DOI: 10.6277/TER.202209_50(3).0002

Welfare Analysis in a Vertically-Related Market with Endogenous Price or Quantity Choice

Hong-Ren Din and Chia-Hung Sun*

This research investigates the endogenous choice of prices versus quantities in a vertically-related market where an integrated firm competes with a downstream firm and they bargain over a two-part tariff input pricing contract. Contrary to the standard result, we show that both Bertrand competition and Cournot competition can be sustained as equilibrium outcomes. First, the Bertrand equilibrium maximizes industry profit, but there is a market failure in the choice of the type of strategic variables. Second, the Cournot equilibrium maximizes social welfare and consumer surplus, but there is a prisoners' dilemma. This paper ends with an extension, showing that our baseline model arises naturally as the equilibrium outcome of a simple game that admits vertical mergers.

Keywords: vertically-related markets, Cournot competition, Bertrand competition, endogenous strategic variables, Nash bargaining solution
JEL classification: L13, D43, D21

1 Introduction

In many industry sectors an integrated firm competes with pure downstream firms. Microsoft, which sells Microsoft Windows and Office software to personal computer vendors such as ASUS and Lenovo, is an integrated technology company that sells its own personal computers (Microsoft Surface) and at the same time also competes with pure downstream firms. Starbucks,

*Department of Economics, Soochow University and Corresponding author. Department of Economics, Soochow University, E-mail: jerry52.sun@msa.hinet.net.

經濟論文叢刊 (*Taiwan Economic Review*), 50:3 (2022), 303–327。
國立台灣大學經濟學系出版

which sells both coffee drinks and coffee beans, is best known for its integrated chain of coffee shops. It not only competes with other chains, but also with trendy independent coffee shops that buy their own coffee beans to make coffee. British Petroleum is a multinational vertically-integrated oil company operating in all areas of its industry and also sells crude oil to Irving Oil Refinery, which is the largest oil refinery in Canada. Auto assemblers in the U.S. are highly vertically integrated, exhibiting one major difference from their less integrated Japanese competitors that source most components externally.[1]

We observe in such vertically-related markets that differences exist in the type of strategic variables among firms in the same industry as well as across industry sectors. For instance, firms compete in prices in the catering industry, whereas firms compete in quantities in the oil industry. In the small car industry, we observe a mixture of Cournot and Bertrand behaviors.[2] These observations motivate us to investigate the endogenous choice of the type of strategic variables for a vertically-related market.

Singh and Vives (1984) first propose in the real world that firms may choose whether to adopt a price contract or a quantity contract they will sign with customers. When a firm chooses the price contract, it will have to supply the quantity the consumers demand at a predetermined price. When a firm chooses the quantity contract, it is committed to supply a predetermined quantity, and the remaining prices are determined to clear markets. They also present the well-known proposition whereby, in a pure final-product market, firms choose quantity contracts if the products are substitutes, while Bertrand competition is the most efficient in terms of consumer surplus and social welfare.

For a vertically-related market with vertical integration, in which an integrated firm competes with a pure downstream firm, Arya, Mittendorf, and Sappington's (2008) demonstrate that Bertrand competition yields higher industry profit, lower consumer surplus, and lower social welfare versus Cournot competition.[3] Fanti and Scrimitore (2019) further show in a vertically-related market with vertical integration that it is the dominant strategy

[1] See Ouchi (1981) and Monteverde and Teece (1982) for a detailed discussion.

[2] Firms such as Saturn and Scion compete in prices, while firms such as Honda and Subaru compete in quantities, as Tremblay and Tremblay (2011) discuss.

[3] Based on Arya, Mittendorf, and Sappington's (2008) vertically-related market with vertical integration, Din and Sun (2018) investigate an endogenous timing game.

for each firm to choose a price contract when the integrated firm pushes a "take it or leave it" input pricing contract, involving only a per-unit input price.

In a vertically-related market with vertical separation, where an upstream monopolist bargains with two downstream firms via two-part tariffs, Alipranti, Milliou, and Petrakis' (2014) demonstrate under Cournot competition (Bertrand competition) that the input prices are lower (higher) than the upstream firm's marginal cost and that Cournot competition yields higher social welfare than Bertrand competition.[4] Basak and Wang's (2016) further show in a vertically-related market with vertical separation that choosing the price contract is a dominant strategy for both downstream firms when the two-part tariff pricing contract is determined through centralized Nash bargaining.

The results of Fanti and Scrimitore (2019) and Basak and Wang's (2016) suggest that Bertrand competition in a vertically-related market is the unique equilibrium outcome, whether the integrated firm charges a per-unit input price or a two-part tariff of input price, as well as for a vertically-related market with vertical integration or for a vertically-related market with vertical separation. Going against such intuition, this present study shows that both Bertrand competition and Cournot competition maximizing social welfare can be sustained as the equilibrium outcomes in a vertically-related market if there is an integrated firm and when the firms bargain over a two-part tariff input pricing contract.[5]

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 executes equilibrium analysis. Section 4 offers welfare analysis. Section 5 extends the model to vertical separation and an endogenous market structure. Section 6 concludes the paper.

[4]For subsequent applications of Alipranti, Milliou, and Petrakis' (2014) vertically-related market with vertical separation, Rozanova (2015) generalizes the result by allowing for a general demand function and for an arbitrary number of retailers. Basak (2017) considers the situation where an input pricing contract is determined through centralized bargaining. Basak and Mukherjee (2017) impose a non-negativity constraint on the input prices.

[5]Villas-Boas (2007) empirically shows that trading among vertically-related firms is commonly via two-part tariff input pricing contracts. For instance, we observe in the vertically-related market that Qualcomm sells chips to Samsung and that they bargain over a two-part tariff input pricing contract.

2 The model

Firm 1, which is integrated, produces an input at zero marginal cost and sells it to downstream firm 2, while at the same time it also competes with firm 2 in the product market. For simplicity, we assume the production of each unit of output requires one unit of input and no other costs.

Following Dixit (1979), the quasi-linear utility function of a representative consumer is:

$$U(q_1, q_2) = \left[a(q_1 + q_2) - \left(q_1^2 + 2\beta q_1 q_2 + q_2^2\right)/2\right] - p_1 q_1 - p_2 q_2, \quad (1)$$

where $a > 0$ is a constant, p_i is the price of firm i's product, q_i is the amount of good i, and $\beta \in (0, 1)$ denotes the degree of substitutability between the two products. The inverse demand function for good i is:

$$p_i = a - q_i - \beta q_j, \quad \text{for} \quad i, j = 1, 2 \quad \text{and} \quad i \neq j. \quad (2)$$

The game runs as follows. In the first stage, each firm chooses whether to adopt a price contract (P) or a quantity contract (Q). In the second stage, the integrated firm bargains with the downstream firm over the two-part tariff input pricing contracts (w, f), involving a per-unit input price w and a fixed fee f. In the third stage, each firm simultaneously chooses either price p_i or quantity q_i contingent upon the decisions made in the first stage.

