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Public Information and Information Aggregation in Committees

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We investigate how public information affects the voting behavior of the voters and the error (information efficiency) of the equilibrium decision of a committee. The influence of public information on efficiency can be decomposed into the information effect and the equilibrium effect. Public information provides an extra piece of information that helps the voters better match their votes with the received signals, and hence may reduce the error. However, it also affects the equilibrium behavior of the strategic voters, and this may increase the error. We show that for any non-unanimous voting rule, when the precision of public information is marginally higher than that of private information, the information effect is negligible, while the equilibrium effect has negative impact on efficiency. Hence, public information with low precision may make the decision of the committee worse. By contrast, for the unanimous rule, the equilibrium effect has positive impact and public information will always improve the efficiency. Furthermore, we find that the existence of non-strategic voters may cause asymptotic inefficiency when the voting rule is too stringent in the sense that it is far away from the simple majority rule. Public information can *de facto* alter the voting rule to be less stringent through its influence on the votes of the non-strategic voters. Thus, the efficiency can be improved.

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1 Introduction

What is the effect of public information on collective decision-making? In a recruiting committee, a report by an outside reviewer provides some useful information that helps committee members decide whether or not a job candidate is qualified. However, the reviewer is too influential in the sense that his/her opinion affects the beliefs of all members and may have a disproportionate influence on the outcome of the decision. In an election, the position of mass media or opinion leaders affects the voting behaviors of many voters and may have significant impacts on the election. In a trial jury, the appearance of new evidence changes the jurors' assessment regarding whether a defendant is guilty and the verdict may hence be turned around. In this paper, we investigate the effects of public information on the voting behaviors of the voters and the information efficiency of the equilibrium decision of a committee.

We consider a game consisting of n voters (jurors) who must decide whether to acquit or convict a defendant. Every voter can observe a private and a public signal (if public information is available) before casting her vote. When the number of votes for a conviction is greater or equal to k, which denotes the voting rule, the defendant is convicted, otherwise he is acquitted. In the voting game, we first analyze the equilibria in which the voters either vote sincerely, i.e., their votes replicate the private signals, or vote uninformatively, i.e., always vote to convict or acquit. These kinds of equilibria are characterized by the number of uninformative votes e.[1] We find that the number e is proportional to the difference between the voting rule k and the statistical rule k^*.[2] When the difference $|k-k^*|$ is larger, there are more uninformative votes in equilibrium. Public information affects the equilibrium strategy of voters through its influence on k^*. We further examine the relation between the precision of the public information and the statistical rule k^*, and solve for the equilibrium of the voting game with public information.

Second, we investigate whether public information can improve the information efficiency by comparing the equilibrium errors with and without

[1] This is the most efficient monotonic equilibrium studied by McLennan (1998), Dekel and Piccione (2000), and Persico (2004).

[2] The statistical rule is the rule that a statistician will choose if she can observe all private signals before making a decision. It corresponds to the most efficient test of a sample size n.

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public information. The influence of public information on the efficiency can be decomposed into the information effect and the equilibrium effect. Public information provides an extra piece of information that helps the voters better match their votes with the received signals, and may reduce the error. However, it also changes the equilibrium number of uninformative votes e, and this may increase the error. We show that for any non-unanimous voting rule, when the precision of public information is marginally higher than that of private information, the information effect is negligible, while the equilibrium effect is not. Hence, public information with low precision may make the decision of the jury worse. By contrast, for the unanimous rule, the equilibrium effect has positive impacts and public information will always improve the efficiency.

Finally, we find that the existence of non-strategic voters may cause asymptotic inefficiency when the voting rule is too stringent in the sense that it is far away from the simple majority rule. Under these circumstances, public information can de facto alter the voting rule to be less stringent through its influence on the votes of non-strategic voters. This may improve the overall efficiency.

The formal analysis on collective decision problems can at least date back to Condorcet (1743–1794). The Condorcet jury theorem claims that the simple majority rule can achieve asymptotic information efficiency, i.e., the error of the joint decision almost surely approaches zero, when the size of the jury increases to infinity. The result holds under the following simplified presumptions: (1) the voters always vote sincerely, (2) the voters obtain their private information at zero cost, and (3) the voters do not share information before voting. There are many subsequent studies that have tried to relax or modify these presumptions. Austen-Smith and Banks (1996) examine the validity of the presumption that voters always vote sincerely. The authors show that the non-strategic voting behaviors may not be consistent with the concept of a Nash equilibrium. Hence, the Condorcet jury theorem may not hold if voters vote strategically. They identify the condition under which they are consistent and find that the strategy profile that all voters vote sincerely constitutes an equilibrium if and only if the voting rule is identical to the statistical rule, i.e., $k = k^*$. Feddersen and Pesendorfer (1997) and Feddersen and Pesendorfer (1998) further investigate the efficiency property of the symmetric equilibria under different voting rules. The authors show that the unanimous rule is inferior to any other non-unanimous rules in the sense that it is the only rule that is asymptotically inefficient. Persico (2004)

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relaxes the presumption that the cost of information acquisition is zero, and uses a two-stage voting game to investigate the design of an optimal committee. In the first stage, all voters simultaneously decide whether to acquire a noisy signal or to remain uninformed. In the second stage, they cast their votes based on their private signals and the information they infer from being pivotal. The author finds that a voting rule that requires a large plurality (in the extreme, unanimity) to change the status quo can be optimal only if the precision of the private information is sufficiently high. These papers focus on how the private information of the voters is aggregated under different voting rules or different acquisition costs. They do not address the issues of how public information can affect the behaviors of voters and the information efficiency. On the other hand, Ladha (1992) and Berg (1993) generalize the Condorcet jury theorem by allowing for the possibility that the private signals received by the voters may be correlated. This correlation can be induced by public information.[3] The authors use the generalized theorem as an analytical basis for free speech, and show that more diversified public opinions, i.e., less correlation between votes, are better for the information efficiency. These articles study similar issues on public information to ours. However, their analyses are based on the decision models in which all voters are assumed to vote sincerely, and thus ignore the possible strategic interaction between the voters.

The paper is organized as follows. In section 2, we present the model. In section 3, we solve for the equilibrium of the voting game with public information and investigate whether public information can improve the information efficiency. In section 4, we examine the asymptotic results. Section 5 concludes.

2 Model

Consider a jury with n voters (jurors), $j = 1, \cdots, n$, who must decide whether a defendant is innocent or guilty. Let $\omega \in \{G, I\}$ denote the state of the world, where G and I represent the state that the defendant is guilty and innocent, respectively. Voters can be either non-strategic or strategic. A non-strategic voter casts her vote based on the (private or public) signals she receives, while a strategic one casts her vote, based on not only the signals

[3] For example, the beliefs of voters may be influenced by the position of mass media or opinion leaders. This may affect the kind of information they subsequently acquire.

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received but also the information she infers from other voters' votes. We assume that all voters assign the same prior probability of guilty $P(G)$ and innocent $P(I) = 1 - P(G)$ to the states $\omega = G$ and $\omega = I$, respectively.

Voters care about the outcome $D \in \{C, A\}$ of the trial, where C (Conviction) and A (Acquittal) indicate that the jury convicts and acquits the defendant, respectively. The cost of convicting the innocent is $q \in [0, 1]$, the cost of acquitting the guilty is $1 - q$, and taking the right decision gives 0. Therefore, the common utility function of the voters is

$$U(D, \omega) = \begin{cases} 0, & \text{when } (D, \omega) = (A, I) \text{ or } (C, G), \\ -q, & \text{when } (D, \omega) = (C, I), \\ -(1 - q), & \text{when } (D, \omega) = (A, G). \end{cases}$$

Note that the parameter q exactly characterizes a voter's threshold of reasonable doubt. A voter who believes the defendant is guilty with a probability higher than q prefers the defendant to be convicted.

Each voter j can observe a private signal $s_j \in S_j = \{g, i\}$. The conditional probability of the signals is $\Pr(s_j = g|\omega = G) = \Pr(s_j = i|\omega = I) = p$. The parameter $p \in ((1/2), 1)$ is the probability that a voter receives the correct signal. She may also observe a public signal $s_0 \in S_0 = \{g_0, i_0, \emptyset\}$ which changes her prior belief to $\Pr(G|s_0)$.[4] The conditional probability of the public signals is $\Pr(s_0 = g_0|\omega = G) = \Pr(s_0 = i_0|\omega = I) = p_0$. We assume that the public information is more informative than the private information in the sense that its precision is higher, i.e., $p < p_0$. Furthermore, both private and public signals occur with equal probability, i.e., $\Pr(g) = \Pr(i) = \Pr(g_0) = \Pr(i_0) = 1/2$. All signals $S_0, S_1, \cdots, S_n$ are assumed to be conditionally independent of the states.

