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Currency Substitution, Trade Elasticities, Real Exchange Rate Dynamics, and Monetary Independence

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This paper constructs a three-currency generalized macroeconomic model to analyze the effects of domestic monetary expansion on the macroeconomic economy in a small open system. This dynamic general equilibrium model shows that whether these foreign currencies are complementary or substitutable with the domestic currency plays an important role in assessing monetary independence and real exchange rate dynamics. The magnitude of trade elasticities also crucially affects the dynamic consequences of exchange rates under currency substitution. When the modified Marshall-Lerner condition holds, the exchange rate adjustment exhibits a unique saddle path. However, if the modified Marshall-Lerner condition does not hold, the exchange rate adjustment may be either unstable or indeterminate.

Keywords: currency substitution, trade elasticity, exchange rate dynamics, monetary independence, (in) determinacy
JEL classification: E5, F3, F31, F41

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1 Introduction

On 30 November 2015, the Executive Board of the International Monetary Fund (IMF) completed a regular five-yearly review of the basket of currencies that comprise the Special Drawing Right (SDR), marking a milestone in China's push to internationalise its currency. A key outcome of the Board's review was that the Chinese renminbi (RMB) would be effectively included in the SDR basket from 1 October 2016 along with the U.S. dollar (USD), the euro, the Japanese yen, and the British pound.

IMF Press Release No. 15/540 (available on the IMF website) states, 'Authorities of all currencies represented in the SDR basket, which now includes the Chinese authorities, are expected to maintain a policy framework that facilitates operations for the IMF, its membership and other SDR users in their currencies'. Although any direct impact may not be immediately apparent, being the first developing country to join the SDR-basket group will spur the currency towards further reform to more closely integrate with the world economy. In particular, admission to the SDR currency club symbolises that the IMF is lending its imprimatur to the RMB as a reserve currency. As a result, it will encourage more central banks to hold RMB as part of their foreign exchange reserves as well as build confidence in people's demand for RMB and yuan-denominated assets. To be specific, Wu (2015) quoted Huang Yiping, Peking University economist and central bank monetary policy adviser: 'The [reform] direction is very clear, [that is, towards] a more flexible yuan and market-driven interest rates, further opening up of the capital account and greater global use of the yuan.' This point of view is consistent with Frankel (2012), who said, 'To attain international currency status, capital and money markets in the home country must be not only open and free of controls, but also deep and well developed.'

The well-known Mundellian trilemma (also known as the impossible trinity; Mundell (1963)) states that simultaneously having the following is impossible: a stable exchange rate, capital mobility, and an independent monetary policy. Therefore, if Chinese financial reform is leaning towards loosening capital control and making the RMB more flexible, the Chinese government can retain monetary independence (Tyers, Bain, and Bu, 2008). However, if the People's Bank of China (PBOC) stabilises the RMB through foreign exchange rate intervention, the Chinese authority is likely to lose its ability to control capital.

By contrast, currency substitution might occur in Taiwan's portfolio al-

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location for the following reasons: First, China and Taiwan share a similar cultural background and language. Second, Taiwanese investors are deeply involved in supply chain management in the Great China area. Third, the Central Bank of the Republic of China (Taiwan, CBC) announced on 6 February 2013 that it would permit local banks' RMB business in Taiwan. Last, the increased flexibility of the RMB changes expectations regarding future exchange rates, which in turn affects individuals' portfolio choices in their holding of monies of different denominations. According to CBC press release No. 101 (June 18, 2024) on a monthly overview of RMB business conducted by banks, there were 65 authorized domestic banking units and 56 offshore banking units conducting RMB business. The amount of RMB settlements handled by the Taipei branch of the Bank of China in May 2024 was RMB 478 billion. In comparison, the amount of RMB settlements handled by the Taipei branch of the Bank of China in November 2015 was RMB 319 billion.[1]

An increase in the importance of the RMB in Taiwan's portfolio along with the expectation of increased RMB volatility and loosened capital control in China complicate the physical environment faced by the small open economy, Taiwan. Specifically, whether the RMB and the USD exhibit complementarity or substitutability plays a critical role in the effectiveness and independence of the CBC's monetary policy. Therefore, the relationship among currency substitution, independence of monetary policy, and real exchange rate dynamics is a central issue worth studying. This paper thus constructs a three-currency macroeconomic model to analyze the effects of domestic monetary policy on the macroeconomic economy in a small open system. In addition, how the results depend crucially on the relationship between gross substitutability and gross complementarity among the three currencies are carefully explored.

The rest of the paper proceeds as follows. Section 2 reviews related literature on currency substitution and monetary independence. Section 3 describes the construction of a three-currency dynamic general equilibrium model. Section 4 presents an analysis of long-run effects on the degree of monetary independence. Section 5 examines the behaviour of real exchange rates through the dynamic system. Section 6 concludes.

[1] In order to understand the importance of RMB in Taiwan's portfolio, a time-series data on the ratio of RMB deposits to total foreign exchange deposits is presented in Appendix 1.

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2 Literature review

Previous studies on currency substitution and exchange rate dynamics issues (e.g., Calvo and Rodriguez (1977); Niehans (1977); Liviatan (1981); Chen and Tsaur (1983); Park (1987); Chen, Tsaur, and Liu (1989); Engel (1989); Chan (2002); Airaudo (2014) and Kumamoto and Kumamoto (2014))[2] almost without exception imply that including holdings of two monies (the domestic currency and the foreign currency) in a portfolio is desirable.[3] The essential notion behind all currency-substitution models is that individuals' demand for money should consist of their demand for not only the domestic currency but also a group of foreign currencies. As a result, individuals' demand for the domestic currency may or may not be a well-defined and stable function (Krueger, 1983, p.91). Intuitively, the wide variability of exchange rates would be reasonably explained with a model of diversified foreign-currency holdings.

The specification of a money demand function is crucial. For example, both Calvo and Rodriguez (1977) and Liviatan (1981) apply a similar two-sector, small open economy framework to study the effects of currency substitution on real exchange rate dynamics. However, Calvo and Rodriguez (1977) predict that an increase in the rate of money growth leads to a real depreciation, whereas Liviatan (1981) expects the opposite. The key difference is the following: Calvo and Rodriguez (1977) assume that there is a 'competitive' relationship between home and foreign currencies, whereas Liviatan (1981) assumes that domestic and foreign currencies are held similarly to 'cooperant' factors in the production function of liquidity services under the long-run perfect-foresight model. Calvo (1985) and Engel (1989) synthesise the differing results between Calvo and Rodriguez (1977) and Liviatan (1981). Végh (2013) concludes that whether an increase in the rate of monetary expansion induces a real appreciation or depreciation depends on two factors. One is the relative size of the elasticity of substitution between the domestic and the foreign currencies, and the other is the elasticity of substitution between consumption and liquidity services. These studies

[2]Another main question explored in the currency substitution literature is how currency substitution, through its impact on the demand for domestic and/or foreign currencies, may affect the determination of seigniorage and/or inflation tax (e.g., Végh (1989); Özbilgin (2012)).

[3]Rey (2001), which develops a three-country and three-currency model of the world economy, is one of few notable exceptions.

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clearly indicate that the degree of substitutability and/or complementarity between currencies plays a considerable role in real exchange rate fluctuations.

Despite the apparent connexion, the role of international interdependence is surprisingly not generally recognised in the currency substitution literature. For example, Niehans (1977) and Park (1987) construct economies that are immune from external shocks by assuming a fixed foreign price for traded goods or imports. By contrast, the impact of inflation abroad is fully accounted for by Chen and Tsaur (1983) and Chen, Tsaur, and Liu (1989). Both Chen and Tsaur (1983) and Chen, Tsaur, and Liu (1989) conclude that the terms of trade are affected by a change in the foreign rate of inflation and that money exhibits non-neutrality in the long run. Moreover, through real exchange rate dynamics, Chen, Tsaur, and Liu (1989) demonstrates that adjustment paths rely on trade elasticity, substitution elasticity, and cost elasticity of currency diversification.