We assume in the second stage that the two firms determine w and f by maximizing the following generalized Nash bargaining expression:

$$\max_{w,\, f} \; [(p_1 q_1 + w q_2 + f) - d]^{\alpha} [(p_2 - w) q_2 - f]^{1-\alpha}, \quad (3)$$

where $d = a^2/4$ is the integrated firm's disagreement profit, referring to its monopoly profit, and $\alpha \in (0, 1)$ ($(1 - \alpha)$, respectively) indicates the bargaining power of the integrated firm (downstream firm, respectively).[6]

[6]We note that most assumptions in this model are standard in the related literature of a vertically-related market. Specifically, Alipranti, Milliou, and Petrakis' (2014), Basak and Wang's (2016), and Din and Sun (2018) analyze a two-part tariff input pricing contract. Alipranti, Milliou, and Petrakis' (2014), Rozanova (2015), Basak and Wang's (2016), Basak (2017), and Basak and Mukherjee (2017) discuss the centralized or decentralized Nash bargaining of the input price in a vertically-related market.

3 Equilibrium analysis

We now analyze product market competition in the game's third stage.

Case 1. Q-Q game

We first discuss the Q-Q game in which both firms choose quantity contracts. For a given input pricing contract, (w, f), solving two first-order conditions simultaneously yields the equilibrium prices $p_i^{QQ}(w)$, quantities $q_i^{QQ}(w)$, and profits $\pi_i^{QQ}(w, f)$, for $i = 1, 2$:

$$\begin{aligned}
p_1^{QQ}(w) &= \frac{2a - a\beta + w\beta}{(2+\beta)(2-\beta)},\ p_2^{QQ}(w) = \frac{2a - a\beta + 2w - w\beta^2}{(2+\beta)(2-\beta)},\\
q_1^{QQ}(w) &= \frac{2a - a\beta + w\beta}{(2+\beta)(2-\beta)},\ q_2^{QQ}(w) = \frac{2a - a\beta - 2w}{(2+\beta)(2-\beta)},\\
\pi_1^{QQ}(w, f) &= p_1^{QQ}(w)q_1^{QQ}(w) + wq_2^{QQ}(w) + f,\\
\pi_2^{QQ}(w, f) &= \left[p_2^{QQ}(w) - w\right]q_2^{QQ}(w) - f. \qquad (4)
\end{aligned}$$

Case 2. P-P game

We next consider the P-P game in which both firms choose price contracts. Solving two first-order conditions simultaneously yields the equilibrium prices $p_i^{PP}(w)$, quantities $q_i^{PP}(w)$, and profits $\pi_i^{PP}(w, f)$, for $i = 1, 2$:

$$\begin{aligned}
p_1^{PP}(w) &= \frac{2a - a\beta + 3w\beta - a\beta^2}{(2+\beta)(2-\beta)},\\
p_2^{PP}(w) &= \frac{2a - a\beta + 2w - a\beta^2 + w\beta^2}{(2+\beta)(2-\beta)},\\
q_1^{PP}(w) &= \frac{2a + a\beta - w\beta - w\beta^2}{(1+\beta)(2+\beta)(2-\beta)},\\
q_2^{PP}(w) &= \frac{2a + a\beta - 2w - 2w\beta}{(1+\beta)(2+\beta)(2-\beta)},\\
\pi_1^{PP}(w, f) &= p_1^{PP}(w)q_1^{PP}(w) + wq_2^{PP}(w) + f,\\
\pi_2^{PP}(w, f) &= \left[p_2^{PP}(w) - w\right]q_2^{PP}(w) - f. \qquad (5)
\end{aligned}$$

Case 3. P-Q game

We now investigate the P-Q game in which the integrated firm chooses the price contract and the downstream firm chooses the quantity contract. We obtain the equilibrium prices $p_i^{PQ}(w)$, quantities $q_i^{PQ}(w)$, and profits $\pi_i^{PQ}(w, f)$, for $i = 1, 2$:

$$
\begin{aligned}
p_1^{PQ}(w) &= \frac{2a - a\beta + w\beta - a\beta^2}{4 - 3\beta^2}, \\
p_2^{PQ}(w) &= \frac{2a - a\beta + 2w - 2a\beta^2 - w\beta^2 + a\beta^3}{4 - 3\beta^2}, \\
q_1^{PQ}(w) &= \frac{2a - a\beta + w\beta - a\beta^2}{4 - 3\beta^2},\ q_2^{PQ}(w) = \frac{2a - a\beta - 2w}{4 - 3\beta^2}, \\
\pi_1^{PQ}(w, f) &= p_1^{PQ}(w)q_1^{PQ}(w) + wq_2^{PQ}(w) + f, \\
\pi_2^{PQ}(w, f) &= \left[p_2^{PQ}(w) - w\right]q_2^{PQ}(w) - f.
\end{aligned} \tag{6}
$$

Case 4. Q-P game

We finally investigate the Q-P game in which the integrated firm chooses the quantity contract and the downstream firm chooses the price contract. We obtain the equilibrium prices $p_i^{QP}(w)$, quantities $q_i^{QP}(w)$, and profits $\pi_i^{QP}(w, f)$, for $i = 1, 2$:

$$
\begin{aligned}
p_1^{QP}(w) &= \frac{2a - a\beta + 3w\beta - 2a\beta^2 + a\beta^3 - 2w\beta^3}{4 - 3\beta^2}, \\
p_2^{QP}(w) &= \frac{2a - a\beta + 2w - a\beta^2 - w\beta^2}{4 - 3\beta^2}, \\
q_1^{QP}(w) &= \frac{2a - a\beta - w\beta}{4 - 3\beta^2}, \\
q_2^{QP}(w) &= \frac{(1 - \beta)(2a + a\beta - 2w - 2w\beta)}{4 - 3\beta^2}, \\
\pi_1^{QP}(w, f) &= p_1^{QP}(w)q_1^{QP}(w) + wq_2^{QP}(w) + f, \\
\pi_2^{QP}(w, f) &= [p_2^{QP}(w) - w]q_2^{QP}(w) - f.
\end{aligned} \tag{7}
$$

In the game's second stage, maximizing equation (3) with respect to f yields:

$$f^j = \alpha\left[p_1^j(w)q_1^j(w) + p_2^j(w)q_2^j(w) - a^2/4\right] - \left[p_1^j(w)q_1^j(w) + wq_2^j(w) - a^2/4\right], \tag{8}$$

for $j \in \{QQ, PP, PQ, QP\}$. Substituting equation (8) into equation (3), we obtain the maximization problem as:

$$\max_w \alpha^\alpha(1-\alpha)^{1-\alpha}\left[p_1^j(w)q_1^j(w) + p_2^j(w)q_2^j(w) - a^2/4\right]. \tag{9}$$

Solving the first-order condition gives the equilibrium unit input prices and the resulting fixed fees as:[7]

$$\left(w^{QQ} = \frac{a\beta\left(4-4\beta+\beta^2\right)}{2\left(4-3\beta^2\right)},\ f^{QQ} = \frac{a^2(1-\beta)^2\left(4\alpha-3\alpha\beta^2+3\beta^2\right)}{\left(4-3\beta^2\right)^2}\right),$$

$$\left(w^{PP} = \frac{a\beta\left(4+4\beta+\beta^2\right)}{2\left(4+5\beta^2\right)},\right.$$

$$\left. f^{PP} = \frac{a^2(1-\beta)\left(4\alpha+9\alpha\beta^2+5\alpha\beta^4-5\beta^2-4\beta^4\right)}{(1+\beta)\left(4+5\beta^2\right)^2}\right),$$

$$\left(w^{PQ} = \frac{a\beta}{2},\ f^{PQ} = \frac{a^2(1-\beta)^2\left(4\alpha-3\alpha\beta^2-\beta^2\right)}{\left(4-3\beta^2\right)^2}\right),$$

$$\left(w^{QP} = \frac{a\beta}{2},\ f^{QP} = \frac{a^2(1-\beta)^2\left(4\alpha-3\alpha\beta^2-\beta^2+\beta^4\right)}{\left(4-3\beta^2\right)^2}\right). \tag{10}$$

We thus arrive at the following Lemma 1.