A voting game of n voters with a voting rule k, denoted by a game of (n, k), is the game in which each voter votes to convict or acquit, and if k or more voters vote to convict, the defendant is convicted. Otherwise, the defendant is acquitted. A strategy of voter j is a function d_j: $\{g, i\} \to [0, 1]$, where $d_j(s_j)$ is the probability of voting to convict for $s_j = g, i$. We say that a voter votes sincerely if the voter's votes replicate her private signals, i.e., she uses the pure strategy: $d_j(g) = 1$ and $d_j(i) = 0$. A voter votes uninformatively if the voter ignores her private information by always voting to convict or acquit. A strategic voter may not want to vote sincerely, and her vote is relevant only when she is pivotal. Hence, she may just vote

[4] The case where $s_0 = \emptyset$ indicates that public information is not absent.

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uninformatively if the information inferred from being pivotal dominates her private information. The timing of the game is as follows:

1. A public signal is realized and the voters update their prior beliefs to $\Pr(G|s_0)$ and $\Pr(I|s_0)$ for $s_0 = g_0$ and i_0, respectively. For the case where public information is absent, i.e., $s_0 = \emptyset$, the prior beliefs are $\Pr(G)$ and $\Pr(I)$.
2. The voters receive their private signals and cast a vote based on their posteriors. The non-strategic voters vote sincerely if public information is absent. Furthermore, since $p_0 > p$, their votes will replicate the public signals if public information is available. The strategic voters cast their votes based on their private signals, the public signals and the information they infer from being pivotal.
3. If k or more voters vote to convict, the defendant is convicted. Otherwise, the defendant is acquitted. Then the voters receive the payoff.

Given a (pure) strategy profile of the voters $d = (d_1, \cdots, d_n)$, the outcome of the voting game is determined by the function $D : [0,1]^n \to \{A, C\}$:

$$D(d) = D(d_1, \cdots, d_n) = \begin{cases} C, & \text{when } \#\{d_j\} \geq k, \\ A, & \text{when } \#\{d_j\} < k, \end{cases}$$

where $\#\{d_j\}$ represents the number of votes for conviction. The common payoff of the voters, denoted by $u_{s_0}(d, (n,k))$, is equal to the expected cost of the type I and type II error. Let $d^s(n)$ denote the sincere strategy profile that all voters vote sincerely.[5] Then the payoff under $d^s(n)$ is

$$\begin{aligned} u_{s_0}\left(d^s, (n,k)\right) &= -q \Pr(I|s_0) \Pr\left(D\left(d^s\right) = C|\omega = I\right) \\ &\quad - (1-q) \Pr(G|s_0) \Pr\left(D\left(d^s\right) = A|\omega = G\right) \\ &= -q \Pr(I|s_0) \sum_{x \geq k} \binom{n}{x} (1-p)^x p^{n-x} \\ &\quad - (1-q) \Pr(G|s_0) \sum_{x < k} \binom{n}{x} (1-p)^{n-x} p^x, \end{aligned}$$

where $\Pr(D(d^s) = C|\omega = I)$ is the probability of convicting the innocent and $\Pr(D(d^s) = A|\omega = G)$ is the probability of acquitting the guilty.

[5] Note that the superscript s stands for sincere.

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we call the rule

$$k^*_{s_0}(n) \in \arg \max_k u_{s_0}\left(d^s, (n,k)\right),$$

the statistical rule under public signal s_0. It corresponds to the most efficient test of a sample size n for a statistician[6] as if he can observe all private signals. The sincere strategy profile $d^s(n)$ and the statistical rule $k^*_{s_0}(n)$ play important roles in the construction of the equilibria of the voting game. Specifically, we shall focus on the analyzes of the most efficient monotonic equilibrium (hereafter, *the equilibrium*) which is the Nash equilibrium satisfying three extra conditions. First, all voters use pure strategies, i.e., $d_j(s_j) = 0$ or 1. Second, the strategies are monotonic in the sense that $d_j(g) \geq d_j(i)$ for all j. Third, the number of uninformative votes is the smallest. When these conditions are imposed, the voters either vote sincerely or vote uninformatively. The existence of such equilibria is discussed in McLennan (1998), Dekel and Piccione (2000), and Persico (2004). This equilibrium is characterized by the smallest non-zero integer of uninformative votes e_{s_0} such that there are e_{s_0} voters who vote uninformatively, while the other $(n - e_{s_0})$ voters vote sincerely. The equilibrium strategy profile is thus denoted by either $(d^s(n-e_{s_0}), 1(e_{s_0}))$ or $(d^s(n-e_{s_0}), 0(e_{s_0}))$, where $1(e_{s_0})(0(e_{s_0}))$ is the vector of 1 (0) with an e_{s_0} dimension that represents e_{s_0} uninformative votes for conviction (acquittal). Given an equilibrium $d^{**} = (d^s(n-e_{s_0}), 1(e_{s_0}))$ or $(d^s(n-e_{s_0}), 0(e_{s_0}))$, we define the error $\mathrm{Err}_{s_0}(n,k)$ to be the negative of the common equilibrium payoff, i.e., the expected cost of making incorrect decisions

$$\mathrm{Err}_{s_0}(n,k) = -u_{s_0}\left(d^{**}, (n,k)\right).$$

Note that for $d^{**} = (d^s(n-e_{s_0}), 1(e_{s_0}))$, the error $\mathrm{Err}_{s_0}(n,k)$ is equal to the error of the game of (a smaller) size $(n - e_{s_0})$ with the voting rule $(k - e_{s_0})$, in which all $(n - e_{s_0})$ voters vote sincerely. Therefore,

$$\begin{aligned}\mathrm{Err}_{s_0}(n,k) &= \mathrm{Err}_{s_0}\left(n - e_{s_0}, k - e_{s_0}\right) \\ &= q \Pr(I|s_0) \sum_{x \geq (k-e_{s_0})}^{n-e_{s_0}} \binom{n-e_{s_0}}{x} (1-p)^x p^{n-x}\end{aligned}$$

[6]That is, convicting the defendant if and only if the total number of the guilty signals is greater or equal to $k^*_{s_0}(n)$.

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$$+ (1-q)\Pr(G|s_0) \sum_{x<(k-e_{s_0})} \binom{n-e_{s_0}}{x} (1-p)^{n-x} p^x.$$

The error for $d^{**} = (d^s(n-e_{s_0}), 0(e_{s_0}))$ is similar except that $(k-e_{s_0})$ is replaced by k.[7]

In the next section, we shall investigate the relation between the number of uninformative votes e_{s_0} and the statistical rule $k^*_{s_0}(n)$, and how the precision of public information p_0 affects $k^*_{s_0}(n)$. Then we will solve for the equilibrium of the voting game with public information and examine whether public information can help the jury make a better decision.

3 Equilibrium with Public Information

Throughout this section, we assume that all voters are strategic. We shall solve for the equilibrium of the voting game with public information and investigate the effect of public information on the information efficiency, i.e., whether it increases or decreases the expected error of the decision made by the jury. Let $\mathrm{Err}^P(n,k)$ ($\mathrm{Err}(n,k)$) denote the equilibrium error of the voting game with (without) public information. Thus, we shall compare $\mathrm{Err}^P(n,k) = \Pr(g_0)\mathrm{Err}_{g_0}(n,k) + \Pr(i_0)\mathrm{Err}_{i_0}(n,k)$ with $\mathrm{Err}(n,k)$.

Consider a game of (n,k) where a public signal s_0 is realized. Suppose that all other voters, except for voter j, vote sincerely. Let $Piv_j(d^s_{-j}, (n,k))$ denote the event that voter j is pivotal. It is straightforward to compute the posterior belief of guilty for voter j conditional on the event that she observes the private signal g, the public signal s_0, and she is pivotal:

$$\Pr\left(G|I_g(n,k)\right) = \frac{1}{1 + \frac{1-\Pr(G|s_0)}{\Pr(G|s_0)}\left(\frac{p}{1-p}\right)^{n-2k}},$$

where $I_g(n,k) = (g, s_0, Piv_j(d^s_{-j}, (n,k)))$ is the information set of voter

[7] Here we abuse the notation a bit. $\mathrm{Err}_{s_0}(n,k)$ is the equilibrium error under $d^{**} = (d^s(n-e_{s_0}), 1(e_{s_0}))$, while $\mathrm{Err}_{s_0}(n-e_{s_0}, k-e_{s_0})$ is the equilibrium error under $d^s(n-e_{s_0})$. Since we consider only the most efficient monotonic equilibrium, which is either $(d^s(n-e_{s_0}), 1(e_{s_0}))$ or $(d^s(n-e_{s_0}), 0(e_{s_0}))$, the equilibrium error $\mathrm{Err}_{s_0}(n,k)$ is equal to the error of the game of $(n-e_{s_0}, k-e_{s_0})$ or $(n-e_{s_0}, k)$, in which all $(n-e_{s_0})$ voters vote sincerely.