In addition to the influences of currency substitution on exchange rate fluctuations, the role of currency substitution in the impact of monetary policy is widely investigated. For instance, Felices and Tuesta (2013) and Airaudo (2014) demonstrate a negative relationship between the degree of currency substitution and the effectiveness of monetary policy. By contrast, Kumamoto and Kumamoto (2014) build a dynamic stochastic general equilibrium model with an incomplete financial market and determine that the degree of currency substitution does not have a severe impact on the effects of domestic monetary policy shock. Nevertheless, Kumamoto and Kumamoto (2014) report that a higher degree of currency substitution is associated with a larger influence of foreign monetary policy shock on the domestic economy. Recently, Chen (2023) shows that diversified currency holdings operate as an automatic stabilizer that mitigates belief-driven cyclical fluctuations in Farmer (1997) indeterminate monetary economy. This contradicts the existing argument that currency substitution is detrimental for domestic aggregate stability. Although the channel through which currency substitution affects monetary policy is examined in the aforementioned studies, these results are based on a two-country, two-currency, small open economy model. Consequently, these models cannot be used to explore how the degree of currency substitution may influence the monetary policy of a small open economy that has two large trading partners. In particular, we consider the impact of substitutability and/or complementarity among currencies (the domestic currency and two foreign currencies) on the

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effectiveness and independence of the home country's monetary policy.

Our paper is most closely related to Tsaur and Tang (2016), who construct a dynamic general equilibrium model that features three currencies to explore the impact of currency substitution and exchange rate 'renminbisation' on exchange rate volatility and currency diversification. Specifically, Tsaur and Tang (2016) investigate how an expansionary monetary policy by the PBOC (the foreign monetary policy) would eventually lead to smaller RMB (the foreign currency) holdings for domestic residents. However, whether China's expansionary monetary policy increases or decreases Taiwan residents' New Taiwan dollar (NTD) and USD holdings depends on the degree of substitution between currencies. For example, if both the RMB and the NTD, as well as the RMB and the USD, exhibit complementarity, then the Chinese expansionary monetary policy would lead to smaller NTD and smaller USD holdings. Alongside currency holdings, Tsaur and Tang (2016) examine the influence of U.S. and Chinese monetary policies on the long-run real exchange rate in Taiwan. The findings are consistent with Chen and Tsaur (1983) and Chen, Tsaur, and Liu (1989), who conclude that under currency substitution, flexible exchange rates may no longer provide a cushion against external shocks.

Although Tsaur and Tang (2016) conduct comparative static analysis of steady states and investigated the behaviour of relevant macroeconomic variables, three important aspects distinguish Tsaur and Tang (2016) from this paper: First, in contrast to Tsaur and Tang (2016) who discuss the impact of foreign (the PBOC's) monetary policy on home (Taiwan) currency diversification, this paper investigates the effectiveness and independence of *domestic* (the CBC's) monetary policy. An increase in the importance of the RMB in Taiwan's portfolio along with the expectation of increased RMB volatility and loosened capital control in China complicates the physical environment faced by Taiwan. Specifically, whether the RMB and the USD exhibit complementarity or substitutability determines the effectiveness and independence of the CBC's monetary policy. Second, in contrast with Tsaur and Tang (2016), this paper emphasizes that monetary disturbances originate inside. After analyzzing the effects of domestic monetary expansion on the terms of trade in a small open economy, an interesting result that money is not neutral even in the long run is obtained. Third, Tsaur and Tang (2016) perform real exchange rate dynamic analysis in response to *foreign* (the PBOC's) monetary policy. By contrast, the dynamic adjustment path of relevant macroeconomic variables, such as real exchange rates, and

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domestic (the CBC's) monetary policy are investigated in this study.

3 The model

Suppose that there are two large foreign countries and a small home country in the world. The small home country is a price-taker in the world markets for its imports (i.e., Commodity 2) but is sufficiently substantial to affect world prices for its exportables (i.e., Commodity 1). Let the two foreign countries be known as Country 1 and Country 2. Suppose that Country 1 shares a similar cultural background and consumption habits with the home country. Therefore, we assume that Country 1 and the home country trade only in Commodity 1, and that purchasing power parity holds between the two countries. However, citizens of the home country also consume Commodity 2, which is completely imported from Country 2. To visualise the conceptual setting, consider China and the United States as examples of Country 1 and Country 2, respectively. Taiwan and Korea are examples of the small home country in our generalized macroeconomic model.

We use the following notation to facilitate our presentation: Let P be the domestic price of Commodity 1, P_i^*, $i = 1, 2$ be the price of Commodity i in terms of Country $i's$ currency, and E_i, $i = 1, 2$ be the domestic price of Country $i's$ currency. Because purchasing power parity holds between the home country and Country 1, $P = E_1 P_1^*$. Let $Q \equiv E_2 P_2^*/P$ be the real exchange rate in terms of the relative prices of imports with respect to exportables. To capture the concept of currency diversification, we assume that citizens in the home country hold domestic and foreign currencies at the same time. However, citizens of foreign countries do not hold the small home country's currency. Let H be the nominal domestic balance and F_i, $i = 1, 2$ be the nominal holdings of Country $i's$ currency by citizens in the home country. It follows that the real domestic balance M is defined as H/P and the real balance of Country $i's$ currency is defined as $M_i \equiv E_i F_i/P$, $i = 1, 2$.

A representative citizen in the home country maximises the following utility function

$$\max \int_0^\infty \left[C_1 + \frac{C_2^{1-a}}{1-a} + \frac{L^{1-k}}{1-k} \right] e^{-\rho t} dt$$

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subject to the budget constraint

$$\dot{M} + \dot{M}_1 + \dot{M}_2 = S - \pi M - \pi_1 M_1 - \left(\pi_2 - \hat{Q}\right) M_2 + (Y - C_1 - QC_2), \tag{1}$$

where $a, k \in (0, 1)$, $\rho > 0$, $C_1(C_2)$ is the domestic consumption of Commodity 1 (Commodity 2), and L is the currency mix function. The currency mix function, L, is specified with the three imperfectly substitutive currencies in the formulation of the constant elasticity of substitution function

$$L = M_1^{\alpha_1}\left[\beta M_2^{\frac{\sigma-1}{\sigma}} + (1-\beta)M^{\frac{\sigma-1}{\sigma}}\right]^{\frac{(1-\alpha_1)\sigma}{\sigma-1}}, \quad \alpha_1, \beta \in (0, 1),\ \sigma \in (0, \infty). \tag{2}$$

The currency mix function (2) illustrates that the partial elasticity of substitution between Country 1's currency and the domestic currency (Country 2's currency) is equal to one, and the partial elasticity of substitution between the domestic currency and Country 2's currency is σ.

Let S be the lump-sum transfer to the citizen and Y be the level of exportables produced domestically at full employment. Let the time derivative of a variable x be denoted by $\dot{x}$. Thus, $\pi \equiv \dot{P}/P$ represents the rate of domestic inflation, $\hat{Q} \equiv \dot{Q}/Q$ denotes the rate of change of real exchange rates, and for each $i = 1, 2$, $\pi_i \equiv \dot{P}_i^*/P_i^*$ indicates the rate of Country $i's$ inflation. The budget constraint (1) states that there are three sources of accumulation of assets: government lump-sum transfer (S), inflation gains on M, M_1, and M_2 (namely $-\pi M$, $-\pi_1 M_1$ and $(\hat{Q} - \pi_2)M_2$), and the trade surplus $(Y - C_1 - QC_2)$. The first-order conditions for the optimisation problem yield

$$(1-\alpha_1)(1-\beta)L^{1-k}\left[\beta M_2^{\frac{\sigma-1}{\sigma}} + (1-\beta)M^{\frac{\sigma-1}{\sigma}}\right]^{-1} M^{-\frac{1}{\sigma}} = \rho + \pi, \tag{3a}$$

$$\alpha_1 L^{1-k} M_1^{-1} = \rho + \pi_1, \tag{3b}$$

$$(1-\alpha_1)\beta L^{1-k}\left[\beta M_2^{\frac{\sigma-1}{\sigma}} + (1-\beta)M^{\frac{\sigma-1}{\sigma}}\right]^{-1} M_2^{-\frac{1}{\sigma}} = \rho + \pi_2 - \hat{Q}. \tag{3c}$$