Lemma 1. The unit input price w^j, for $j \in \{QQ, PP, PQ, QP\}$ and by maximizing industry profits $(p_1^j(w)q_1^j(w) + p_2^j(w)q_2^j(w))$, is independent of α, and the fixed fee f^j is strictly increasing in α and negative for a small α under the P-P, P-Q, and Q-P games.

We now turn to the game's first stage. Given the opponent firm chooses a quantity contract (Q), we compare the profit functions for a firm when

[7] We allow for negative fixed fees, in which the integrated firm subsidizes the downstream firm's production via a fixed fee, which is similar to Alipranti, Milliou, and Petrakis' (2014) and Basak and Wang's (2016).

choosing a price contract and that when choosing a quantity contract as:

$$\pi_1^{PQ} = \pi_1^{QQ} = \frac{a^2\left(4 + 4\alpha - 8\alpha\beta - 3\beta^2 + 4\alpha\beta^2\right)}{4\left(4 - 3\beta^2\right)},$$
$$\pi_2^{QP} = \pi_2^{QQ} = \frac{a^2(1-\alpha)(1-\beta)^2}{4 - 3\beta^2}. \tag{11}$$

We obtain the following Lemma 2.

Lemma 2. Given the opponent firm chooses a quantity contract (Q) in the first stage, a firm's profits when choosing a price contract (P) and its profits when choosing a quantity contract (Q) are the same.

The intuition behind Lemma 2 runs as follows. Substituting equation (8) respectively into $\pi_1^j(w, f^j) = p_1^j(w)q_1^j(w) + wq_2^j(w) + f^j$ and $\pi_2^j(w, f^j) = [p_2^j(w) - w]q_2^j(w) - f^j$ yields:

$$\pi_1^j\left(w, f^j\right) = \alpha\left[p_1^j(w)q_1^j(w) + p_2^j(w)q_2^j(w) - a^2/4\right] + a^2/4,$$
$$\pi_2^j\left(w, f^j\right) = (1-\alpha)\left[p_1^j(w)q_1^j(w) + p_2^j(w)q_2^j(w) - a^2/4\right]. \tag{12}$$

In such a vertically-related market with vertical integration and a two-part tariff input pricing contract, the bargaining solution of the unit input price and fixed fee, (w^j, f^j) for $j \in \{QQ, PP, PQ, QP\}$, maximizes industry profit, and the profits of the two firms are both functions of the industry profit. In this case, for a given quantity supplied by the opponent firm, a firm faces a residual demand function that depends only on its own action, and its action behavior is just like what the monopoly firm does, meaning the prices and quantities are independent of its choice of quantity (Q) or price (P) as the strategic variable. Therefore, industry profit and the profit of each firm are also irrelevant to a firm's choice of quantity (Q) or price (P) as the strategic variable.

Given that opponent firm 2 chooses a quantity contract satisfying $q_2^{QQ}(w^{QQ}) = q_2^{PQ}(w^{PQ})$, firm 1 faces a residual demand function that depends only on its own action, and its prices, quantities, and profits are the same, whether it chooses a quantity contract (Q) or a price contract (P), which means $p_1^{QQ}(w^{QQ}) = p_1^{PQ}(w^{PQ})$ and $q_1^{QQ}(w^{QQ}) = q_1^{PQ}(w^{PQ})$. It follows that $p_2^{QQ}(w^{QQ}) = p_2^{PQ}(w^{PQ})$, $[p_1^{QQ}(w^{QQ})q_1^{QQ}(w^{QQ}) + p_2^{QQ}(w^{QQ})q_2^{QQ}(w^{QQ})] = [p_1^{PQ}(w^{PQ})q_1^{PQ}(w^{PQ}) + p_2^{PQ}(w^{PQ})q_2^{PQ}(w^{PQ})]$, and

$\pi_1^{PQ} = \pi_1^{QQ}$. The same argument applies to show that $\pi_2^{QP} = \pi_2^{QQ}$. Lemma 2 implies that one of the equilibrium outcomes is Cournot competition, (Q, Q).

Given the opponent firm conversely chooses a price contract (P) in the first stage, we compare the profit functions for a firm when it chooses a price contract and that when it chooses a quantity contract as:

$$\pi_1^{PP} - \pi_1^{QP} = \frac{2a^2\alpha\beta^4(1-\beta)}{(1+\beta)(4+5\beta^2)(4-3\beta^2)} > 0,$$
$$\pi_2^{PP} - \pi_2^{PQ} = \frac{2a^2\beta^4(1-\alpha)(1-\beta)}{(1+\beta)(4+5\beta^2)(4-3\beta^2)} > 0. \tag{13}$$

We summarize the results in the following Proposition 1.

Proposition 1. The equilibrium outcomes in the game's first stage are Bertrand competition, (P, P), and Cournot competition, (Q, Q).

We find here that both Bertrand competition and Cournot competition can be sustained as the equilibrium outcomes. These results are somewhat different from those derived in the related literature, such as Basak and Wang's (2016) and Fanti and Scrimitore (2019), in which choosing the price contract is the dominant strategy for each firm and Bertrand competition is the unique equilibrium outcome. Under the conditions of whether the market structure is vertical integration or vertical separation as well as whether the input pricing contract involves only a per-unit input price or a two-part tariff of input price, we conclude that they all play a crucial role in determining the endogenous mode of competition in a vertically-related market.

The intuition underlying Bertrand competition, (P, P), as the equilibrium outcome goes as follows. Given the downstream firm chooses a price contract (P), under Bertrand competition a higher input price w induces the downstream firm to choose a higher price p_2, thus subsequently increasing the integrated firm's price p_1 and profit in the product market. Furthermore, the integrated firm's higher price p_1 raises the downstream firm's output q_2 and thereby increases the integrated firm's input revenue wq_2.[8]

[8]In contrast, a higher input price w induces firm 2 to choose a higher price p_2, which subsequently increases firm 1's output q_1 if firm 1 chooses a quantity contract. Firm 1's higher quantity reduces firm 2's output and thereby decreases firm 1's input revenue wq_2.

As a result, the integrated firm's best response to the downstream firm's price contract is to choose the price contract.