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j.[8] Similarly, the posterior belief of guilty conditional on the information set $I_i(n,k) = (i, s_0, Piv_j(d^s_{-j}, (n,k)))$ is

$$\Pr(G|I_i(n,k)) = \frac{1}{1 + \frac{1-\Pr(G|s_0)}{\Pr(G|s_0)}\left(\frac{p}{1-p}\right)^{n-2k+2}}.$$

How voter j shall vote depends on the relation between these posterior beliefs and the threshold of reasonable doubt q. When the inequality $\Pr(G|I_g) > q > \Pr(G|I_i)$ holds, voter j will also vote sincerely. Therefore, the strategy profile $d^s(n)$ is a Nash equilibrium. When it does not hold, voter j will either always vote to convict or acquit regardless of the private signals.[9] Therefore, $d^s(n)$ cannot be an equilibrium. Thus, $d^s(n)$ is a Nash equilibrium if and only if the inequality holds. Note that Austen-Smith and Banks (1996) show that $d^s(n)$ is a Nash equilibrium if and only if the voting rule is identical to the statistical rule, i.e., $k = k^*_{s_0}(n)$. The follow lemma establishes the equivalent relation between the condition for posterior beliefs, i.e., the inequality $\Pr(G|I_g) > q > \Pr(G|I_i)$ and the condition for the voting rule, i.e., $k = k^*_{s_0}(n)$.[10]

Lemma 1. $\Pr(G|I_g(n,k)) > q > \Pr(G|I_i(n,k))$ if and only if $k = k^*_{s_0}(n)$.

proof. See the Appendix. □

Several notes on the lemma are in order.

[8] Note that

$$\begin{aligned}\Pr(G|I_g(n,k)) &= \left(\Pr(G|s_0)\, p^k (1-p)^{n-k}\right) / \left(\Pr(G|s_0)\, p^k (1-p)^{n-k}\right. \\ &\quad \left. + \Pr(I|s_0)\,(1-p)^k p^{n-k}\right) \\ &= 1/\left(1 + \frac{\Pr(I|s_0)}{\Pr(G|s_0)}\left(\frac{p}{1-p}\right)^{n-2k}\right),\end{aligned}$$

and similarly $\Pr(G|I_i(n,k)) = 1/[1 + (\Pr(I|s_0)/\Pr(G|s_0))(p/1-p)^{n-2k+2}]$.

[9] Note that $\Pr(G|I_g) > \Pr(G|I_i)$ i.e., a voter infers that the defendant is more likely to be guilty when she observes signal g. If the inequality $\Pr(G|I_g) > q > \Pr(G|I_i)$ does not hold, then either $\Pr(G|I_g) > \Pr(G|I_i) > q$ or $q > \Pr(G|I_g) > \Pr(G|I_i)$. In the former case, the voter always votes to convict, while in the latter case, she always votes to acquit.

[10] Thus, these three conditions are equivalent: (1) the strategy profile $d^s(n)$ is a Nash equilibrium, (2) $k = k^*_{s_0}(n)$, and (3) $\Pr(G|I_g(n,k)) > q > \Pr(G|I_i(n,k))$.

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(1) Since $\Pr(G|I_g(n,k)) = \Pr(G|I_g(n+2e,k+e))$ and $\Pr(G|I_i(n,k)) = \Pr(G|I_i(n+2e,k+e))$ for any integer e, a voter in a game of (n,k) forms the same posterior beliefs as she does in a game of $(n+2e,k+e)$ given that all other voters in the games vote sincerely. This implies that $d^s(n)$ is an equilibrium in the game of (n,k) if and only if $d^s(n+2e)$ is an equilibrium in the game of $(n+2e,k+e)$, and the statistical rule increases at half the speed of n, i.e., $k^*_{s_0}(n+2e) = k^*_{s_0}(n)+e$.[11] Thus, when n is large, the statistical rule is close to the simple majority rule. We shall use this result to investigate the asymptotic information efficiency of the equilibrium outcome.

(2) When the inequality $\Pr(G|I_g(n,k)) > \Pr(G|I_i(n,k)) > q$ holds, voter j will vote to convict regardless of her private signals. This case occurs when the voting rule k is too stringent in the sense that $k > k^*_{s_0}(n)$. The information she infers from being pivotal, which indicates that the defendant is guilty, outweighs her private information.[12] Since the acquittals from the sincere voting are more frequent than they optimally should be, some voters may try to rectify this by always voting to convict. Suppose that there are e such voters. Then, the rest of the $(n-e)$ voters de facto face a less stringent voting rule $(k-e)$ than k in the sense that the gap between the voting rule and the statistical rule is smaller, i.e., $(k-e)-k^*_{s_0}(n-e) = k-k^*_{s_0}(n)-e/2 < k-k^*_{s_0}(n)$. Therefore, the voter infers that the defendant is less likely to be guilty since the corresponding posterior beliefs of guilty $\Pr(G|I_g(n-e,k-e))$ and $\Pr(G|I_i(n-e,k-e))$ are now lower.[13] Let $e_{s_0} \leq k-1$ denote the smallest integer such that the following inequality holds

[11] Note that since $\Pr(G|I_g(n,k)) = \Pr(G|I_g(n+2e,k+e))$ and $\Pr(G|I_i(n,k)) = \Pr(G|I_i(n+2e,k+e))$, $k = k^*_{s_0}(n)$ if and only if $k+e = k^*_{s_0}(n+2e)$. Therefore, we obtain that $k^*_{s_0}(n)+e = k^*_{s_0}(n+2e)$.

[12] For example, consider a game of $(n,k) = (13,9)$. Note that when $\Pr(G) = 1/2$ and $q = 1/2$, $k^*(13) = 7$. How should voter j vote given that all other voters vote sincerely and she is pivotal? Note that the information she infers from being pivotal is 8 convictions and 4 acquittals. Since the voters vote sincerely, this means that 8 guilty and 4 innocent signals are realized. This information outweighs the private information of voter j who observes only one signal regardless of whether the defendant is guilty or innocent. Therefore, she should vote to convict regardless of the private signals.

[13] Note that both posteriors $\Pr(G|I_g(n-e,k-e)) = 1/(1+(1-\Pr(G|s_0)/\Pr(G|s_0))[(p/1-p)]^{n-2k+e}$ and $\Pr(G|I_i(n-e,k-e)) = 1/(1+(1-\Pr(G|s_0)/\Pr(G|s_0))[(p/1-p)]^{n-2k+2+e}$ decrease in e.

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$$\Pr\left(G|I_g\left(n-e_{s_0},k-e_{s_0}\right)\right) > q > \Pr\left(G|I_i\left(n-e_{s_0},k-e_{s_0}\right)\right). \quad (1)$$

Then, given that there are e_{s_0} voters who always vote to convict, the strategy profile $d^s(n-e_{s_0})$ that all voters in the rest of the jury vote sincerely is an equilibrium of the game of $(n-e_{s_0},k-e_{s_0})$. Thus, the strategy profile $(d^s(n-e_{s_0}),1(e_{s_0}))$ is the equilibrium and $\mathrm{Err}_{s_0}(n,k) = \mathrm{Err}_{s_0}(n-e_{s_0}, k-e_{s_0})$. Furthermore, since $k^*_{s_0}(n-e_{s_0}) = k^*_{s_0}(n) - e_{s_0}/2$ and from lemma 1 $k^*_{s_0}(n-e_{s_0}) = k-e_{s_0}$, the equilibrium uninformative votes are equal to twice the difference between k and $k^*_{s_0}$, i.e., $e_{s_0} = 2(k-k^*_{s_0}(n))$.

If there exists no such e_{s_0}, i.e., $\Pr(G|I_g(n-e_{s_0},k-e_{s_0})) > \Pr(G|I_i(n-e_{s_0},k-e_{s_0})) > q$ for all $e_{s_0} \leq k-1$, the voting rule is so stringent that always voting to convict is a best response for the voters. Then, the strategy profile for which at least k voters always vote to convict is an equilibrium. In equilibrium, the jury always convicts the defendant and incurs the cost of the type I error, which is equal to $\mathrm{Err}_{s_0}(n,k) = q\Pr(I|s_0)$.

(3) By analogy, when the inequality $q > \Pr(G|I_g(n,k)) > \Pr(G|I_i(n,k))$ holds, voter j will vote to acquit regardless of her private signals. This case occurs when the voting rule k is too loose in the sense that $k < k^*_{s_0}(n)$. Suppose that there exist $e_{s_0} \leq n-k-1$ voters trying to rectify this by always voting to acquit, and the number e_{s_0} satisfies the following inequality

$$\Pr\left(G|I_g\left(n-e_{s_0},k\right)\right) > q > \Pr\left(G|I_i\left(n-e_{s_0},k\right)\right). \quad (2)$$

Thus, the equilibrium strategic profile is $(d^s(n-e_{s_0}),0(e_{s_0}))$ and the error $\mathrm{Err}_{s_0}(n,k) = \mathrm{Err}_{s_0}(n-e_{s_0},k)$. Furthermore, for this case, $e_{s_0} = 2(k^*_{s_0}(n)-k)$.

If there exists no such e_{s_0}, i.e., $q > \Pr(G|I_g(n-e_{s_0},k)) > \Pr(G|I_i(n-e_{s_0},k)) > q$ for $e_{s_0} \leq n-k$, the strategy profile for which at least $n-k+1$ voters always vote to acquit is an equilibrium. In equilibrium, the jury always acquits the defendant and incurs the cost of the type II error, which is equal to $\mathrm{Err}_{s_0}(n,k) = (1-q)\Pr(G|s_0)$.

We summarize the above discussions below.