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Through log linearization and the omission of constant terms, Equations (3a), (3b), and (3c) yield the following three log-linear equations revealing the properties of the demand function of each currency

$$\begin{aligned} m = & -[(1-\alpha_1)(1-\beta_2)\varepsilon + \beta_2\sigma + \alpha_1(1-\beta_2)](\rho+\pi) \\ & -\alpha_1(\varepsilon-1)(\rho+\pi_1) + \beta_2[(\sigma-\varepsilon)+\alpha_1(\varepsilon-1)] \\ & (\rho+\pi_2-\dot{q}), \end{aligned} \tag{4a}$$

$$\begin{aligned} m_1 = & -(1-\alpha_1)(1-\beta_2)(\varepsilon-1)(\rho+\pi) - [\alpha_1(\varepsilon-1)+1] \\ & (\rho+\pi_1) - (1-\alpha_1)\beta_2(\varepsilon-1)(\rho+\pi_1-\dot{q}), \end{aligned} \tag{4b}$$

$$\begin{aligned} m_2 = & (1-\beta_2)[(\sigma-\varepsilon)+\alpha_1(\varepsilon-1)](\rho+\pi) - \alpha_1(\varepsilon-1) \\ & (\rho+\pi_1) - [(1-\alpha_1)\beta_2\varepsilon + (1-\beta_2)\sigma + \alpha_1\beta_2] \\ & (\rho+\pi_2-\dot{q}), \end{aligned} \tag{4c}$$

where $m \equiv \ln M$, for each $i = 1, 2$, $m_i \equiv \ln M_i$, $\dot{q} \equiv d\ln Q/dt$, $\varepsilon = 1/k$ (the opportunity cost elasticity of holding the currency mix, L), $\beta_2 \equiv \beta M_{20}^{(\sigma-1)/\sigma}/[\beta M_{20}^{(\sigma-1)/\sigma} + (1-\beta)M_0^{(\sigma-1)/\sigma}]$ (the share of Country 2's currency with respect to the opportunity cost of holding the currency mix consisting of the domestic currency and Country 2's currency), and a subscript of a variable zero denotes its initial equilibrium value.

The growth of the domestic money supply, $\dot{H}/H$, is assumed to be exogenously given and constant at μ. Hence, $\dot{M} = \mu M - \pi M$. Assume that seigniorage is a lump-sum transfer to the citizen. Namely, $S = \mu M$. The budget constraint (1) can then be written as

$$\dot{M}_1 + \dot{M}_2 + \pi_1 M_1 + \left(\pi_2 - \hat{Q}\right) M_2 = (Y - C_1 - QC_2). \tag{1'}$$

The net inflow of foreign currencies, $\dot{M}_1 + \dot{M}_2 + \pi_1 M_1 + (\pi_2 - \hat{Q})M_2$, clearly must be 'financed' by a trade balance T.

The trade balance T comprises exports $(Y - C_1)$ and imports (QC_2). The former consists of X_1 (exports to Country 1) and X_2 (exports to Country 2). The export demand function of Country 2 is assumed to be the following form:

$$X_2 = Q^{1/b}, \quad b > 0. \tag{5}$$

From the first-order conditions for the representative citizen's optimisation, we have

$$C_2 = Q^{-1/a}. \tag{6}$$

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Equations (5) and (6) imply that the trade balance can be written as

$$T = X_1 + Q^{1/b} - Q^{1-1/a} = T(Q). \tag{7}$$

Whether $T' \equiv (dT/dQ) = C_{20}[(X_{20}/Q_0C_{20})(1/b) + (1/a) - 1] \gtrless 0$ depends on whether the modified Marshall–Lerner condition is satisfied. Substituting Equation (7) into Equation (1′), and dividing the resulting equation by M_2, we obtain

$$\frac{T(Q)}{M_2} - \frac{M_1}{M_2}\dot{m}_1 - \frac{M_1}{M_2}\pi_1 = \dot{m}_2 - \dot{q} + \pi_2. \tag{1″}$$

The left-hand side of Equation (1″) can be approximately log-linearised by $\delta q - \pi_{20}M_2 - \gamma\dot{m}_1 - \gamma\pi_{10}m_1 - \gamma\pi_1$, where $\delta \equiv Q_0T'/M_{20} \gtrless 0$ accordingly when $T' \gtrless 0$, and $\gamma \equiv M_{10}/M_{20} > 0$. The budget constraint (1) can then be rewritten as

$$\gamma\dot{m}_1 + \dot{m}_2 - \dot{q} = \delta q - \gamma\pi_{10}m_1 - \pi_{20}m_2 - \gamma\pi_1 - \pi_2. \tag{4d}$$

4 Long-run effects of domestic monetary expansion

We now analyse the long-run effects of domestic monetary expansion on the real balance of the domestic and foreign currencies and real exchange rates. Expansions of the domestic money supply lead to a rise in the relative cost of holding the domestic currency, which in turn results in a change in the holdings of foreign currencies. The home country can alter the holdings of foreign currencies only through changing the trade balance, which requires an accommodating real currency depreciation or appreciation.

4.1 Real holdings of currencies

Suppose that the system is initially in a steady-state equilibrium corresponding to a certain value of μ. Suddenly at $t = 0^+$, a higher rate of expansion in the domestic money supply, μ', is instituted. What would happen to the new steady-state value of the three endogenous variables $\bar{m}$, $\bar{m}_1$, and $\bar{m}_2$ in response to an increase in μ? The steady-state equilibrium is characterised by $\dot{m} = \dot{m}_1 = \dot{m}_2 = \dot{q} = 0$. It follows directly from Equation (4a) to Equation (4c) that

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$$\frac{\Delta\bar{m}}{\Delta\mu} = -\left[(1-\alpha_1)(1-\beta_2)\varepsilon + \beta_2\sigma + \alpha_1(1-\beta_2)\right] < 0, \tag{8a}$$

$$\frac{\Delta\bar{m}_1}{\Delta\mu} = -(1-\alpha_1)(1-\beta_2)(\varepsilon-1) < 0, \tag{8b}$$

$$\frac{\Delta\bar{m}_2}{\Delta\mu} = (1-\beta_2)\left[(\sigma-\varepsilon) + \alpha_1(\varepsilon-1)\right] > 0 \quad \text{if} \quad \sigma > \varepsilon. \tag{8c}$$

From Equations (8a) and (8b), the real holdings of the domestic currency and Country 1's currency would be smaller in the new steady state. An increase in the rate of domestic money growth leads to a rise in the relative cost of holding the domestic currency. There are two effects: the substitution effect and the scale effect. The substitution effect refers to the relatively higher cost of the domestic currency reducing the proportion of the holding of domestic currency relative to foreign currencies in the portfolio; the scale effect refers to the higher cost of the domestic currency raising the weighted average cost of holding the entire portfolio, which reduces the size of the overall portfolio demanded. Both effects reduce the quantity of domestic currency held. The holding of Country 1's currency also decreases because the scale effect exceeds the substitution effect ($\varepsilon > 1$). Equation (8c) clearly shows that in the new steady state, the holding of Country 2's currency is larger when the substitution effect relatively exceeds the scale effect.[4]

Calvo and Rodriguez (1977) and Liviatan (1981) analyse the effect of an increase in domestic money growth on the long-run level of foreign currency holdings. Calvo and Rodriguez conclude that the long-run level of foreign currency holdings increases, whereas Liviatan obtains a contrary result. The key to the diametric results is the different assumptions on the (substitutive or complementary) relationship between the domestic and foreign currency. Liviatan assumes that 'the two assets behave like "cooperant" factors of production in the theory of the firm', (p. 1224). By contrast, Calvo and Rodriguez implicitly assume the two currencies to be substitutes. Assuming the long-run constancy of total assets, an individual's steady-state allocation of money demand between the domestic currency and the foreign currency, in effect, can be viewed as the firm's choice of 'perfect competitive' inputs in a two-factor production function.[5]

[4]This problem, related to Gresham's law and Hayek's free currency system, is comprehensively discussed in Chen, Tsaur, and Chou (1981).

[5]These arguments briefly appear in Chen and Tsaur (1992, pp.320–321).

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The effects on net inflow of foreign currencies (NF) are investigated if the home country raises its money growth rate in our three-currency model. This problem related to the monetary independence proposition is discussed by Miles (1978). Miles argues that when currency substitution exists, even perfectly flexible exchange rates may not guarantee monetary independence. The monetary policy induces changes in the interest rate that result in offsetting money flows, and the magnitude of these offsetting flows is directly proportional to the degree of substitution between currencies.