Given the integrated firm chooses a price contract (P) in the first stage, if the downstream firm chooses a price contract, then the integrated firm has an incentive to behave less aggressively in the product market in order to raise the downstream firm's output q_2. This increases the downstream firm's profit, meaning that the downstream firm's best response to the integrated firm's price contract is also to choose the price contract, and hence Bertrand competition is the equilibrium outcome.

Lemma 2 implies Cournot competition, (Q, Q), is the equilibrium outcome. We find that the two-part tariff pricing and the market structure of vertical integration both play important roles for the Cournot outcome to emerge in the equilibrium. Recall that under our two-part tariff pricing model, a firm's profit is the same no matter whether it chooses a price contract (P) or a quantity contract (Q), given the opponent firm chooses a quantity contract (Q). On the other hand, under the unit pricing model of Fanti and Scrimitore (2019), a firm's best response to its opponent's quantity contract (Q) is to choose a price contract (P).[9] Therefore, the two-part tariff pricing model generates the equilibrium outcome of Cournot competition (Q, Q), compared with the unit pricing model.

Under our vertical integration model, a firm's profit is the same no matter whether it chooses a price contract (P) or a quantity contract (Q), given the opponent chooses a quantity contract (Q). On the other hand, under Basak and Wang's (2016) vertical separation model with centralized Nash bargaining, a firm's best response to its opponent's quantity contract (Q) is to choose a price contract (P).[10] In this sense, Cournot competition (Q, Q) in the vertical integration model is sustained as the equilibrium outcome, compared with the vertical separation model.

[9]The intuition is that including a positive unit input price under a price contract allows the integrated firm to control the pure downstream firm's marginal cost and makes firms less aggressive in the product market.

[10]The intuition is that for a given opponent's price or quantity strategy, a downstream firm's output loss (price increase, respectively) following an increase in its own input price is smaller (larger, respectively) under a price contract than under a quantity contract. These output reducing and price increasing effects imply that choosing a price contract is the dominant strategy for both downstream firms.

4 Welfare analysis

We now discuss which mode of competition is the most efficient in terms of social welfare and consumer surplus and which mode of competition is the most profitable in terms of industry profit. We calculate consumer surplus under all the Q-Q, P-P, P-Q, and Q-P games respectively as:

$$\begin{aligned} CS^{QQ} &= \frac{a^2\left(8-4\beta-3\beta^2\right)}{8\left(4-3\beta^2\right)}, \\ CS^{PP} &= \frac{a^2\left(8+4\beta+5\beta^2+\beta^3\right)}{8(1+\beta)\left(4+5\beta^2\right)}, \\ CS^{PQ} &= \frac{a^2\left(8-4\beta-3\beta^2\right)}{8\left(4-3\beta^2\right)}, \\ CS^{QP} &= \frac{a^2\left(8-4\beta-7\beta^2+4\beta^3\right)}{8\left(4-3\beta^2\right)}. \end{aligned} \tag{14}$$

We compare consumer surplus under all these games by:

$$\begin{aligned} CS^{QQ}-CS^{PQ} &= 0, \\ CS^{QQ}-CS^{QP} &= \frac{a^2\beta^2(1-\beta)}{2\left(4-3\beta^2\right)} > 0, \\ CS^{QQ}-CS^{PP} &= \frac{a^2\beta^2(1-\beta)(2+\beta)(2+3\beta)}{2(1+\beta)\left(4-3\beta^2\right)\left(4+5\beta^2\right)} > 0, \\ CS^{QP}-CS^{PP} &= \frac{a^2\beta^3(1-\beta)\left(4-2\beta-5\beta^2\right)}{2(1+\beta)\left(4-3\beta^2\right)\left(4+5\beta^2\right)} > 0, \quad \text{for } \beta < 0.717. \end{aligned} \tag{15}$$

We calculate industry profits under all the Q-Q, P-P, P-Q, and Q-P games as:

$$\begin{aligned} \pi^{QQ} &= \frac{a^2\left(8-8\beta+\beta^2\right)}{4\left(4-3\beta^2\right)}, \\ \pi^{PP} &= \frac{a^2\left(8+9\beta^2+\beta^3\right)}{4(1+\beta)\left(4+5\beta^2\right)}, \\ \pi^{PO} &= \frac{a^2\left(8-8\beta+\beta^2\right)}{4\left(4-3\beta^2\right)}, \\ \pi^{QP} &= \frac{a^2\left(8-8\beta+\beta^2\right)}{4\left(4-3\beta^2\right)}. \end{aligned} \tag{16}$$

We compare industry profits under all these games by:

$$\begin{aligned}
\pi^{PP} - \pi^{QQ} &= \frac{2a^2\beta^4(1-\beta)}{(1+\beta)\,(4-3\beta^2)\,(4+5\beta^2)} > 0,\\
\pi^{QQ} - \pi^{PQ} &= 0,\\
\pi^{PQ} - \pi^{QP} &= 0.
\end{aligned} \tag{17}$$

We calculate social welfare under all the Q-Q, P-P, P-Q, and Q-P games as:

$$\begin{aligned}
SW^{QQ} &= \frac{a^2\left(24-20\beta-\beta^2\right)}{8\,(4-3\beta^2)},\\
SW^{PP} &= \frac{a^2\left(24+4\beta+23\beta^2+3\beta^3\right)}{8(1+\beta)\,(4+5\beta^2)},\\
SW^{PQ} &= \frac{a^2\left(24-20\beta-\beta^2\right)}{8\,(4-3\beta^2)},\\
SW^{QP} &= \frac{a^2\left(24-20\beta-5\beta^2+4\beta^3\right)}{8\,(4-3\beta^2)}.
\end{aligned} \tag{18}$$

We compare social welfare under all these games by:

$$\begin{aligned}
SW^{QQ} - SW^{PQ} &= 0,\\
SW^{QQ} - SW^{QP} &= \frac{a^2\beta^2(1-\beta)}{2\,(4-3\beta^2)} > 0,\\
SW^{QQ} - SW^{PP} &= \frac{a^2\beta^2(1-\beta)\left(4+8\beta-\beta^2\right)}{2(1+\beta)\,(4-3\beta^2)\,(4+5\beta^2)} > 0,\\
SW^{QP} - SW^{PP} &= \frac{a^2\beta^3(1-\beta)\left(4-6\beta-5\beta^2\right)}{2(1+\beta)\,(4-3\beta^2)\,(4+5\beta^2)} > 0, \quad \text{for } \beta < 0.477.
\end{aligned} \tag{19}$$

We summarize the findings in the following Proposition 2.

Proposition 2. (1) The rankings of social welfare are $SW^{QQ} = SW^{PQ} > SW^{QP} > SW^{PP}$ for $\beta < 0.477$ and $SW^{QQ} = SW^{PQ} > SW^{PP} > SW^{QP}$ for $\beta > 0.477$.

(2) The rankings of consumer surplus are $CS^{QQ} = CS^{PQ} > CS^{QP} > CS^{PP}$ for $\beta < 0.717$ and $CS^{QQ} = CS^{PQ} > CS^{PP} > CS^{QP}$ for $\beta > 0.717$.

