,t})(T-t) \right] \end{array} \right\}, 0 \right\} \right|.$$

臺灣大學學術期刊資料庫

The absolute VaR formula can be obtained by

$$VaR(0) = \left| Min\left\{ V_t\left\{ \exp\left[(\gamma_t \mu_{d,t} + \sum_{i=1}^{n} \beta_{i,t}(\mu_{e_i,t} + \mu_{f_j,t} + \rho_{f_j,e_j}\sigma_{f_j,t}\sigma_{e_j,t}) - \frac{1}{2}\sigma_t^2)(T-t) + Z_\alpha \sigma_t \sqrt{T-t} \right] - 1 \right\}, 0 \right\} \right|.$$

臺灣大學學術期刊資料庫

作者簡介

* 陳芬英

國立政治大學金融博士，同時為國立政治大學風險管理與保險學系博士候選人，目前任教於世新大學財務金融學系。主要研究領域為財務工程學、利率模型和固定收益證券分析。學術論文刊載於 Economic Modelling、Journal of Risk Finance、Applied Mathematics and Computation、International Journal of Innovative Computing、Information and Control、Computers and Mathematics with Applications、The Journal of Risk Model Validation、《中山管理評論》、《管理學報》、《亞太經濟管理評論》、《台灣期貨與衍生性商品學刊》等期刊。

The author thanks the editor and referees for their valuable comments and suggestions. The author also thanks the financial support from Social Science Research Center of Taiwan.
科技部計畫編碼：NSC96-2416-H-128-008
* E-mail: fyichen@cc.shu.edu.tw

臺灣大學學術期刊資料庫