In our three-currency model, whereas the net substitutability between M_1 and M is small so that the two currencies are gross complements ($1 < \varepsilon$), M_2 and M are gross substitutes for each other ($\sigma > \varepsilon$). More rapid money growth increases inflation in domestic currency prices, which leads to a decrease in real demand for both the domestic currency and Country 1's currency and an increase in real demand for Country 2's currency. In the long-run equilibrium, net inflow of foreign currencies can be derived using a weighed sum of Equations (8b) and (8c)

$$\begin{aligned}\frac{\Delta NF}{\Delta\mu} &= \alpha_1\left(\frac{\Delta\bar{m}_1}{\Delta\mu}\right) + (1-\alpha_1)\,\beta_2\left(\frac{\Delta\bar{m}_2}{\Delta\mu}\right)\\ &= (1-\alpha_1)\,(1-\beta_2)\,[\beta_2(\sigma-\varepsilon) - \alpha_1\,(1-\beta_2)\,(\varepsilon-1)] \gtreqless 0\\ &\quad \text{according as} \quad \sigma^* \equiv \omega\sigma + (1-\omega)\cdot\ 1 \gtreqless \varepsilon, \qquad (9)\end{aligned}$$

where $0 < \omega \equiv \beta_2/[\beta_2 + \alpha_1(1-\beta_1)] < 1$. The critical condition of Equation (9) is worth explanation. The σ^* can be interpreted as a *combined* elasticity of substitution between the domestic currency and these foreign currencies, defined as a weighted average of partial elasticity of substitution between M and M_2 (which is σ) and partial elasticity of substitution between M and M_1 (which is 1). The weights of σ and 1 are ω and $(1-\omega)$, respectively.[6] This condition can be further simplified as $\sigma \gtreqless \varepsilon$ when the two partial elasticities are equal to the same value, σ. An expansionary money policy leads to a rise (fall) in the quantity of foreign currencies held when domestic currency and foreign currencies are gross substitutes (complements). Domestic monetary expansion may not necessarily induce an offsetting inflow of foreign currencies, unless a certain relative value of

[6] In our $\sigma > \varepsilon > 1$ case, this condition reduces to $\sigma \gtreqless \varepsilon + [\alpha_1(1-\beta_2)(\varepsilon-1)/\beta_2]$. In other words, the partial elasticity between M and M_2 must be greater than ε to offset the decrease in cooperant currencies M and M_1.

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σ to ε is that the appropriate gross substitutability among currencies yields $\Delta NF/\Delta\mu = 0$. Otherwise, the inherent monetary policy autonomy of flexible exchange rates would not exist. The distinction between the substitutive (competitive) and complementary (cooperant) relationship among currencies is crucial in determining whether monetary independence exists.[7] In other words, the gross substitutability, rather than the elasticity of substitution itself, is substantively important in influencing the validity of monetary independence under flexible exchange rates. Miles (1978, pp.432–433) comment on the Cobb-Douglas demand for money function assumed by Chen (1973) 'unfortunately constrains the elasticity of substitution to equal one', may have missed the point.

4.2 Real exchange rates

Substituting $\dot{m}_1 = \dot{m}_2 = \dot{q} = 0$ and the results of Equations (8b) and (8c) into Equation (4d), we obtain the steady- state effect of domestic monetary expansion on q as follows:

$$\frac{\Delta\bar{q}}{\Delta\mu} = \frac{1-\beta_2}{\delta}\Big\{\pi_{20}\sigma - (\pi_{20}+\gamma\pi_{10})(1-\alpha_1)\varepsilon - (\pi_{20}+\gamma\pi_{10})\alpha_1 + \gamma\pi_{10}\Big\} \gtreqless 0,$$

according as

$$\sigma \gtreqless \left(\lesseqgtr\right)(1-\alpha_1)\varepsilon + \alpha_1 + (\gamma\pi_{10}/\pi_{20})(1-\alpha_1)(\varepsilon-1) \equiv \phi > \varepsilon, \quad \text{if} \quad \delta > (<)\, 0. \tag{10}$$

As mentioned previously, an increase in μ causes a currency portfolio to shift from the domestic currency and Country 1's currency to Country 2's currency. Net inflow of foreign currencies may then increase or decrease. Given $\delta > 0$, when $\sigma > \phi$, gross substitutability between the domestic currency and Country 2's currency appears, and the net foreign inflation tax to pay is greater because of the increased tax base. By contrast, when $\sigma < \phi$, gross substitutability between the domestic currency and Country 2's currency disappears, and the net foreign inflation tax to pay is smaller because of the

[7] The relative value of σ to ε rather than the absolute value of σ is important. See Chen and Tsaur (1983).

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smaller tax base, because a higher (lower) value of trade balance is now required to pay for the greater (smaller) net foreign inflation tax. The terms of trade in turn must deteriorate (improve) to sustain the higher (lower) level of trade balance, which leads to a real depreciation (appreciation) in exchange rates. The case of $\delta < 0$ can be similarly illustrated under the same inference. (This is addressed in subsection 5.3.) Therefore, the real exchange rate (terms of trade) is affected by a change in domestic money growth. Money thus is not neutral in the context of inflation equilibrium.[8]

5 Real exchange rate dynamics

5.1 Dynamic system

By applying log-linearisation to Equation (3b), we obtain

$$(1/\varepsilon)\Big\{(1-\beta_2)(1-\alpha_1)(\varepsilon-1)m - [(1-\alpha_1)\varepsilon+\alpha_1]m_1 + \beta_2(1-\alpha_1)(\varepsilon-1)m_2\Big\} = \rho+\pi_1. \tag{3b$'$}$$

Differentiating Equation (3b$'$) with respect to time and rearranging the resulting equation, we then have

$$\dot{m}_1 = \frac{(1-\alpha_1)(1-\beta_2)(\varepsilon-1)}{(1-\alpha_1)\varepsilon+\alpha_1}\dot{m} + \frac{(1-\alpha_1)\beta_2(\varepsilon-1)}{(1-\alpha_1)\varepsilon+\alpha_1}\dot{m}_2.$$

We substitute the preceding relationships and $\pi = \mu - \dot{m}$ into Equations (4a), (4b), (4c), and (4d) to obtain the dynamic system, which is expressed by the following three differential equations in q, m, and m_2:

$$\begin{bmatrix}\dot{q}\\ \dot{m}\\ \dot{m}_2\end{bmatrix} = \begin{bmatrix}0 & J_{12} & J_{13}\\ 0 & J_{22} & J_{23}\\ J_{31} & J_{32} & J_{33}\end{bmatrix}\begin{bmatrix}q\\ m\\ m_2\end{bmatrix} + \begin{bmatrix}J_{14} & 0 & J_{16} & 1\\ J_{24} & 1 & J_{26} & 0\\ J_{34} & J_{35} & J_{36} & 0\end{bmatrix}\begin{bmatrix}\rho\\ \mu\\ \pi_1\\ \pi_2\end{bmatrix}, \tag{11}$$

where the definitions of J_{ij} ($i = 1, 2, 3$; $j = 1, 2, \cdots, 6$) are shown in part (A) of Appendix 2.

[8] The strong results obtained by Calvo and Rodriguez (1977) and Liviatan (1981) evidencing that money is neutral come from the limiting assumption that the inflation rates of foreign countries are not only constant but also nil ($\pi_{10} = \pi_{20} = 0$). See also Chen and Tsaur (1983).

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Let λ be the characteristic root of the dynamic system (11). The characteristic equation for the dynamic system (11) is

$$-\lambda^3 + \text{TrJ}\,\lambda^2 - \text{BJ}\,\lambda + \text{Det}\,J = 0, \tag{12}$$

where the definitions of TrJ, BJ, and Det J are shown in part (B) of Appendix 2.