(3) The rankings of industry profits are $\pi^{PP} > \pi^{QQ} = \pi^{PQ} = \pi^{QP}$.

Including a positive unit price w under Bertrand competition, (P, P), pushes both firms to be less aggressive in the product market, implying Bertrand competition maximizes industry profit. For a given downstream firm's quantity contract, the integrated firm's incentive to choose a higher price or a lower quantity is absent, because the downstream firm's demand is not affected by the price or quantity decision taken up by the integrated firm. Consumer surplus and social welfare are thus both higher whenever the downstream firm chooses a quantity contract versus whenever it chooses a price contract (i.e., $CS^{PQ} > CS^{PP}$, $CS^{QQ} > CS^{QP}$, $SW^{PQ} > SW^{PP}$, and $SW^{QQ} > SW^{QP}$). Lemma 1 implies that both consumer surplus and social welfare are the highest when the downstream firm chooses a quantity contract, whether the integrated firm chooses a quantity contract or a price contract (i.e., $CS^{QQ} = CS^{PQ}$ and $SW^{QQ} = SW^{PQ}$). Combining the results of Propositions 1 and 2 yields the following Corollary 1.[11]

Corollary 1. (1) The equilibrium outcome (P, P) maximizes industry profit, whereas there is a market failure in the choices of the type of strategic variables.

(2) The equilibrium outcome (Q, Q) maximizes social welfare, whereas there is a prisoners' dilemma in the choices of the type of strategic variables.[12]

Singh and Vives (1984) show in the pure final-product market that firms choose quantity contracts in equilibrium, whereas there is a market failure in the choices of the type of strategic variables. Compared to the results of Singh and Vives (1984), we conclude that Cournot competition, which maximizes social welfare, can be sustained as an equilibrium outcome in a vertically-related market if there is an integrated firm and when the firms bargain over a two-part tariff input pricing contract. On the other hand, one policy implication of Lemma 2 and Proposition 2 is that the most efficient competition mode will be achieved if the downstream firm is forced, by a

[11] A comparison of firms' profits at the Bertrand equilibrium and Cournot equilibrium yields $\pi_1^{PP} - \pi_1^{QQ} = [2a^2\alpha\beta^4(1-\beta)]/[(1+\beta)(4+5\beta^2)(4-3\beta^2)] > 0$ and $\pi_2^{PP} - \pi_2^{QQ} = [2a^2\beta^4(1-\alpha)(1-\beta)]/[(1+\beta)(4+5\beta^2)(4-3\beta^2)] > 0$, implying Bertrand equilibrium Pareto dominates Cournot equilibrium.

[12] A market failure in the choices of the type of strategic variables refers to the situation in which the equilibrium mode of competition is not the most efficient mode of competition in terms of social welfare. On the other hand, a prisoners' dilemma in the choices of the type of strategic variables corresponds to the situation in which the equilibrium mode of competition is not the most profitable mode of competition in terms of industry profit.

social planner or government, to initiate a quantity contract (Q) with its customers.

5 Extensions

The previous analysis takes the market structure of vertical integration as given. This section extends the previous basic model in two main directions in order to make the analysis as general as possible. The first considers the market structure of vertical separation, and the second looks at an endogenous market structure.[13]

5.1 Vertical separation

Up until now, we have adopted a model of a vertically-related market with vertical integration. It is possible that vertical integration is an equilibrium outcome from vertical separation to the endogenous market structure game. To investigate this case of vertical separation, we simply assume a model where two downstream firms, denoted by D_i, for $i = 1, 2$, produce differentiated products. The downstream firm D_i purchases a critical input for production from an upstream firm U, through a two-part tariff contract involving a fixed fee, f_i, and a per-unit price, w_i, for $i = 1, 2$. The upstream firm U produces the input at a constant marginal cost of production, which is assumed to be zero.

The profit functions of U and D_i are respectively:

$$\begin{aligned} \bar{\pi}_U &= \sum_{i=1}^{2} (w_i q_i + f_i), \\ \bar{\pi}_{Di} &= (p_i - w_i) q_i - f_i, \end{aligned} \tag{20}$$

where $i = 1, 2$. We consider the following game. In the first stage, each D_i simultaneously chooses whether to adopt a quantity contract (Q) or a price contract (P). In the second stage, U involves itself in centralized or decentralized Nash bargaining with D_i to determine the terms of the two-part tariff contract, f_i and w_i.[14] In the third stage, each D_i simultaneously

[13] We thank an anonymous referee for the suggestion of the extensions.

[14] If an upstream firm bargains with multiple competing downstream firms, then it may involve itself in centralized or decentralized Nash bargaining with downstream firms. We

chooses either price p_i or quantity q_i contingent upon the decisions made in the first stage.

We first denote the difference between centralized Nash bargaining and decentralized Nash bargaining. A critical assumption of decentralized Nash bargaining is that U bargains with the downstream firms simultaneously and separately. We assume in the second stage that the upstream firm U and each downstream firm D_i determine w_i and f_i by maximizing the following decentralized Nash bargaining expression:

$$\max_{w_i,\, f_i} \left[\sum_{i=1}^{2} (w_i q_i + f_i) - \bar{d}\right]^{\alpha} [(p_i - w_i)\, q_i - f_i]^{1-\alpha}, \qquad (21)$$

where $\bar{d} = w_j(a - w_j)/2 + f_j$, for $i, j = 1, 2$ and $i \neq j$, is U's disagreement profit. In this case, if U's negotiation with D_i breaks down, then D_j acts as a monopolist in the downstream market. This implies during the negotiations between U and D_i that each one takes as given the outcome of the simultaneous negotiations between U and D_j.

One critical assumption of the centralized Nash bargaining is that contracts are public and each downstream firm observes both its own contract terms and the terms of its opponent before deciding to accept them. We assume in the second stage that w_i and f_i are determined by the following centralized Nash bargaining expression:

$$\max_{w_i,\, f_i} \left[\sum_{i=1}^{2} (w_i q_i + f_i)\right]^{\alpha} \left[\sum_{i=1}^{2} (p_i - w_i)\, q_i - f_i\right]^{1-\alpha}, \qquad (22)$$

where $i = 1, 2$. Upstream firm U's disagreement profit is zero in the first term of equation (22), because U bargains with a representative of D_1 and D_2 over the two-part tariff contract. If U's negotiation with a representative of D_1 and D_2 breaks down, then both D_1 and D_2 cannot survive in the downstream market.

Basak and Wang's (2016) show that if the two-part tariff pricing contract is determined through centralized bargaining, then Bertrand competition (P, P) is the unique equilibrium outcome. Appendix A presents in the case of decentralized Nash bargaining that each firm choosing a quantity strategy is a dominant strategy.

---

note that the upstream firm bargains with the two downstream firms in the model of a vertically-related market with vertical separation.

Proposition 3. Given a vertically-related market with vertical separation, Cournot competition (Q, Q) is the unique equilibrium outcome when the two-part tariff pricing contract is determined through decentralized bargaining.