Lemma 2. For $k \geq k^*_{s_0}(n)$, if $e_{s_0} = 2(k-k^*_{s_0}(n)) \leq k-1$, then the strategy profile $(d^s(n-e_{s_0}),1(e_{s_0}))$ is the equilibrium. Otherwise, the strategy profile for which at least k voters always vote to convict is an equilibrium. For $k < k^*_{s_0}(n)$, if $e_{s_0} = 2(k^*_{s_0}(n)-k) \leq n-k-1$, the strategy profile $(d^s(n-e_{s_0}),0(e_{s_0}))$ is the equilibrium. Otherwise, the strategy profile for which at least $n-k+1$ voters always vote to acquit is an equilibrium. In

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both cases, in equilibrium the number of uninformative votes e_{s_0} is equal to $2|k - k^*_{s_0}(n)|$.

proof. See the Appendix. □

Next we investigate how the precision of public information p_0 affects the statistical rule $k^*_{s_0}(n)$ and hence the equilibrium number of uninformative votes e_{s_0}. To simplify the exposition, we assume that $\Pr(G) = q = 1/2$.[14] Note that this assumption implies that the statistical rule when public information is absent is $k^*(n) = (n+1)/2$, i.e., the simple majority rule.[15] Let integer m represent the amount of the adjustment for the statistical rule when public information is available. When the precision of the public information is higher, the corresponding statistical rules $k^*_{g_0}(n)$ and $k^*_{i_0}(n)$ move further away from $k^*(n)$. In the following lemma, we characterize the relationship between p_0 and m.

Lemma 3. Assume $\Pr(G) = q = 1/2$. For $p_0 \in (p_m, p_{m+1}]$, where $m \geq 1$ and p_m is determined by

$$\frac{1-p_m}{p_m} = \left(\frac{1-p}{p}\right)^{2m-1},$$

we have $k^*_{g_0}(n) = k^*(n) - m$ and $k^*_{i_0}(n) = k^*(n) + m$.

proof. See the Appendix. □

Public information with precision $p_0 \in (p_m, p_{m+1}]$ will change the statistical rule by the amount m, and the corresponding number of uninformative votes $e_{s_0} = 2|k - k^*_{s_0}(n)| = 2|k - (n+1)/2 \pm m|$ for $s_0 = g_0, i_0$. With the results in lemmas 2 and 3, we can solve for the equilibrium of the voting game with public information.

Proposition 1. Assume $\Pr(G) = q = 1/2$ and the precision of public information $p_0 \in (p_m, p_{m+1}]$. Then $e_{s_0} = 2|k - k^*_{s_0}(n)| = 2|k - (n+1)/2 \pm m|$ for $s_0 = g_0, i_0$. The strategy profiles $(d^s(n - e_{s_0}), 1(e_{s_0}))$ and

[14] The formula below will be more complicated for the general case, but this assumption will not affect our conclusions qualitatively.

[15] Note that when $k = (n+1)/2$, we have $1/[1 + (1 - \Pr(G|s_0)/\Pr(G|s_0))(p/1-p)^{n-2k}] = p > q = 1/2 > 1/[1+(1-\Pr(G|s_0)/\Pr(G|s_0))(p/1-p)^{n-2k+2}] = (1-p)$. Hence, from lemma 1, we obtain $k^*(n) = (n+1)/2$.

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$(d^s(n - e_{s_0}), 0(e_{s_0}))$ are the equilibria for $k \geq k^*_{s_0}(n)$ and $k \leq k^*_{s_0}(n)$, respectively. Furthermore, the expected error with public information is

$$\begin{aligned}\mathrm{Err}^P &\equiv \Pr(g_0)\,\mathrm{Err}_{g_0}(n,k) + \Pr(i_0)\,\mathrm{Err}_{i_0}(n,k)\\ &= \begin{cases} \frac{1}{2}\left(\mathrm{Err}_{g_0}\left(n-e_{g_0},k-e_{g_0}\right)+\mathrm{Err}_{i_0}\left(n-e_{i_0},k-e_{i_0}\right)\right) & \text{for}\\ \quad k \geq k^*_{g_0}(n) \geq k^*_{i_0}(n) & \\ \frac{1}{2}\left(\mathrm{Err}_{g_0}\left(n-e_{g_0},k-e_{g_0}\right)+\mathrm{Err}_{i_0}\left(n-e_{i_0},k\right)\right) & \text{for}\\ \quad k^*_{g_0}(n) \geq k \geq k^*_{i_0}(n) & \\ \frac{1}{2}\left(\mathrm{Err}_{g_0}\left(n-e_{g_0},k\right)+\mathrm{Err}_{i_0}\left(n-e_{i_0},k\right)\right) & \text{for}\\ \quad k^*_{g_0}(n) \geq k^*_{i_0}(n) \geq k. & \end{cases}\end{aligned}$$

proof. See the Appendix. □

Now we can compare the error under public information (Err^P) with the error when the public information is absent (Err). Public information affects the error through two channels. First, it provides additional information that may help the voters make better decisions. We call this *the information effect* (IE) and define it as follows

$$\begin{aligned}\mathrm{IE} \equiv &\left\{\Pr(g_0)\,\mathrm{Err}_{g_0}\left(n,k^*_{g_0}(n)\right)+\Pr(i_0)\,\mathrm{Err}_{i_0}\left(n,k^*_{i_0}(n)\right)\right\}\\ &-\mathrm{Err}\left(n,k^*(n)\right),\end{aligned}$$

where the term in the first (second) basket is the error with (without) public information. Note that when computing the information effect, we allow the voting rule to be optimally adjusted, i.e., we use the corresponding statistical rule to compute the error. The IE captures the beneficial effect that public information provides an extra piece of information that helps the voters better match their decisions with the received signals. Hence, the error may be reduced due to IE.

On the other hand, public information changes the equilibrium number of the uninformative votes e_{s_0} by changing the statistical rule (from k^* to $k^*_{s_0}$). As a result, the equilibrium error may change. We call the effect of public information on e_{s_0} *the equilibrium effect* (EE), and define it as follows

$$\begin{aligned}\mathrm{EE} \equiv &\left\{\left[\Pr(g_0)\,\mathrm{Err}_{g_0}(n,k)+\Pr(i_0)\,\mathrm{Err}_{i_0}(n,k)\right]\right.\\ &-\left[\Pr(g_0)\,\mathrm{Err}_{g_0}\left(n,k^*_{g_0}(n)\right)+\Pr(i_0)\,\mathrm{Err}_{i_0}\left(n,k^*_{i_0}(n)\right)\right]\Big\}\\ &-\left\{\mathrm{Err}(n,k)-\mathrm{Err}\left(n,k^*(n)\right)\right\},\end{aligned}$$

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where the terms in the first (second) basket represent the difference between the error under the actual voting rule k and that under the statistical rule with (without) public information. From lemma 1, we know that if the voting rule can be (optimally) adjusted to the statistical rule $k^*_{s_0}$, all voters will vote sincerely and the number of the uninformative votes will be 0. However, since the voting rule k is fixed, it is the behaviors of the voters that have to adjust in equilibrium. From lemma 2, we know that there are $e_{s_0} = 2|k - k^*_{s_0}|$ voters who have to vote uninformatively. Therefore, the number of uninformative votes changes from $e = 2|k-k^*|$ to $e_{s_0} = 2|k-k^*_{s_0}|$ when public signal s_0 is realized. This results in the changes of the equilibrium error. The EE captures the effect of public information on the equilibrium adjustments of the voters to the changes in the statistical rules.

Thus, the difference in the errors $\text{TE} \equiv \text{Err}^P - \text{Err}$ can be decomposed into the information effect and the equilibrium effect:

$$\begin{aligned}\text{TE} &\equiv \text{Err}^P - \text{Err} \\ &= \Pr(g_0)\,\text{Err}_{g_0}(n,k) + \Pr(i_0)\,\text{Err}_{i_0}(n,k) - \text{Err}(n,k) \\ &= \text{IE} + \text{EE}.\end{aligned}$$

Note that the information effect IE is a piecewise linear function in the precision of the public information p_0. It has kinks at $p_0 = p_m$, is decreasing for $p_0 \in (p_1, 1)$, and $\lim_{p_0 \downarrow p} \text{IE} = 0$. Therefore, public information always (weakly) reduces the error due to the information effect, and the effect is negligible when p_0 is close to $p = p_1$.