Let $f \equiv \ln(F_2/P_2^*) = \ln(M_2/Q) = m_2 - q$. Note that m_2 is Country 2's currency stock in terms of domestic exportables, and f is that in terms of domestic imports. Because the foreign price of domestic imports is exogenously determined by Country 2, f is essentially a predetermined variable in this dynamic system. Indeed, the dynamic system (11) can be reformulated through a linear transformation to obtain a new dynamic framework comprising q, m, and f. Both dynamic systems have identical characteristic polynomials and hence the same characteristic equations expressed as Equation (12).[9]

5.2 The modified marshall-lerner condition holds

Suppose that the modified Marshall-Lerner condition is satisfied ($\delta > 0$, and hence Det $J < 0$). The characteristic equation (12) then has either three negative roots, or one negative real root and the other two with positive real parts. The rates of foreign inflation are generally considered smaller than 10%, and the elasticities of substitution between the domestic and foreign currencies are generally considered smaller than 10. Under these conditions, TrJ is likely to be positive. Therefore, having three negative roots is unlikely. A reasonable conclusion is that there is one negative real root and the other two roots have positive real parts.

In this dynamic system, besides a predetermined variable f, there are two jump variables q and m. As long as $\delta > 0$, the number of (positive) unstable roots equals the number of (independent) jump variables.[10] The system exhibits saddle path stability and equilibrium uniqueness, which is common in perfect-foresight models (e.g., Burmeister (1980); Buiter (1984); Turnovsky (1995)). Let $\lambda_1 < 0$. The saddle path solutions of the system are

[9] See Ostaszewski (1993, p.234) for a detailed discussion.

[10] Because $f = m_2 - q$, m_2 is the sum of f and q, whereas m_1 is a linear combination of m_2 and m. Hence, m_1 and m_2 are not independent variables in this dynamic system.

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described by

$$f(t) - \bar{f} = \left[f(0) - \bar{f}\right] e^{\lambda_1 t}, \tag{13a}$$

$$q(t) - \bar{q} = \nu_1 \left[f(t) - \bar{f}\right], \tag{13b}$$

$$m(t) - \bar{m} = \nu_2 \left[f(t) - \bar{f}\right], \tag{13c}$$

$$m_1(t) - \bar{m}_1 = \nu_3 \left[f(t) - \bar{f}\right], \tag{13d}$$

$$m_2(t) - \bar{m}_2 = \left[f(t) - \bar{f}\right] + [q(t) - \bar{q}] = (1 + \nu_1)\left[f(t) - \bar{f}\right], \tag{13e}$$

where $\bar{x}$ denotes the steady-state equilibrium value for variable x, and definitions of $v_i (i = 1, 2, 3)$ are shown in part (C) of Appendix 2.

When $\delta > 0$, trade elasticities and currency-substitutability play the key role in determining the impact effect and the dynamic adjustment path. Before examining the dynamics of real exchange rates in this case, let us interpret the reason underlying the determinacy of macroeconomic stability. Consider individuals' optimistic expectations about the rate of return on Country 2's currency (i.e., $\dot{q}^e > 0$) that lead to an increase in the holding of Country 2's currency. A higher level of balance of trade is required to accumulate Country 2's currency balance. Given that the modified Marshall-Lerner condition holds, the real exchange rate must rise to improve the trade balance (i.e., $\dot{f}(t) > 0$). However, Equation (13b) clearly states that $\dot{f}(t) > 0$ accordingly follows $q(t) > \bar{q}$ given that v_1 is negative. This in turn implies that to attain steady-state equilibrium, the change rate of q must be negative (i.e., $\dot{q}(t) < 0$). Individuals' initial optimistic expectations are clearly not fulfilled, leading to determinacy of equilibrium.

We know that f is the only predetermined variable in this dynamic system. Immediately after the shock of an increase in domestic money growth from μ to $\dot{\mu}$ at $t = 0^+$, f remains at its original steady-state value (i.e., $f(0^+) = f(0) = \bar{f}(\mu)$). However, the new steady-state equilibrium value changes to $\bar{f}(\mu')$. The initial steady-state value of Country 2's currency stock in terms of Commodity 2 $\bar{f}(\mu)$ is smaller or greater than the new steady-state equilibrium value $\bar{f}(\mu')$, denoted $\Delta \bar{f} \equiv \bar{f}(\mu') - \bar{f}(\mu) \gtrless 0$, and induces $\dot{f}(0^+) \gtrless 0$.[11] $\dot{f}(0^+)$ is a crucial factor in determining the variables q and m_2 jumping to a new level $q(0^+)$ and $m_2(0^+)$, and in permitting convergence to the new steady-state equilibrium values $\bar{q}(\mu')$ and

[11] Differentiating (13a) with respect to time and taking $t = 0^+$, we then have $\dot{f}(0^+) = \lambda_1[\bar{f}(\mu) - \bar{f}(\mu')]$. Because $\lambda_1 < 0$, $\dot{f}(0^+) \gtrless 0$ depends on $\Delta \bar{f} \gtrless 0$ (namely, $\bar{f}(\mu') \gtrless \bar{f}(\mu)$).

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$\bar{m}_2(\mu')$. These relationships can be illustrated by taking $t = 0^+$ and rewriting Equations (13b) and (13e) as

$$q(0^+) - \bar{q}(\mu') = (\nu_1/\lambda_1)\,\dot{f}(0^+), \tag{13b$'$}$$

$$m_2(0^+) - \bar{m}_2(\mu') = [(1+\nu_1)/\lambda_1]\,\dot{f}(0^+). \tag{13e$'$}$$

The sign of $\dot{f}(0^+)$ is governed by $\Delta\bar{f}$, which depends on the relative magnitude between $\Delta\bar{m}_2(\equiv \bar{m}_2(\mu') - m_2(\mu))$ and $\Delta\bar{q}(\equiv \bar{q}(\mu') - \bar{q}(\mu))$. On the basis of Equations (8c) and (10), the sign of $\Delta\bar{f}$ corresponding to $\delta > 0$ can be classified according to the following two cases.

5.2.1 The $\sigma > \phi$ case

Where a higher currency substitutability ($\sigma > \phi$) exists, we obtain

$$\frac{\Delta\bar{f}}{\Delta\mu} \gtrless 0 \quad \text{according as} \quad \delta \gtrless \frac{\pi_2(\sigma-\phi)}{(\sigma-\varepsilon)+\alpha_1(\varepsilon-1)} \equiv \theta > 0. \tag{14}$$

If the trade elasticities are relatively sensitive ($\delta > \theta$), then $\Delta\bar{f} > 0$. Hence, $\dot{f}(0^+) > 0$. Referring to Equations (8e), (10), (13b$'$), and (13e$'$) and $\Delta f(0^+) \equiv f(0^+) - \bar{f}(\mu) = 0$, we have $\Delta m_2(0^+) = \Delta q(0^+)$, where $\Delta m_2(0^+) \equiv m_2(0^+) - \bar{m}_2(\mu)$ and $\Delta q(0^+) \equiv q(0^+) - \bar{q}(\mu)$. It follows that

$$q(0^+) > \bar{q}(\mu') > \bar{q}(\mu), \tag{15a}$$

$$\bar{m}_2(\mu') > m_2(0^+) > \bar{m}_2(\mu). \tag{15b}$$

In other words, q overshoots, whereas m_2 undershoots.

If the trade elasticities are relatively insensitive ($\delta < \theta$), then $\Delta\bar{f} < 0$. Hence, $\dot{f}(0^+) < 0$. We obtain the opposite results

$$\bar{q}(\mu') > q(0^+) > \bar{q}(\mu), \tag{16a}$$

$$m_2(0^+) > \bar{m}_2(\mu') > \bar{m}_2(\mu). \tag{16b}$$

That is, q undershoots, whereas m_2 overshoots.