The intuition behind Proposition 3 is as follows. The input price is higher under Bertrand competition than that under Cournot competition, which is even lower than the upstream firm U's marginal cost. This implies that the opportunistic behavior of the upstream firm is significant under Cournot competition. Each downstream firm's marginal cost is lower and obtains a larger profit under Cournot competition than under Bertrand competition. Therefore, choosing a quantity contract is a dominant strategy for both downstream firms.

Alipranti, Milliou, and Petrakis' (2014) compare Cournot and Bertrand equilibria in a vertically-related market with vertical separation, showing that the equilibrium downstream profits are higher under Cournot than under Bertrand competition when the two-part tariff pricing contract is determined through decentralized bargaining. We further show that both downstream firms choosing a quantity strategy (Q, Q) are the unique equilibrium outcome when the two downstream firms' types of strategic variables are endogenously determined. Proposition 3 further implies that Basak and Wang's (2016) assumption of centralized Nash bargaining also plays a crucial role in determining their unique result of Bertrand competition in a vertically-related market with vertical separation.

5.2 Endogenous market structure

We now demonstrate that the market structure in the basic model can arise naturally as the equilibrium outcome of a simple game. We consider a four-stage game as follows. In the first stage, both D_i announce simultaneously the amount they are willing to pay to merge with U, and then U decides whether to choose a vertical merger or vertical separation. In the second stage, both D_i choose whether to adopt a price contract or a quantity contract. In the third stage, U is involved in centralized or decentralized Nash bargaining with D_i to determine the terms of the two-part tariff contract, f_i and w_i. In the fourth stage, both downstream firms simultaneously choose either price or quantity contingent upon the decisions made in the second stage.

In the game's first stage, U announces that it will merge with D_i by using a first-price auction with minimum bid ($\bar{\pi}_U$), which is the profit the upstream firm can secure under vertical separation. Each D_i submits a bid it is willing to pay to merge with U. If at least one of the bids exceeds the minimum $\bar{\pi}_U$, then the high bidder wins the rights to the merger at the bidding price. If the bids are equal and exceed $\bar{\pi}_U$, then U will merge at random with one of the two downstream firms, D_1 and D_2. If neither bid exceeds $\bar{\pi}_U$, then no merger takes place, and vertical separation prevails.

We turn to the game's first stage to determine the endogenous market structure. We note that both Bertrand competition (P, P) and Cournot competition (Q, Q) can be sustained as the equilibrium outcomes in our baseline model. We first consider the case where Cournot competition is the equilibrium outcome in a vertically-related market with vertical integration.

For the case of decentralized Nash bargaining under vertical separation, we show that D_1 will merge with U if, in the absence of their merger, U does not merge with D_2. The profit of the merged firm in a vertically-related market with vertical integration is π_1 in equation (4). If U merges with neither D_1 nor D_2, then the profits of U and D_1 are $\bar{\pi}_U$ and $\bar{\pi}_{D1}$ in equation (20), respectively, in a vertically-related market with vertical separation. The incremental joint profit that U and D_1 secure from a merger in this case is:

$$\begin{aligned}&\pi_1 - (\bar{\pi}_U + \bar{\pi}_{D1}) \\ &\quad = \frac{a^2\beta\left[\begin{array}{l}(1-\alpha)\left(32 - 24\beta - 24\beta^2 + 22\beta^3 - 3\beta^5\right) \\ +2\alpha\beta\left(4 - 2\beta^2 - 2\beta^3 + \beta^4\right)\end{array}\right]}{8\left(4 - 3\beta^2\right)\left(2 - \beta^2\right)^2} > 0, \qquad (23)\end{aligned}$$

which is unambiguously positive. Therefore, D_1 will merge with U.

We now show that D_1 will also merge with U if, in the absence of their merger, U merges with D_2. If D_1 merges with U, then their combined profit is π_1 in equation (4). If D_2 merges with U, then D_1's profit is as given by π_2 in equation (4), which is denoted by $\tilde{\pi}_1$. These expressions indicate that D_1's maximum bid is:

$$\pi_1 - \tilde{\pi}_1 = \frac{a^2\left[\beta(1-\alpha)(8-7\beta) + \alpha\beta^2 + 8\alpha(1-\beta)\right]}{4\left(4 - 3\beta^2\right)} > 0. \qquad (24)$$

By similar argument, D_2's maximum bid is the same as equation (24). Hence, U will at random merge with either of the two downstream firms.

We next consider the case of centralized bargaining under vertical separation and obtain the following:

$$\pi_1 - (\bar{\pi}_U + \bar{\pi}_{D1}) = \frac{a^2\beta\left[(1-\alpha)\left(4-3\beta^2\right)-\alpha\beta(1-\beta)\right]}{8(1+\beta)\left(4+5\beta^2\right)} > 0, \quad \text{for}$$
$$\alpha < \frac{4-3\beta^2}{4+\beta-4\beta^2} \equiv \bar{\alpha},$$
$$\pi_1 - \tilde{\pi}_1 = \frac{a^2\left[\beta(1-\alpha)(8-7\beta)+\alpha\beta^2+8\alpha(1-\beta)\right]}{4\left(4-3\beta^2\right)} > 0. \tag{25}$$

We conclude that one of the two downstream firms will merge with U for $\alpha < \bar{\alpha}$, and neither downstream firm will merge with U for $\alpha > \bar{\alpha}$. Appendix B shows that given Bertrand competition is the equilibrium outcome in a vertically-related market with vertical integration, the results are similar to the previous analysis of Cournot competition. We summarize the results in the following Proposition 4.

Proposition 4. (1) Allowing decentralized Nash bargaining under vertical separation, one of the two downstream firms merges with the upstream firm, and the vertical merger is the unique equilibrium outcome.

(2) Allowing centralized Nash bargaining under vertical separation, the vertical merger is the unique equilibrium outcome when the upstream firm's bargaining power α is relatively small. Vertical separation is the unique equilibrium outcome when the upstream firm's bargaining power α is relatively large.

The intuition behind Proposition 4 is as follows. As explained by Proposition 3, in the case of decentralized Nash bargaining the input price is lower than the upstream firm U's marginal cost under Cournot competition, which results in lower profit for upstream firm U. In this sense, the sum of the profits of U and D_i is lower than a merged firm's profit and the vertical merger is the unique equilibrium outcome in the case of decentralized Nash bargaining.

We next discuss the case of centralized Nash bargaining. When the upstream firm's bargaining power, α, is relatively large, U has more incentive to behave opportunistically, which means that the sum of the profits of U and D_i is higher than a merged firm's profit and that vertical separation is the unique equilibrium outcome. In contrast, when the upstream firm's

bargaining power, α, is relatively small, U's opportunistic behavior is less pronounced and the vertical merger is the unique equilibrium outcome.

The baseline model arises naturally as the equilibrium outcome of a simple game that admits vertical mergers. We conclude that vertical integration will arise in equilibrium since it increases the joint profit of the merging firms. The equilibrium vertical merger between the upstream firm U and a downstream firm D_i in the present setting helps to justify our focus on the previous baseline model, where an integrated firm competes with a pure downstream firm.