The equilibrium effect EE is also piecewise linear in p_0. However, it has a discrete jump at the point p_m. Therefore, the error may increase at the points near p_m due to the equilibrium effect. The reason why EE has a discrete jump at the point p_m can be clearly illustrated by the case of $k = k^* = (n+1)/2$. When public information is absent, since the voting rule is equal to the statistical rule, the strategy profile that all voters vote sincerely is the equilibrium. Hence, there is no private information lost in equilibrium. On the other hand, when public information is available, from lemma 3, we know that for $p_0 \in (p_m, p_{m+1}]$, $k^*_{g_0} = k^* - m$ and $k^*_{i_0} = k^* + m$ and the corresponding number of uninformative votes is equal to $e_{s_0} = 2|k - k^*_{s_0}| = 2m$ for $s_0 = g_0, i_0$. Heuristically, when $p_0 \uparrow p_m$, there are on average $2(m-1)$ pieces of private information lost and 1 piece of public information with the precision close to p_m gained.[16] On the other hand, when $p_0 \downarrow p_m$,

[16]The average uninformative votes under public information is $\Pr(g_0)e_{g_0} + \Pr(i_0)e_{i_0} =$

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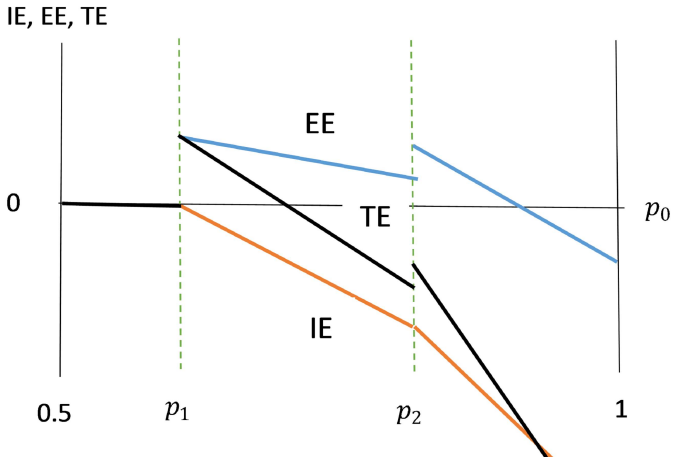


Figure 1: The graphs of IE (bottom), EE (top) and TE (middle)

there are on average $2m$ pieces of private information lost and 1 piece of public information gained. Hence, $\lim_{p_0 \downarrow p_m} \mathrm{EE} > \lim_{p_0 \uparrow p_m} \mathrm{EE}$, i.e., EE has a discrete jump at the point p_m. We show that the property that EE has a discrete jump at the point $p_1 = p$ continues to hold for any non-unanimous voting rule k due to the result that the function $\mathrm{Err}(n, (n+1)/2)$ satisfies the property of the increasing difference.[17] Consequently, when the precision of public information is marginally higher than that of private information, since the information effect IE is negligible, while the equilibrium effect EE stays positive, public information will make the decision of the jury worse.

We illustrate IE, EE, and TE in the following Figure 1:

Note that for $p_0 \in (0.5, p_1]$, since the precision of the public information is lower than that of the private information $p = p_1$, we have $k^*_{g_0} = k^*_{i_0} = k^*$ and the decision of the voters will not be affected by the public signals. Therefore, IE, EE, and hence TE are 0. For $p_0 \in (p_1, p_2]$, from lemma 3, we know that $k^*_{g_0} = k^* + 1$ and $k^*_{i_0} = k^* - 1$. There are on average 2 uninformative votes in equilibrium, and hence the graph of EE jumps at $p_0 = p_1$, and then decreases in the range. On the other hand,

$(1/2)(2m + 2m) = 2m$ when $p_0 \in (p_m, p_{m+1}]$.

[17] See Claim 2 in the proof of Proposition 5 and the example below the proposition.

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IE $= 0$ at $p_0 = p_1$, and then decreases in $(p_1, 1)$. Note that EE jumps while IE has a kink at p_m. Furthermore, TE is positive, i.e., $\text{Err}^P > \text{Err}$ when the precision of the public information p_0 is close to p_1. In addition, when the precision p_0 is far away from p_1, the information effect IE is not negligible, and IE may dominate EE. Then, public information will reduce the error. This situation occurs when the graph of TE lies below the p_0 axis.

Proposition 2. Suppose $k^* = (n+1)/2$. For any non-unanimous rule $k < n$, as $p_0 \downarrow p$, the information effect approaches 0, while the equilibrium effect remains positive. Hence, public information with low precision will make the decision of the jury worse. Furthermore, the loss of efficiency due to public information is negligible when the size of the jury $n \to \infty$.

proof. See the Appendix. □

We use an example to illustrate the result in the proposition and relegate the formal arguments to the Appendix. Consider a jury of size $n = 13$ with a voting rule $k = 9$. Note that for $\Pr(G) = (1/2)$ and $q = (1/2)$, the statistical rule is $k^*(13) = 7$. Since the rule is suboptimal $k \neq 7$, in the equilibrium of the game without public information, there are $e = 2(k - k^*) = 4$ voters who always vote to convict and the other 9 voters vote sincerely. Hence, the number of uninformative votes for conviction is 4 and the equilibrium error is

$$\text{Err}(13, 9) = \text{Err}(9, 5).$$

Assume that the precision of private information $p = 2/3$. From lemma 3, we obtain $p_1 = p = 2/3$ and $p_2 = 8/9$, etc. When public information with precision $p_0 \in (p_1, p_2]$ is available, $k^*_{g_0}(13) = k^*(13) - 1 = 6$ and $k^*_{i_0}(13) = k^*(13) + 1 = 8$. The numbers of uninformative votes for conviction are $e_{g_0} = 2(k - k^*_{g_0}) = 6$ and $e_{i_0} = 2(k - k^*_{i_0}) = 2$, respectively, and $\text{Err}_{g_0}(13, 9) = \text{Err}_{g_0}(7, 3)$ and $\text{Err}_{i_0}(13, 9) = \text{Err}_{i_0}(11, 7)$. Hence, from Proposition 4, we obtain

$$\begin{aligned}\text{Err}^P(13, 9) &= \frac{1}{2}\text{Err}_{g_0}(13, 9) + \frac{1}{2}\text{Err}_{i_0}(13, 9) \\ &= \frac{1}{2}\text{Err}_{g_0}(7, 3) + \frac{1}{2}\text{Err}_{i_0}(11, 7).\end{aligned}$$

Now the difference between the errors $\text{TE} = \text{Err}^P(13, 9) - \text{Err}(13, 9)$ can be decomposed into the information effect (IE) and the equilibrium

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effect (EE):

$$\begin{aligned}\mathrm{IE} &= \left\{\frac{1}{2}\mathrm{Err}_{g_0}\left(n, k^*_{g_0}(n)\right) + \frac{1}{2}\mathrm{Err}_{i_0}\left(n, k^*_{i_0}(n)\right)\right\} - k^*(n)\\ &= \left\{\frac{1}{2}\mathrm{Err}_{g_0}(13,6) + \frac{1}{2}\mathrm{Err}_{i_0}(13,8)\right\} - \mathrm{Err}(13,7),\end{aligned}$$

and

$$\begin{aligned}\mathrm{EE} &= \left\{\left[\frac{1}{2}\mathrm{Err}_{g_0}(13,9) + \frac{1}{2}\mathrm{Err}_{i_0}(13,9)\right] - \left[\frac{1}{2}\mathrm{Err}_{g_0}(13,6)\right.\right.\\ &\quad \left.\left.+\frac{1}{2}\mathrm{Err}_{i_0}(13,8)\right]\right\} - \{\mathrm{Err}(13,9) - \mathrm{Err}(13,7)\}\\ &= \left\{\left[\frac{1}{2}\mathrm{Err}_{g_0}(7,3) + \frac{1}{2}\mathrm{Err}_{i_0}(11,7)\right] - \left[\frac{1}{2}\mathrm{Err}_{g_0}(13,6)\right.\right.\\ &\quad \left.\left.+\frac{1}{2}\mathrm{Err}_{i_0}(13,8)\right]\right\} - \{\mathrm{Err}(9,5) - \mathrm{Err}(13,7)\}.\end{aligned}$$

Note that when computing IE, we use the corresponding statistical rules. On the other hand, since the voting rule is fixed, some voters have to vote uninformatively to accommodate it in equilibrium. EE captures the residual error (EE = TE − IE) caused by the equilibrium adjustment of the voters. When $p_0 \downarrow p$, from Claim 1 in the proof of Proposition 5, we obtain $\mathrm{Err}_{g_0}(13,6) = \mathrm{Err}(13,7) = \mathrm{Err}_{i_0}(13,8)$.[18] Therefore, IE = 0 at $p_0 = p$. Then,

$$\begin{aligned}\mathrm{EE} &= \frac{1}{2}\mathrm{Err}_{g_0}(7,3) + \frac{1}{2}\mathrm{Err}_{i_0}(11,7) - \mathrm{Err}(9,5)\\ &= \frac{1}{2}\mathrm{Err}(7,4) + \frac{1}{2}\mathrm{Err}(11,6) - \mathrm{Err}(9,5),\end{aligned}$$

where the second equality is from Claim 1. Observe that the voting rule k is more stringent under the public signal g_0 than that without public information i.e., $k - k^*_{g_0} = 3 > k - k^* = 2$. As a result, more voters (from 4 to 6) rectify this by always voting to convict and the error increases from $\mathrm{Err}(9,5)$ to $\mathrm{Err}_{g_0}(7,3) = \mathrm{Err}(7,4)$. On the other hand, it is less stringent

[18] That is, when the precision of public information is close to that of private information, the errors (under the statistical rules) with public signal $s_0 = g_0, i_0$ are also close to the error without public information.