These results are illustrated graphically in Figure 1 and 2, respectively. In all figures, Line M_2M_2 represents $m_2(t) - \bar{m}_2 = [(1+\nu_1)/\lambda_1]\dot{f}(t)$, whereas Line QQ represents $q(t) - \bar{q} = (\nu_1/\lambda_1)\dot{f}(t)$. q increases from right to left on the horizontal axis. As μ increases to μ', we have a new $\bar{m}_2(\bar{m}_2(\mu'))$

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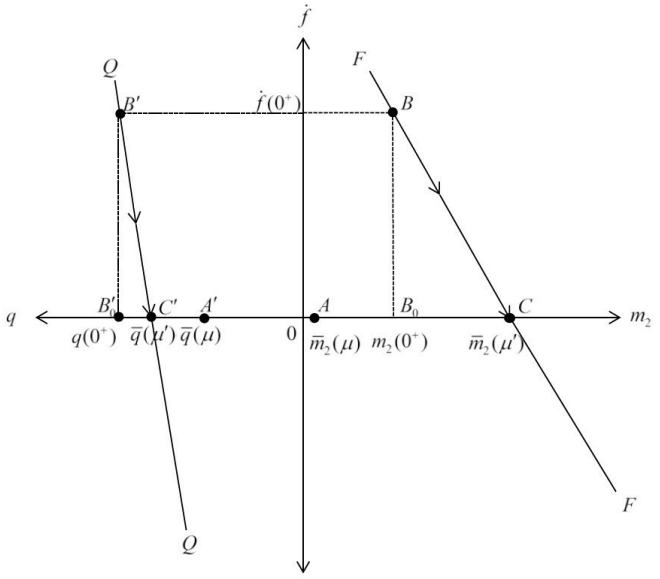


Figure 1: Real Exchange Rate Overshooting

and $\bar{q}(\bar{q}((\mu'))$. Because momentarily $f(0^+) = \bar{f}(\mu)$, we have the change rate of $f(t)$ increasing (decreasing) from zero to a positive (negative) $\dot{f}(0^+)$ in Figure 1 (2). This corresponds on Line QQ to an increase in q from $\bar{q}(\mu)$ to $q(0^+)$, which is higher (lower) than $\bar{q}(\mu')$ in Figure 1 (2). The real exchange rate overshoots (undershoots) its new steady-state equilibrium value. The dynamic paths are represented by movements along BC and $B'C'$. Thus along the dynamic adjustment path BC in Figure 1 (2), we have real currency appreciation (depreciation), whereas q remains above (below) $\bar{q}(\mu')$ throughout the figure.

Following Chen, Tsaur, and Liu (1989), the reason for these results can be explained extensively. We first discuss the case of sensitive trade elasticities ($\delta > \theta$). When the trade elasticities are relatively high, the real exchange rate rises so moderately that the new steady-state stock of Country 2's currency in terms of imports also increases (i.e., $\Delta\dot{f} > 0$, hence $\dot{f}(0^+) > 0$). To accumulate a stock of Country 2's currency, a higher level of trade balance in transition is required. It can be attained through a transitory higher value of the real exchange rate. From Equation (13b′), we already know that

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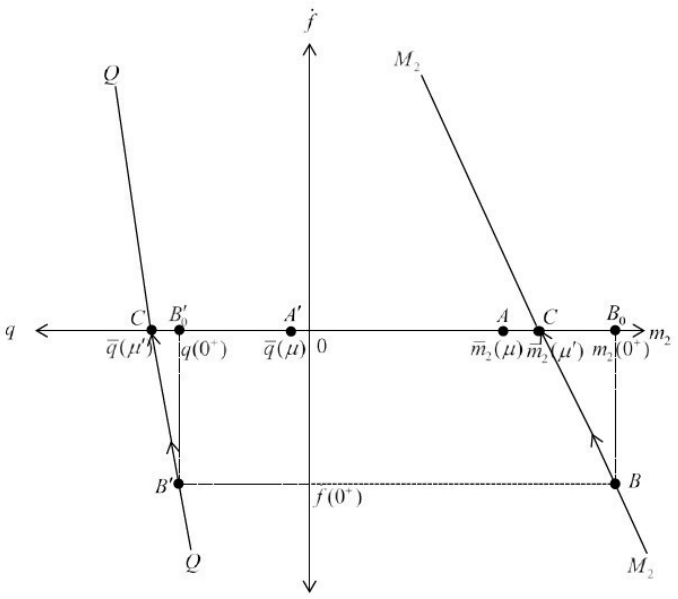


Figure 2: Real Exchange Rate Undershooting

$q(0^+) > \bar{q}(\mu')$. This implies that the real exchange rate jumps initially to a higher level, and then it decreases gradually to approach its long-run level in the transition period. The real exchange rate q overshoots because, after the initial jump, q continues falling until it reaches the new steady-state value. By contrast, the real stock of Country 2's currency in term of exports m_2 undershoots because, after the initial jump, m_2 continues rising until it reaches the steady-state value.

We now address the case of insensitive trade elasticities ($\delta < \theta$). Because the trade elasticities are relatively low, the real exchange rate must rise drastically to pay for increased foreign inflation tax in the new steady state. Because the increase in the real exchange rate ($\Delta\bar{q}$) outweighs the increase in the real stock of Country 2's currency ($\Delta\bar{m}_2$), the existing stock exceeds the new steady-state stock in terms of imports ($\Delta\bar{f} < 0$). However, in terms of exportables, we have already seen that the existing stock at the moment falls short of the new steady-state stock ($\bar{m}_2(\mu) < \bar{m}_2(\mu')$ i.e., $\Delta\bar{m}_2 > 0$). The result is inconsistent. Thus, m_2 must attain a new level $m_2(0^+)$ to exceed the new steady-state level $\bar{m}_2(\mu')$. Now in both terms, the new initial value

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of Country 2's currency stock exceeds the new steady-state value. To dispose of the redundant stock of Country 2's currency, a lower level of trade balance in transition is required. It can be attained through a transitory lower value of q. Therefore, the new initial level of the real exchange rate $q(0^+)$ must fall short of the new steady-state value $\bar{q}(\mu')$. This explains why m_2 overshoots and q undershoots.[12]

5.2.2 The $\sigma < \phi$ case

When currency-substitutability is relatively low ($\sigma < \phi$), from Equation (10), we have $\Delta\bar{q}/\Delta\mu < 0$ and $\Delta\bar{f}/\Delta\mu > 0$. It follows that $\dot{f}(0^+) > 0$. From Equations (8c), (10), (13b′), and (13e′), we have

$$q(0^+) > \bar{q}(\mu) > \bar{q}(\mu'), \tag{17a}$$

$$\bar{m}_2(\mu') > m_2(0^+) > \bar{m}_2(\mu). \tag{17b}$$

These results show that q countershoots (misjumps), whereas m_2 undershoots. We explain the reason for the results in Figure 3.

The countershooting of the real exchange rate to its new steady-state equilibrium value means that the direction of momentary real exchange rate movement is opposite to its long-run movement. Although the frequently occurring countershooting phenomenon may be empirically more relevant, the interpretation of Equation (17a) seems theoretically more meaningful. When the net substitutability between Country 2's currency and the domestic currency is small so that $\Delta\bar{q}/\Delta\mu < 0$, we obtain $\Delta\bar{f}/\Delta\mu > 0$. As a result, $f(t)$ monotonously converges from $\bar{f}(\mu)$ to the greater new steady-state value, $\bar{f}(\mu')$. The rapidly adjusting variable m_2 accommodates the new situation and also jumps to a new level $m_2(0^+)$. After the jump, $m_2(t)$ continues rising, similar to the adjustment path of $f(t)$, until it reaches the greater new steady-state value. We can therefore conclude that m_2 undershoots its equilibrium value in response to an increase of domestic money growth, that is, $\Delta m_2(0^+)/\Delta\mu > 0$. Because f is assumed to be a predetermined variable, on the shock of an increase in μ, $f(0^+)$ remains at its original steady-state value immediately after the change, and therefore $\Delta f(0^+)/\Delta\mu = 0$. Combining these results, we obtain $\Delta q(0^+)/\Delta\mu < 0$.[13] It implies that the real exchange rate countershoots its new steady-state equilibrium value in response to domestic monetary expansion.

[12] The reason is explained in great depth in Chen, Tsaur, and Liu (1989, pp.961–962).

[13] This result comes from $\Delta f(0^+) = \Delta m_2(0^+) - \Delta q(0^+)$.