6 Conclusion

From the viewpoint of firms' mode of competition, quantity competition and price competition are the two classical models in the literature of industrial organization. It is often exogenously assumed that all firms within the same industry engage in either quantity or price competition. We observe in the real world that an integrated firm competes with downstream firms in many industry sectors. While there are also some commonly observed facts for firms' type of strategic variables in vertically-related markets, differences do exist in the types of strategic variables among firms in the same industry as well as across industry sectors.

This paper investigates the endogenous choice of prices versus quantities by taking into account a vertically-related market where an integrated firm competes with a downstream firm and they bargain over a two-part tariff input pricing contract. We also investigate whether or not there is a prisoners' dilemma or a market failure in the choices of the type of strategic variables.

Contrary to the standard result, we show that both Bertrand competition and also Cournot competition can be sustained as equilibrium outcomes. The Bertrand equilibrium maximizes industry profit, but there is a market failure in the choices of the type of strategic variables. Central to our finding is that Cournot competition, which maximizes social welfare, can also be sustained as an equilibrium outcome in a vertically-related market if there is an integrated firm and when the firms bargain over a two-part tariff input pricing contract. The paper ends with an extension, showing that our baseline model arises naturally as the equilibrium outcome of a simple game that admits vertical mergers.

Appendix A

Alipranti, Milliou, and Petrakis' (2014) consider Cournot competition and Bertrand competition under vertical separation and decentralized Nash bargaining. Following Alipranti, Milliou, and Petrakis' (2014), we illustrate the equilibrium unit prices, fixed fees, and profits in the Q-Q game as follows:

$$\begin{aligned}
w_1^{QQ} &= w_2^{QQ} = \frac{-a\beta^2}{2\left(2-\beta^2\right)},\\
f_1^{QQ} &= f_2^{QQ} = \frac{a^2(2-\beta)^2\left(2\alpha-\alpha\beta^2+\beta^2\right)}{8\left(2-\beta^2\right)^2},\\
\bar{\pi}_{D1}^{QQ} &= \bar{\pi}_{D2}^{QQ} = \frac{a^2(1-\alpha)(2-\beta)^2}{8(2-\beta^2)},\\
\bar{\pi}_U^{QQ} &= \frac{a^2(2-\beta)\left[\alpha(2-\beta)\left(2-\beta^2\right)-\beta^3\right]}{4\left(2-\beta^2\right)^2}.
\end{aligned} \tag{A1}$$

From Alipranti, Milliou, and Petrakis' (2014), we illustrate the equilibrium unit prices, fixed fees, and profits in the P-P game as:

$$\begin{aligned}
w_1^{PP} &= w_2^{PP} = \frac{a\beta^2}{4},\\
f_1^{PP} &= f_2^{PP} = \frac{a^2(2+\beta)\left[4\alpha-2\beta(\alpha+\beta)+\beta^3(1-\alpha)(1-\beta)\right]}{32(1+\beta)},\\
\bar{\pi}_{D1}^{PP} &= \bar{\pi}_{D2}^{PP} = \frac{a^2(1-\alpha)(2+\beta)\left(4-2\beta-\beta^3+\beta^4\right)}{32(1+\beta)},\\
\bar{\pi}_U^{PP} &= \frac{a^2(2-\beta)\left[2\alpha(2-\beta)+\beta^3(1-\alpha)(1-\beta)\right]}{4\left(2-\beta^2\right)^2}.
\end{aligned} \tag{A2}$$

We now investigate the Q-P and P-Q games in the model of Alipranti, Milliou, and Petrakis' (2014). With straightforward derivations, we calculate the equilibrium unit prices, fixed fees, and profits in the Q-P game as:

$$\begin{aligned}
w_1^{QP} &= \frac{a\beta^2}{4(1+\beta)},\ w_2^{QP} = \frac{-a\beta^2}{2\left(2+2\beta-\beta^2-\beta^3\right)},\\
f_1^{QP} &= \frac{a^2(2-\beta)^2\left(2\alpha-\alpha\beta^2-\beta^2\right)}{8(2-\beta^2)^2},
\end{aligned}$$

$$
\begin{aligned}
f_2^{QP} &= \Big\{a^2\left[4(1+\beta)\left(4\alpha-3\alpha\beta^2+2\beta^2\right)-2\beta^3(2\alpha+\beta)\right.\\
&\quad\left.-\beta^5(1-\alpha)(4-\beta)\right]/32\left(2-\beta^2\right)(1+\beta)^2\Big\},\\
\bar{\pi}_{D1}^{QP} &= \frac{a^2(1-\alpha)(2-\beta)^2}{8(2-\beta^2)},\\
\bar{\pi}_{D2}^{QP} &= \frac{a^2(1-\alpha)\left(16+16\beta-12\beta^2-16\beta^3+4\beta^5-\beta^6\right)}{32(1+\beta)^2\left(2-\beta^2\right)},\\
\bar{\pi}_{U}^{QP} &= \Big\{a^2\left[4(1+\beta)\left(16\alpha-20\alpha\beta^2+8\alpha\beta^4-2\beta^4-\alpha\beta^6+\beta^6\right)\right.\\
&\quad\left.-\beta^8(1-\alpha)-2\alpha\beta^6\right]/32(1+\beta)^2\left(2-\beta^2\right)^2\Big\}. \qquad \text{(A3)}
\end{aligned}
$$

We calculate the equilibrium unit prices, fixed fees, and profits in the P-Q game as:

$$
\begin{aligned}
w_1^{PQ} &= \frac{-a\beta^2}{2\left(2+2\beta-\beta^2-\beta^3\right)},\ w_2^{PQ}=\frac{a\beta^2}{4(1+\beta)},\\
f_1^{PQ} &= \Big\{a^2\left[4(1+\beta)\left(4\alpha-3\alpha\beta^2+2\beta^2\right)-2\beta^3(2\alpha+\beta)\right.\\
&\quad\left.-\beta^5(1-\alpha)(4-\beta)\right]/32(2-\beta^2)(1+\beta)^2\Big\},\\
f_2^{PQ} &= \frac{a^2(2-\beta)^2\left(2\alpha-\alpha\beta^2-\beta^2\right)}{8\left(2-\beta^2\right)^2},\\
\bar{\pi}_{D1}^{PQ} &= \frac{a^2(1-\alpha)\left(16+16\beta-12\beta^2-16\beta^3+4\beta^5-\beta^6\right)}{32(1+\beta)^2\left(2-\beta^2\right)},\\
\bar{\pi}_{D2}^{PQ} &= \frac{a^2(1-\alpha)(2-\beta)^2}{8\left(2-\beta^2\right)},\\
\bar{\pi}_{U}^{PQ} &= \Big\{a^2\left[4(1+\beta)\left(16\alpha-20\alpha\beta^2+8\alpha\beta^4-2\beta^4-\alpha\beta^6+\beta^6\right)\right.\\
&\quad\left.-\beta^8(1-\alpha)-2\alpha\beta^6\right]/32(1+\beta)^2\left(2-\beta^2\right)^2\Big\}. \qquad \text{(A4)}
\end{aligned}
$$