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under the public signal i_0 i.e., $k - k^*_{i_0} = 1 < k - k^* = 2$. As a result, fewer voters (from 4 to 2) always vote to convict and the error decreases from $\mathrm{Err}(9, 5)$ to $\mathrm{Err}_{i_0}(11, 7) = \mathrm{Err}(11, 6)$. In addition, the average number of uninformative votes under public information is the same as that without public information, which is $4 = (6 + 2)/2$. Thus, Err is equal to a convex combination of the errors under public signals, and EE can be expressed as the *difference* of the difference in errors:

$$\begin{aligned}\mathrm{EE} &= \frac{1}{2}\mathrm{Err}(7, 4) + \frac{1}{2}\mathrm{Err}(11, 6) - \mathrm{Err}(9, 5)\\ &= \frac{1}{2}\{(\mathrm{Err}(11, 6) - \mathrm{Err}(9, 5)) - (\mathrm{Err}(9, 5) - \mathrm{Err}(7, 4))\}.\end{aligned}$$

From Claim 2 in the proof of Proposition 5 that $\mathrm{Err}(n, (n + 1)/2)$ satisfies the property of increasing differences,[19] we thus obtain $\mathrm{EE} > 0$ as $p_0 \downarrow p$.

By contrast, if $k = 13$, i.e., the unanimous rule, then $e = 2(k - k^*) = 12$. The error without public information is $\mathrm{Err}(13, 13) = \mathrm{Err}(1, 1) = (1 - p)/2$, which is the error incurred by a single voter who votes sincerely.[20] On the other hand, for the case where public information with precision $p_0 \downarrow p$ is available, since $k^*_{g_0}(n) = 6$ and $k^*_{i_0}(n) = 8$, which imply $e_{g_0} = n = 13$ and $e_{i_0} = 10$, respectively,[21] the error is equal to

$$\begin{aligned}\mathrm{Err}^P &= \frac{1}{2}\mathrm{Err}_{g_0}(13, 13) + \frac{1}{2}\mathrm{Err}_{i_0}(13, 13)\\ &= \frac{1}{2}\mathrm{Err}_{g_0}(13, 13) + \frac{1}{2}\mathrm{Err}_{i_0}(3, 3)\\ &= \frac{1}{2}\left(\frac{1}{2}(1 - p_0) + \mathrm{Err}(3, 2)\right) < \frac{1}{2}(1 - p) = \mathrm{Err},\end{aligned}$$

where $\mathrm{Err}_{g_0}(13, 13)$ is equal to the error that all voters vote to convict, which is the cost of the type I error, i.e., $\mathrm{Err}_{g_0}(13, 13) = (1 - p_0)/2$, and $\mathrm{Err}_{i_0}(3, 3) = \mathrm{Err}(3, 2)$ is from Claim 1 in Proposition 5. Since $\mathrm{Err}(3, 2)$

[19] Note that if we treat n as a continuous variable, then $\mathrm{Err}(n, (n + 1)/2)$ is a convex function of n.

[20] Note that $\mathrm{Err}(1, 1) = (1 - q)\Pr(G)(1 - p) + q\Pr(I)(1 - p) = (1 - p)/2$.

[21] Note that when the equilibrium number of uninformative votes is larger than the total number of voters, $2(k - k^*_{g_0}) = 2(13 - 6) = 14 > n$, all voters will vote to convict in equilibrium, and $e_{g_0} = 13$. Furthermore, the error $\mathrm{Err}_{g_0}(13, 13)$ is the error that all voters vote to convict, which is the cost of the type I error, i.e., $\mathrm{Err}_{g_0}(13, 13) = (1 - p_0)/2$.

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is smaller than $(1-p)/2$,[22] public information thus always reduces the error under the unanimous rule. This result is different from the case of non-unanimous rules, in which the error with public information increases. The reason is that when the voting rule is unanimous, the average uninformative votes under public information are $\Pr(g_0)e_{g_0} + \Pr(i_0)e_{i_0} = n/2 + (n-3)/2 = (n-3/2)$, which is smaller than the uninformative votes without public information $(n-1)$. Thus, Err is *not* equal to a convex combination of the errors under public signals, and there are on average *fewer* uninformative votes under public information. As a result, the error is reduced under the unanimous voting rule. Formally, we have the following result:

Proposition 3. Suppose $k^* = (n+1)/2$. For the unanimous rule $k = n$, as $p_0 \downarrow p$, both the information effect and the equilibrium effect are (weakly) negative. Hence, the error with public information will only be reduced, i.e. public information will make the decision of the jury better.

proof. See the Appendix. □

Note that the loss of efficiency caused by public information is bounded by the loss when $k = k^*$, in which there is no private information lost in the game without public information. On the other hand, the average number of pieces of private information lost under public information is $e_{s_0} = 2|k - k^*_{s_0}| = 2$ when $p_0 \downarrow p$. Therefore,

$$\mathrm{Err}^P - \mathrm{Err} = \mathrm{Err}\left(n-2, k^*(n)-1\right) - \mathrm{Err}\left(n, k^*n\right),$$

which approaches 0 as $n \to \infty$. Hence, the harmful effect caused by public information is small when n is large.

4 Non-strategic Voters and Asymptotic Information Efficiency

In this section, we analyze the situation in which some of the voters are non-strategic and investigate the effect of public information on the asymptotic information efficiency, i.e., whether public information increases or decreases the error as the number of the voters approaches infinity. Recall that the non-strategic voters' votes are based directly on the signals they observe.

[22] Note that $\mathrm{Err}(3,2) = (1/2)(\sum_{x\geq 2}\binom{3}{x}(1-p)^x p^{3-x}) = (1-p)^2(1+2p)/2 < (1-p)/2$ for $p > 1/2$.

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When public information is absent, their votes replicate their private signals, while when public information is available, their votes replicate the public signals since $p_0 > p$. Let (n_1, n_2) denote the composition of the jury, where n_1 and n_2 are the numbers of non-strategic and strategic voters respectively. Let $r_1 = n_1/n (r_2 = n_2/n)$ denote the share of the non-strategic (strategic) voters, $R = k/n \in [(1/2), 1)$ the voting rule (in terms of the share of votes), and $R^*(n) = k^*(n)/n$ the statistical rule.[23]

Note that from the discussions below lemma 1, the statistical rule increases at half the speed of the size of the jury. This implies that the statistical rules with or without public information all approach the simple majority rule, i.e., $R^*_{s_0}(n) \to (1/2)$ as $n \to \infty$ for $s_0 = \emptyset, i_0, g_0$. Furthermore, when the voting rule R is different from the statistical rule $R^*_{s_0}$, some of the (strategic) voters may want to rectify this by always voting uninformatively. In equilibrium, the share of uninformative votes is equal to $e = 2(R - R^*(n))$. When $e < R$, the other $(1 - e)$ share of voters vote sincerely and the equilibrium error is equal to $\mathrm{Err}(1, (R - e)/(1 - e))$,[24] while when $e \geq R$, the strategy profile that at least an R share of the (strategic) voters vote to convict is an equilibrium, and the error is equal to $\Pr(I)q = q/2$.

We first analyze the asymptotic results for the case where public information is absent. Since $R^*(n) \simeq 1/2$ for a large n, we obtain that $e \simeq 2(R - 1/2)$, which is strictly less than 1 when the rule $R < 1$. That is, when the voting rule R is non-unanimous, an $e = 2R - 1$ mass of voters will vote to convict and the other $1 - e = 2(1 - R)$ mass of voters will vote sincerely. Therefore, the equilibrium error is equal to the error that the $2(1 - R)$ mass of voters vote sincerely under the the simple majority rule $(R - e)/(1 - e) = 1/2$. As a result, when the state is that the defendant is innocent (guilty), by the law of large numbers, there is a $p > 1/2$ share of the voters who vote to acquit (convict). Thus, the jury makes the correct decision in equilibrium and the asymptotic information efficiency is achieved.

The existence of non-strategic voters, however, limits the capability of the jury to rectify a "suboptimal" voting rule $R > 1/2$ by the strategic voters.

[23] In this section, we introduce new notation r_i and R (in terms of the share of voters and votes) to replace n_i and k (in terms of the number of voters and votes) for the purpose of analyzing the asymptotic results.

[24] In some abuse of the notation, note that the error here is denoted by the ratio: $\mathrm{Err}(n/n, k/n)$ instead of by the number: $\mathrm{Err}(n, k)$ as in the previous sections.

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When the share of non-strategic voters is large, i.e., $e = 2R - 1 > r_2$ or equivalently $r_1 > 2(1 - R)$, all strategic voters will vote to convict and the equilibrium error will be equal to the error that all non-strategic voters vote sincerely under the super majority rule $(R - r_2)/r_1 > 1/2$.[25] Therefore, when the precision of the private information is low, i.e., $p < (R - r_2)/r_1$, by the law of large numbers, the decision of the jury is always to acquit the defendant, and hence the error is equal to $\Pr(G)(1 - q) = (1 - q)/2$. Thus, the outcome is asymptotically inefficient. On the other hand, when the share is small, i.e., $r_1 \leq 2(1-R)$, we obtain the same result as in the case where all voters are strategic, i.e., the outcome is asymptotically efficient. In sum, the asymptotic equilibrium error without public information is

$$\text{Err} = \begin{cases} \frac{1}{2}(1 - q) \text{ when } r_1 \geq 2(1 - R) \text{ and } p < \frac{R - r_2}{r_1}, \\ 0 \text{ when } r_1 < 2(1 - R) \text{ or } p \geq \frac{R - r_2}{r_1}. \end{cases}$$

Next we investigate the case where public information is available. As discussed in the previous section, public information changes the statistical rule, and hence affects the equilibrium error through the information effect and the equilibrium effect. When there exist non-strategic voters, in addition, public information affects the error by dictating the votes of the non-strategic voters. Note that since the statistical rule $R^*_{s_0}(n)$ approaches 1/2, the effect of public information on the error through the changes in the statistical rules is negligible. Only the effect on the votes of the non-strategic voters will affect the asymptotic error.