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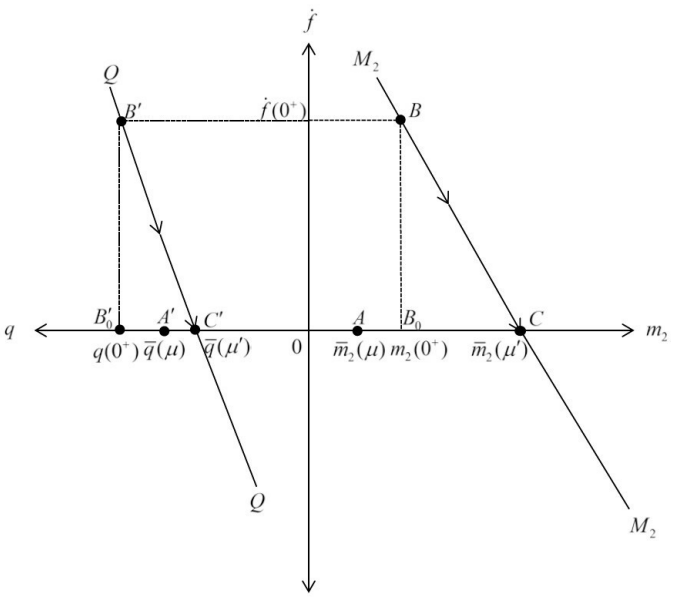


Figure 3: Real Exchange Rate Countershooting

5.3 The modified marshall-lerner condition does not hold

We now address the case in which the modified Marshall-Lerner condition does not hold, so that $\delta < 0$. Equation (10) clearly shows that $\Delta\bar{q}/\Delta\mu < (>)0$, if $\sigma > (<)\phi$. That is, when the substitutability between the domestic currency and Country 2's currency is high (low), an increase in μ causes a relatively greater (smaller) portfolio shift from the domestic currency and Country 1's currency to Country 2's currency. The net inflow of foreign currencies then increases (decreases). The net foreign inflation tax to pay is greater (smaller) because of the increased (decreased) tax base. Thus, in the new steady state, the real exchange rate must decrease (rise) to generate the increased (decreased) trade balance.

Given $\delta < 0$, it follows from Equation (12c) that $Det\,J > 0$. Consequently, the characteristic equation (12) has either three positive roots or two roots with negative real parts and one positive real root. For the three positive real roots scenario, the dynamic system is too locally unstable to reach the new steady state.

By contrast, this dynamic system has only one predetermined variable,

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$f(t)$. As addressed in the literature on dynamic perfect-foresight models (e.g., Benhabib and Farmer (1994); Guo and Lansing (1999); Weder (2001); Chang, Hung, and Huang (2011)), where Equation (12) has two roots with negative real parts so that the number of (negative) stable roots is greater than the number of predetermined variables, then local indeterminacy emerges in the economy.

The reason underlying the indeterminacy of equilibria is worth explaining: Consider individuals' optimistic expectations about the rate of return on Country 2's currency (i.e., $\dot{q}^e > 0$) that leads to an increase in the holding of Country 2's currency. A higher level of balance of trade is required to accumulate Country 2's currency. Given that the modified Marshall-Lerner condition fails, the real exchange rate must fall to improve the trade balance. However, there are two roots with negative real parts in the characteristic equation (12), and thus the change rate of q may be either positive or negative in transition. This implies that individuals' initial optimistic expectations are self-fulfilling under certain conditions. In other words, the economy exhibits indeterminacy of equilibria.

6 Conclusion

Previous studies have shown that when currency substitution exists, even perfectly flexible exchange rates may not guarantee monetary independence. The introduction of a third currency in a small open economy with a dual-currency model deepens a theoretical ambiguity regarding the existence of the inherent monetary policy autonomy of flexible exchange rates. This three-currency framework verifies that the distinction between the substitutive (competitive) and complementary (cooperative) relationships among currencies is substantively important in determining the validity of monetary independence under flexible exchange rates. The steady-state effect of domestic monetary expansion on the real exchange rate (terms of trade) is also investigated in this paper. We have shown that the gross substitutability among currencies is crucial in influencing the direction of response of real exchange rates.

The dynamic behaviour of real exchange rates in response to domestic monetary expansion is also studied by applying a dynamic general equilibrium model. When the Marshall-Lerner condition holds, the adjustment of real exchange rates exhibits a unique saddle path. Additionally, the relative magnitude between currency substitution elasticities and trade elasticities

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critically determines the dynamic properties of real exchange rates. Where there is a higher currency substitutability, the real exchange rate overshoots its new steady-state value. By contrast, where the currency substitutability is relatively low, the real exchange rate countershoots (misjumps) its steady-state equilibrium. Finally, when the Marshall-Lerner condition does not hold, the adjustment of real exchange rates may be either unstable or indeterminate. Under the situation where trade elasticities are so small that the modified Marshall-Lerner condition fails, a certain relative value of currency substitution elasticities to the cost elasticity of holding currency will help fulfil agents' initial optimistic expectations, thus leading to indeterminacy of equilibria and endogenous real exchange-rate fluctuations.

Based on the analysis of this paper, the effects of Taiwan's monetary policy on the macroeconomic economy in the context of RMB internationalization can be examined. For example, He, Liang, and Liu (2024) find that while RMB internationalization reduces Chinese firms' exchange rate exposure to non-US dollar currencies, it increases exchange rate exposure to the US dollar. This can be viewed as exhibiting a complementary relationship between RMB and NTD as well as a substitutive relationship between RMB and USD. In this case, the greater the gross (rather than net) substitutability between different foreign currencies, the larger the loss of degree of Taiwan's monetary autonomy and exchange-rate stability. Thus, small-country policymakers must pay particular attention to the relative magnitude of relevant structural parameters such as these elasticities of substitution among different foreign currencies and the cost (interest) elasticity of holding the currency mix. A comprehensive empirical study on the demand functions for these currencies will be necessary in subsequent research.

Appendix 1

In order to understand the importance of RMB in Taiwan's portfolio, a time-series data on the ratio of RMB deposits to total foreign exchange deposits is presented in Figure A1.

The data shows that the ratio of RMB deposit to total foreign exchange deposit rose from 6.2% in February 2013 to 38.1% in April 2014, peaked at 38.9% in September and October 2014, and then remained high around 37% until August 2015. After that, it continued to decline to 25.4% in August 2018 and maintained its downward trend thereafter. In March 2024, the ratio was 6.1%, which is below the starting point.

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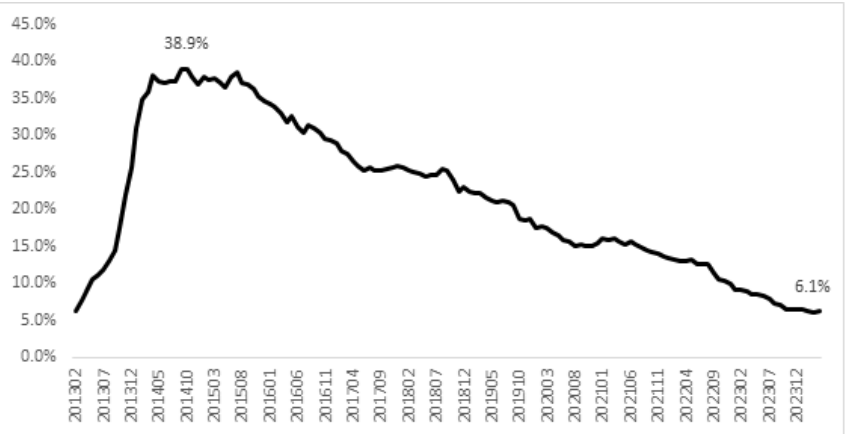


Figure A1: The ratio of RMB deposit to total foreign exchange deposit, February 2013 to April 2024

Source: Financial Statistics Monthly, Republic of China (Taiwan).