We now discuss the choices of the type of strategic variables in the first stage under decentralized Nash bargaining:

$$\bar{\pi}_{D1}^{QP}-\bar{\pi}_{D1}^{PP}=\bar{\pi}_{D2}^{PQ}-\bar{\pi}_{D2}^{PP}$$

$$= \frac{a^2\beta^3(1-\alpha)\left(8-4\beta-4\beta^2+\beta^3+\beta^4\right)}{32(1+\beta)\left(2-\beta^2\right)} > 0,$$

$$\bar{\pi}_{D1}^{QQ} - \bar{\pi}_{D1}^{PQ} = \bar{\pi}_{D2}^{QQ} - \bar{\pi}_{D2}^{QP}$$

$$= \frac{a^2\beta^3(1-\alpha)\left(8+4\beta-4\beta^2+\beta^3\right)}{32(1+\beta)^2\left(2-\beta^2\right)} > 0. \tag{A5}$$

It follows that each firm choosing a quantity strategy is a dominant strategy, and thus choosing a quantity strategy for both downstream firms is the unique equilibrium outcome. Q.E.D.

Appendix B

We now consider the case where Bertrand competition is the equilibrium outcome in a vertically-related market with vertical integration. We first analyze the model of decentralized Nash bargaining under vertical separation. By similar argument of the case of Cournot competition, we obtain the following:

$$\pi_1 - (\bar{\pi}_U + \bar{\pi}_{D1}) =$$

$$\frac{a^2\beta\left[\begin{array}{l}(1-\alpha)\left(32+8\beta^2-18\beta^4+5\beta^5+3\beta^6\right)+8\beta(1+\beta)\\ +8\beta^3(1-\beta)+2\beta^3(3+7\alpha)-2\beta^5(\alpha-\beta)\end{array}\right]}{8(1+\beta)\left(4+5\beta^2\right)\left(2-\beta^2\right)^2} > 0,$$

$$\pi_1 - \tilde{\pi}_1 =$$

$$\frac{a^2\left[\beta^2(1+\beta)+8\left(1+\beta^2\right)(\alpha+\beta-\alpha\beta)\right]}{4(1+\beta)\left(4+5\beta^2\right)} > 0, \tag{A6}$$

which are both unambiguously positive, implying that a vertical merger is the equilibrium outcome.

We next consider the case of centralized Nash bargaining under vertical separation and obtain the following:

$$\pi_1 - (\bar{\pi}_U + \bar{\pi}_{D1}) = \frac{a^2\beta\left[4(1-\alpha)\left(1+\beta^2\right)-\alpha\beta+\beta^2\right]}{4(1+\beta)\left(4+5\beta^2\right)} > 0, \text{ for}$$

$$\alpha < \frac{4+5\beta^2}{4+\beta+4\beta^2} \equiv \bar{\bar{\alpha}},$$

$$\pi_1 - \tilde{\pi}_1 = \frac{a^2\left[\beta^2(1+\beta)+8\left(1+\beta^2\right)(\alpha+\beta-\alpha\beta)\right]}{4(1+\beta)\left(4+5\beta^2\right)} > 0. \tag{A7}$$

We conclude that a vertical merger is the unique equilibrium outcome for $\alpha < \bar{\bar{\alpha}}$ and vertical separation is the unique equilibrium outcome for $\alpha > \bar{\bar{\alpha}}$. Q.E.D.

References

Alipranti, Maria, Chrysovalantou Milliou, and Emmanuel Petrakis' (2014), "Price vs. Quantity Competition in a Vertically Related Market," *Economic Letters*, 124(1), 122–126.

Arya, Anil, Brian Mittendorf, and David E. M. Sappington's (2008), "Outsourcing, Vertical Integration, and Price vs. Quantity Competition," *International Journal of Industrial Organization*, 26(1), 1–16.

Basak, Debasmita (2017), "Cournot vs. Bertrand Under Centralised Bargaining," *Economics Letters*, 154, 124–127.

Basak, Debasmita and Arijit Mukherjee (2017), "Price vs. Quantity Competition in a Vertically Related Market Revisited," *Economics Letters*, 153, 12–14.

Basak, Debasmita and Leonard F. S. Wang's (2016), "Endogenous Choice of Price or Quantity Contract and the Implications of Two-Part-Tariff in a Vertical Structure," *Economics Letters*, 138, 53–56.

Din, Hong-Ren and Chia-Hung Sun (2018), "Endogenous Timing in Vertically-Related Markets," *The B. E. Journal of Theoretical Economics*, 18(20 160103, ISSN (Online)), 1935–1704, **URL**: `http://doi.org/10.1515/bejte-2016-0103`.

Dixit, Avinash (1979), "A Model of Duopoly Suggesting a Theory of Entry Barriers," *The Bell Journal of Economics*, 10(1), 20–32.

Fanti, Luciano and Marcella Scrimitore (2019), "How to Compete? Cournot Versus Bertrand in a Vertical Structure with an Integrated Input Supplier," *Southern Economic Journal*, 85(3), 796–820.

Monteverde, Kirk and David J. Teece (1982), "Supplier Switching Costs and Vertical Integration in the Automobile Industry," *The Bell Journal of Economics*, 13(1), 206–213.

Ouchi, William (1981), *Theory Z. How American Business Can Meet the Japanese Challenge*, Reading, MA: Addison-Wesley.

Rozanova, Olga (2015), "Price vs. Quantity Competition in Vertically Related Markets. Generalization," *Economics Letters*, 135, 92–95.

Singh, Nirvikar and Xavier Vives (1984), "Price and Quantity Competition in a Differentiated Duopoly," *Rand Journal of Economics*, 15(4), 546–554.

Tremblay, Carol Horton and Victor J. Tremblay (2011), "The Cournot-Bertrand Model and the Degree of Product Differentiation," *Economics Letters*, 111(3), 233–235.

Villas-Boas, Sofia Berto (2007), "Vertical Relationships Between Manufacturers and Retailers: Inference With Limited Data," *Review of Economic Studies*, 74(2), 625–652.

投稿日期: 2020年6月2日, 接受日期: 2021年2月11日

內生化價格或數量選擇下垂直相關市場的福利分析

丁虹仁
東吳大學經濟學系
孫嘉宏
東吳大學經濟學系

本文在垂直相關市場下, 研究內生的廠商價格或數量選擇。假設在垂直相關市場中存在一家垂直整合廠商與一家獨立下游廠商, 兩家廠商就中間財契約進行二部定價的談判。相異於文獻上的研究發現, 本文證明 Bertrand 競爭與 Cournot 競爭都有可能是此賽局的均衡結果。其中, Bertrand 均衡極大化產業利潤, 然而卻存在策略變數類型選擇上的市場失靈。Cournot 均衡極大化社會福利和消費者剩餘, 然而卻存在策略變數類型選擇上的囚犯困境。我們最後證明, 本文的市場架構, 是允許廠商垂直整合的垂直相關市場下之均衡結果。

關鍵詞: 垂直相關市場, Cournot 競爭, Bertrand 競爭, 內生化策略變數, Nash 談判解
JEL 分類代號: L13, D43, D21