Since all the votes from the non-strategic voters replicate the public signals, when $s_0 = g_0$, all non-strategic voters vote to convict. This results in a situation where there is a $(1 - r_1)$ mass of strategic voters who must decide how to vote under the voting rule $(R - r_1)/(1 - r_1)$ and the jury incurs an error equal to $\text{Err}_{g_0}(1, (R - r_1)/(1 - r_1))$. Note that the rule $(R - r_1)/(1 - r_1)$ is less stringent than the one without public information R in the sense that it is closer to the statistical rule 1/2. On the other hand, when $s_0 = i_0$, all non-strategic voters vote to acquit. This leads to a situation where there is a $(1 - r_1)$ mass of strategic voters who must decide how to vote under the voting rule $R/(1 - r_1)$ and the jury incurs an error equal to $\text{Err}_{i_0}(1, R/(1 - r_1))$. Note that, on the contrary, the rule $R/(1 - r_1)$ is

[25] Note that since $r_2 < 2R - 1$, $(R - r_2)/r_1 > (R - (2R - 1))/(1 - (2R - 1)) = 1/2$.

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more stringent than R. Thus, the expected error with public information is

$$\mathrm{Err}^P = \frac{1}{2}\mathrm{Err}_{g_0}\left(1, \frac{R - r_1}{1 - r_1}\right) + \frac{1}{2}\mathrm{Err}_{i_0}\left(1, \frac{R}{1 - r_1}\right).$$

Depending on the share of non-strategic voters r_1 and the stringency of the voting rule R, there are three possible cases to consider.

Case (1) $r_1 \geq R$. Since the votes from the non-strategic voters are large enough to determine the outcome, the decisions of the jury replicate the public signals and all private information is lost. The jury thus incurs an error equal to the expected cost that the public signals are wrong, i.e., $\mathrm{Err}^P = (1 - p_0)/2$.[26]

Case (2) $R > r_1 > (1 - R)$. Since $r_1 > (1 - R)$, when $s_0 = i_0$, the votes from non-strategic voters alone are enough to acquit the defendant. Therefore, $\mathrm{Err}_{i_0}(1, R/(1 - r_1)) = (1 - p_0)(1 - q) > 0$. On the other hand, since $R > r_1$, when $s_0 = g_0$, the mass of uninformative votes e_{g_0} is smaller than r_2.[27] That is, the mass of strategic voters is large enough to rectify any non-statistical voting rule $(R - r_1)/(1 - r_1) \geq 1/2$. Therefore, the asymptotic efficiency is reached.[28] Hence, the expected error is $\mathrm{Err}^P = (1 - p_0)(1 - q)/2$. Note that when $s_0 = g_0$, the equilibrium error is computed under a less stringent rule i.e., $(R - r_1)/(1 - r_1) < R$. This is the reason why public information can potentially improve the efficiency.

Case (3) $r_1 \leq (1 - R)$. The share of non-strategic voters is so small that there is a sufficient mass of strategic voters to rectify the more stringent voting rule $R/(1 - r_1)$ (than R) when $s_0 = i_0$.[29] Therefore, the decisions of the jury for $s_0 = i_0$ and $s_0 = g_0$ are both asymptotically correct and $\mathrm{Err}_{i_0}(1, R/(1 - r_1)) = \mathrm{Err}_{g_0}(1, (R - r_1)/(1 - r_1)) = 0$.

In sum, the asymptotic equilibrium error with public information is

$$\mathrm{Err}^P = \begin{cases} \frac{1}{2}(1 - p_0) & \text{when } r_1 \geq R, \\ \frac{1}{2}(1 - p_0)(1 - q) & \text{when } R > r_1 > (1 - R), \\ 0 & \text{when } r_1 \leq (1 - R). \end{cases}$$

[26] Note that $\mathrm{Err}_{i_0}(1, R/(1 - r_1))$ is equal to the cost of the type II error: $\Pr(G|s_0 = i_0)(1 - q) = (1 - p_0)(1 - q)$. Furthermore, $\mathrm{Err}_{g_0}(1, (R - r_1)/(1 - r_1))$ is equal to the cost of the type I error $(1 - p_0)q$. Hence, $\mathrm{Err}^P = (1 - p_0)/2$.

[27] Note that $e_{g_0} = 2(R - r_1 - (1 - r_1)/2) = 2R - 1 - r_1 < r_2 \Leftrightarrow R < 1$.

[28] We have $\mathrm{Err}_{g_0}(1, (R - r_1)/(1 - r_1)) = \mathrm{Err}_{g_0}(1, (R - r_1 - e_{g_0})/(1 - r_1 - e_{g_0})) = \mathrm{Err}_{g_0}(1, 1/2) = 0$.

[29] Note that $e_{i_0} = 2(R - (1 - r_1)/2) = r_1 + (2R - 1) \leq 1 - R + (2R - 1) \leq r_2$.

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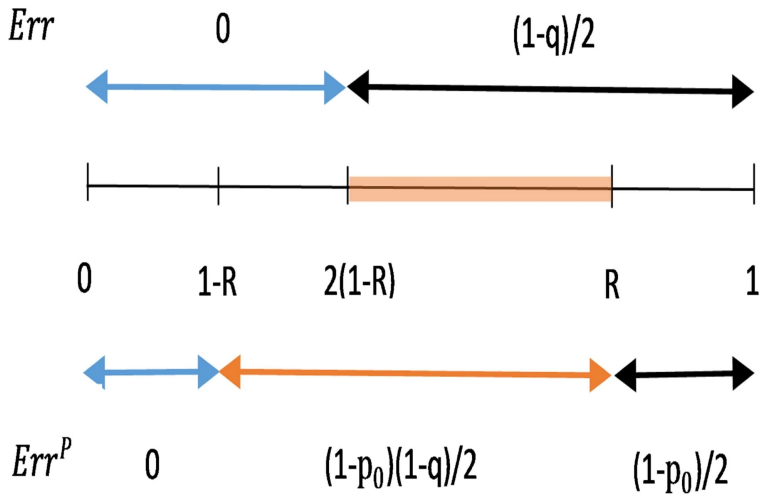


Figure 2: The asymptotic error for different ranges of r_1

By comparing Err with Err^P (see the following figure), we find that when the share of non-strategic voters is small, i.e., $r_1 < 2(1-R)$ or the precision of the private information is relatively high, i.e., $p \geq (R-r_2)/r_1$, the asymptotic information efficiency can be achieved without public information. Hence, public information cannot improve the efficiency. On the other hand, when the information efficiency is not achievable, i.e., $r_1 \geq 2(1-R)$ and $p < (R-r_2)/r_1$, public information can improve the efficiency under two circumstances. First, it can be improved when the precision of public information is higher than the threshold of reasonable doubt, i.e., $p_0 > q$. For this case, even if the share of non-strategic voters is large and the outcomes replicate the public signals, the error is still reduced, i.e., $\text{Err}^P = (1-p_0)/2 < \text{Err} = (1-q)/2$. Second, even when the precision p_0 is lower than q, the error can still be reduced if the share of non-strategic voters is in the range of $R > r_1 > 2(1-R)$ (the shaded area in the figure). In this case, pubic information de facto alters the voting rule faced by the strategic voters to be less stringent by its influence on the votes of the non-strategic voters. Consequently, there is a large enough mass of strategic voters to rectify this less stringent voting rule $(R-r_1)/(1-r_1)$ and the information efficiency is achieved when $s_0 = g_0$. Thus, the overall efficiency is improved.

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Proposition 4. Asymptotic information efficiency can be achieved without public information for any non-unanimous rule $R \in [(1/2), 1)$ as long as the share of non-strategic voters is small, i.e., $r_1 < 2(1-R)$ or the precision of the private information is high, i.e., $p \geq (R - r_2)/r_1$. Otherwise, public information can improve the efficiency when $p_0 > q$ or $r_1 \in (2(1-R), R)$.

proof. See the Appendix. □

Feddersen and Pesendorfer (1997) first establish the result that a jury with a non-unanimous voting rule can achieve the asymptotic information efficiency by studying symmetric mixed-strategy equilibria. Thus, we provide an alternative proof (by using the asymmetric monotonic equilibrium) for this result. We further extend the analysis to the case where some voters are non-strategic and show that as long as the share of non-strategic voters is small, the result continues to hold. Furthermore, when the efficiency is not achievable, pubic information can through its dictating influence on the non-strategic voters improve the efficiency by "loosening" the voting rule faced by the strategic voters.