Appendix 2

(A) Definitions of J_{ij} $(i = 1, 2, 3;\ j = 1, 2, \cdots, 6)$ (Equation 11, page 370) are as follows:

$$J_{12} = \frac{(1-\beta_2)\left[(\sigma-\varepsilon)+\alpha_1(\varepsilon-1)\right]}{\sigma\left[(1-\alpha_1)\varepsilon+\alpha_1\right]};$$

$$J_{13} = \frac{\beta_2\sigma+(1-\beta_2)\left[(1-\alpha_1)\varepsilon+\alpha_1\right]}{\sigma\left[(1-\alpha_1)\varepsilon+\alpha_1\right]};$$

$$J_{22} = \frac{(1-\beta_2)\sigma+\beta_2\left[(1-\alpha_1)\varepsilon+\alpha_1\right]}{\sigma\left[(1-\alpha_1)\varepsilon+\alpha_1\right]};$$

$$J_{23} = \frac{\beta_2\left[(\sigma-\varepsilon)+\alpha_1(\varepsilon-1)\right]}{\sigma\left[(1-\alpha_1)\varepsilon+\alpha_1\right]};$$

$$J_{31} = \frac{\delta\left[(1-\alpha_1)\varepsilon+\alpha_1\right]}{(1-\alpha_1)\varepsilon+\alpha_1+\gamma(1-\alpha_1)\beta_2(\varepsilon-1)};$$

$$J_{32} = \frac{(1-\beta_2)\left[(\sigma-\varepsilon)+\alpha_1(\varepsilon-1)\right]}{\sigma\left[(1-\alpha_1)\varepsilon+\alpha_1+\gamma(1-\alpha_1)\beta_2(\varepsilon-1)\right]} - \frac{\pi_{10}\gamma(1-\alpha_1)(1-\beta_2)(\varepsilon-1)}{(1-\alpha_1)\varepsilon+\alpha_1+\gamma(1-\alpha_1)\beta_2(\varepsilon-1)}$$

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$$- \frac{\gamma\,(1-\alpha_1)\,(1-\beta_2)\,(\varepsilon-1)A}{\sigma\,[(1-\alpha_1)\,\varepsilon+\alpha_1+\gamma\,(1-\alpha_1)\,\beta_2(\varepsilon-1)]\,B},$$

where $A = [(1-\alpha_1)\,\beta_2\varepsilon + (1-\beta_2)\,\sigma + \alpha_1\beta_2]$,

$B = [(1-\alpha_1)\,\varepsilon + \alpha_1]$;

$$J_{33} = \frac{(1-\beta_2)\,[(1-\alpha_1)\,\varepsilon+\alpha_1]\,\beta_2\sigma}{\sigma\,[(1-\alpha_1)\,\varepsilon+\alpha_1+\gamma\,(1-\alpha_1)\,\beta_2(\varepsilon-1)]}$$
$$- \frac{\pi_{20}\,[(1-\alpha_1)\,\varepsilon+\alpha_1]+\gamma\pi_{10}\,(1-\alpha_1)\,\beta_2(\varepsilon-1)}{(1-\alpha_1)\,\varepsilon+\alpha_1+\gamma\,(1-\alpha_1)\,\beta_2(\varepsilon-1)}$$
$$- \frac{\gamma\,(1-\alpha_1)\,(1-\beta_2)\,\beta_2(\varepsilon-1)\,[(\sigma-\varepsilon)+\alpha_1(\varepsilon-1)]}{\sigma\,[(1-\alpha_1)\,\varepsilon+\alpha_1+\gamma\,(1-\alpha_1)\,\beta_2(\varepsilon-1)]\,C},$$

where $C = [(1-\alpha_1)\,\varepsilon+\alpha_1]$;

$$J_{14} = J_{24} = \frac{\varepsilon}{(1-\alpha_1)\,\varepsilon+\alpha_1};$$

$$J_{16} = J_{26} = \frac{\alpha_1(\varepsilon-1)}{(1-\alpha_1)\,\varepsilon+\alpha_1};$$

$$J_{34} = J_{14}\left[1 - \frac{\gamma\,(1-\alpha_1)\,(\varepsilon-1)-\gamma\pi_{10}\,[(1-\alpha_1)\,\varepsilon+\alpha_1]}{(1-\alpha_1)\,\varepsilon+\alpha_1+\gamma\,(1-\alpha_1)\,\beta_2(\varepsilon-1)}\right];$$

$$J_{35} = \frac{-\gamma\,(1-\alpha_1)\,(1-\beta_2)\,(\varepsilon-1)}{(1-\alpha_1)\,\varepsilon+\alpha_1+\gamma\,(1-\alpha_1)\,\beta_2(\varepsilon-1)};$$

$$J_{36} = J_{16}\left[1 - \frac{\gamma\,(1-\alpha_1)\,(\varepsilon-1)}{(1-\alpha_1)\,\varepsilon+\alpha_1+\gamma\,(1-\alpha_1)\,\beta_2(\varepsilon-1)}\right]$$
$$+ \frac{\gamma\,\{\pi_{10}\varepsilon - [(1-\alpha_1)\,\varepsilon+\alpha_1]\}}{(1-\alpha_1)\,\varepsilon+\alpha_1+\gamma\,(1-\alpha_1)\,\beta_2(\varepsilon-1)}.$$

(B) Definitions of TrJ, BJ, and Det J (Equation 12, page 371) are as follows:

$$\mathrm{TrJ} = \frac{[(1-\alpha_1)\,\varepsilon+\alpha_1]\,(1-\sigma\pi_{20})+D+\sigma}{\sigma\,[(1-\alpha_1)\,\varepsilon+\alpha_1+\gamma\,(1-\alpha_1)\,\beta_2(\varepsilon-1)]},$$

where $D = \gamma\,(1-\alpha_1)\,\beta_2(\varepsilon-1)\,(1-\sigma\pi_{10})$;

$$\mathrm{BJ} = \frac{\{(1-\beta_2)\,\sigma+\beta_2\,[(1-\alpha_1)\,\varepsilon+\alpha_1]\}\,J_{33}}{\sigma\,[(1-\alpha_1)\,\varepsilon+\alpha_1]}$$
$$- \frac{\{\beta_2\sigma+(1-\beta_2)\,[(1-\alpha_1)\,\varepsilon+\alpha_1]\}\,J_{31}}{\sigma\,[(1-\alpha_1)\,\varepsilon+\alpha_1]}$$
$$- \frac{\beta_2\,[(\sigma-\varepsilon)+\alpha_1(\varepsilon-1)]\,J_{32}}{\sigma\,[(1-\alpha_1)\,\varepsilon+\alpha_1]};$$

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$$\text{Det } J = \frac{-\delta}{\sigma\left[(1-\alpha_1)\,\varepsilon + \alpha_1 + \gamma\,(1-\alpha_1)\,\beta_2(\varepsilon-1)\right]}.$$

(C) Definitions of ν_i $(i = 1, 2, 3)$ (Equation 13, page 372) are as follows:

$$\begin{aligned}
\nu_1 &= \Omega\left[\lambda_1\left\{\beta_2\sigma + (1-\beta_2)\left[(1-\alpha_1)\,\varepsilon + \alpha_1\right]\right\} - 1\right] < 0;\\
\nu_2 &= \Omega\lambda_1\beta_2\left[(\sigma-\varepsilon) + \alpha_1(\varepsilon-1)\right] < 0 \quad \text{if} \quad \sigma > \varepsilon;\\
\nu_3 &= \Omega\lambda_1\beta_2\,(1-\alpha_1)\,(\varepsilon-1)\,(\lambda_1\sigma - 1) > 0;\\
1+\nu_1 &= \Omega\Big[\lambda_1^2\sigma\left[(1-\alpha_1)\,\varepsilon + \alpha_1\right] - \lambda_1\left\{(1-\beta_2)\,\sigma + \beta_2\left[(1-\alpha_1)\right.\right.\\
&\qquad \left.\left.\varepsilon + \alpha_1\right]\right\}\Big] > 0;\\
\Omega &\equiv \Big\{\lambda_1^2\sigma\left[(1-\alpha_1)\,\varepsilon + \alpha_1\right] - \lambda_1\left[\sigma + (1-\alpha_1)\,\varepsilon + \alpha_1\right]\\
&\qquad + 1\Big\}^{-1} > 0.
\end{aligned}$$

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通貨替代、貿易彈性、實質匯率動態與貨幣政策獨立性

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本文建構一個包括三元通貨的一般化總體分析架構, 探討本國採行擴張的貨幣政策對於小型開放經濟體相關總體變數的影響。動態一般均衡模型顯示: 兩個大國的通貨與小 (本) 國通貨呈現毛替代或毛補充的關係, 是影響本國貨幣獨立性及實質匯率動態路徑的重要因素。同時, 在此通貨替代模型中, 貿易彈性的高低也是左右匯率動態行爲的關鍵。當修正的 Marshall-Lerner 條件成立時, 匯率動態調整具唯一馬鞍路徑的特性; 但若修正的 Marshall-Lerner 條件不成立, 則匯率調整可能呈現動態不穩定或不確定性。

關鍵詞: 通貨替代, 貿易彈性, 匯率動態, 貨幣政策獨立性, (不) 確定性
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