By contrast, for the unanimous rule $R = 1$, since $R^*_{s_0}(n) \to (1/2)$ for $s_0 = \emptyset$, i_0, g_0, as $n \to \infty$, all voters but one will vote to convict in equilibrium regardless of their signals.[30] Therefore, the error is equal to $(1-p)/2$ (without public information), or $(1-p_0)/2$ (with public information). Note that asymptotic information efficiency is not achievable as long as the information is not perfectly accurate i.e., $p < 1$ (or $p_0 < 1$). Furthermore, since $p_0 < p$, public information will reduce the error and the information efficiency will improve. This result can be extended to the case where the share of non-strategic voters is strictly positive, i.e., $r_1 > 0$. First, consider the case without public information. Since non-strategic voters vote sincerely, there must be a strictly positive measure of votes for acquittal as long as the information is not perfectly accurate. Thus, the outcome will always be to acquit the defendant under the unanimous rule, and will incur an error equal to $(1-q)/2$. On the other hand, if public information is available, when $s_0 = i_0$, the outcome will be to acquit the defendant, and will incur an error equal to $(1-p_0)(1-q)$, while when $s_0 = g_0$, all

[30] Note that $e_{s_0} = 2|k - k^*_{s_0}| = 2|n - (n + 1/2)| = n - 1$ as $n \to \infty$. That is, all voters but one will vote to convict in equilibrium regardless of their signals. Hence, the error is equal to the error of a single voter who votes sincerely: $(1-p)/2$ or $(1-p_0)/2$.

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voters will vote to convict, and will incur an error equal to $(1 - p_0)q$.[31] Therefore, the average error with public information is equal to $(1 - p_0)/2$. We summarize the above discussion as follows:

Proposition 5. For the non-unanimous rule $R = 1$, asymptotic information efficiency is not achievable with or without public information. Furthermore, when $p_0 > q$, public information improves the asymptotic information efficiency regardless of the share of non-strategic voters.

5 Conclusion

In a committee consisting of non-strategic and strategic voters, we investigate the effect of public information on the voting behaviors and the information efficiency of the equilibrium outcome. When all voters are strategic, we find that for any non-unanimous voting rule, when the precision of public information is marginally higher than that of private information, public information will make the decision worse, and the loss of efficiency caused by public information will be small when the size of the jury is large. Furthermore, we find that the existence of non-strategic voters may cause asymptotic inefficiency when the voting rule is far away from the simple majority rule. Public information de facto causes the voting rule faced by the strategic voters to become less stringent by its influence on the votes of the non-strategic voters. Thus, the overall efficiency can be improved.

In our model, we assume that the behaviors of the non-strategic voters are the least sophisticated in the sense that they make no inference from the equilibrium behaviors of other players. On the other hand, the behaviors of the strategic voters are the most sophisticated in the sense that they can make perfect inference. In reality, the voters' assessments may lie in between these two extreme cases. Eyster and Rabin (2005) introduce the notion of cursed equilibria, which means that each player correctly predicts the distribution of other players' actions, but underestimates the degree to which these actions are correlated with other players' information. In a cursed equilibrium, players act as if they face their opponents with a positive probability that they will vote sincerely, and with the complementary probability that they

[31] Note that all non-strategic voters will vote to convict and all strategic voters but one will also vote to convict regardless of their signals. The one who votes strategically will also vote to convict since $p_0 > p$. Therefore, the equilibrium outcome is to convict the defendant and incurs an error $(1 - p_0)q$.

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will vote sophisticatedly, i.e., strategically. On the other hand, Esponda and Pouzo (2012) consider a learning process according to which players may learn from a biased sample since they fail to account for the consequences of unchosen past alternatives, i.e., counterfactuals. The authors investigate the extent to which information aggregation is precluded in such a learning environment. It may be worthwhile to explore the effect of public information on the behaviors of the voters and the efficiency property of the equilibrium outcome under the behavioral assumptions by Eyster and Rabin (2005) and Esponda and Pouzo (2012).

Appendix

Proof of Lemma 1

For the "if" part, since $k = k^*_{s_0}(n)$. both $\mathrm{Err}_{s_0}(n,k) = -u_{s_0}(d^s(n),(n,k)) \leq -u_{s_0}(d^s(n),(n,k-1))$ and $\mathrm{Err}_{s_0}(n,k) \leq -u_{s_0}(d^s(n),(n,k+1))$ should hold. Recall that

$$-u_{s_0}(d^s(n),(n,k) = q\Pr(I|s_0)\sum_{x\geq k}\binom{n}{x}(1-p)^x p^{n-x} + (1-q)\Pr(G|s_0)\sum_{x<k}\binom{n}{x}(1-p)^{n-x}p^x.$$

Thus, we obtain

$$\begin{aligned}
\mathrm{Err}_{s_0}(n,k) &\leq -u_{s_0}(d^s(n),(n,k-1)) \\
&\Leftrightarrow (1-q)\Pr(G|s_0)\binom{n}{k-1}(1-p)^{n-k+1}p^{k-1} \\
&\quad \leq q\Pr(I|s_0)\binom{n}{k-1}(1-p)^{k-1}p^{n-k+1} \\
&\Leftrightarrow \frac{q}{1-q} \geq \frac{\Pr(G|s_0)}{\Pr(I|s_0)}\frac{(1-p)^{n-2k+2}}{p^{n-2k+2}} \\
&\Leftrightarrow q \geq \frac{1}{1+\frac{\Pr(I|s_0)}{\Pr(G|s_0)}\left(\frac{p}{1-p}\right)^{n-2k+2}} = \Pr(G|I_i(n,k)).
\end{aligned}$$

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And similarly,

$$\begin{aligned}
\mathrm{Err}_{s_0}(n,k) &\leq -u_{s_0}\left(d^s(n),(n,k+1)\right)\\
&\Leftrightarrow q\Pr(I|s_0)\binom{n}{k}(1-p)^k p^{n-k}\\
&\quad\leq (1-q)\Pr(G|s_0)\binom{n}{k}(1-p)^{n-k}p^k\\
&\Leftrightarrow \frac{q}{1-q}\leq \frac{\Pr(G|s_0)}{\Pr(I|s_0)}\frac{(1-p)^{n-2k}}{p^{n-2k}}\\
&\Leftrightarrow q\leq \frac{1}{1+\frac{\Pr(I|s_0)}{\Pr(G|s_0)}\left(\frac{p}{1-p}\right)^{n-2k}}\\
&=\Pr\left(G|I_g(n,k)\right).
\end{aligned}$$

For the "only if" part, note that from the above arguments, when $\Pr(G|I_g) > q > \Pr(G|I_i)$, both $\mathrm{Err}_{s_0}(n,k) \leq -u_{s_0}(d^s(n),(n,k-1))$ and $\mathrm{Err}_{s_0}(n,k) \leq -u_{s_0}(d^s(n),(n,k+1))$ should hold. Since

$$\begin{aligned}
&-u_{s_0}\left(d^s(n),(n,k-l)\right)-\left(-u_{s_0}\left(d^s(n),(n,k-(l-1))\right)\right)\\
&=q-\frac{1}{1+\frac{\Pr(I|s_0)}{\Pr(G|s_0)}\left(\frac{p}{1-p}\right)^{n-2(k-l)+2}}\\
&>q-\frac{1}{1+\frac{\Pr(I|s_0)}{\Pr(G|s_0)}\left(\frac{p}{1-p}\right)^{n-2k+2}}\geq 0,
\end{aligned}$$

for any integer $l \geq 2$, and similarly,

$$\begin{aligned}
&-u_{s_0}\left(d^s(n),(n,k+l)\right)-\left(-u_{s_0}\left(d^s(n),(n,k+(l-1))\right)\right)\\
&=\frac{1}{1+\frac{\Pr(I|s_0)}{\Pr(G|s_0)}\left(\frac{p}{1-p}\right)^{n-2(k+l)}}-q\\
&>\frac{1}{1+\frac{\Pr(I|s_0)}{\Pr(G|s_0)}\left(\frac{p}{1-p}\right)^{n-2k}}-q\geq 0.
\end{aligned}$$

By induction, $-u_{s_0}(d^s(n),(n,k-l)) > \mathrm{Err}_{s_0}(n,k)$ and $-u_{s_0}(d^s(n),(n,k+l)) > \mathrm{Err}_{s_0}(n,k)$ for all $l \geq 1$. Hence, $k = k^*_{s_0}(n)$.

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Proof of Lemma 2

We should argue that the strategy profile $(d^s(n - e_{s_0}), 1(e_{s_0}))$ is a Nash equilibrium when there exists $e_{s_0} \leq k - 1$ such that inequality (1) holds. The argument for the case of $(d^s(n - e_{s_0}), 0(e_{s_0}))$ when there exists e_{s_0} such that inequality (2) holds is similar.

First, consider a voter among the $(n - e_{s_0})$ voters who vote strategically. Given that there are e_{s_0} voters who always vote to convict and the other $n - e_{s_0} - 1$ voters who vote sincerely, since inequality (1) holds, voting sincerely is a best response of the voter if she is pivotal. Note that if she is not pivotal, any vote will be a best response. Hence, voting sincerely is a best response of the voter.

Second, for a voter among the e_{s_0} voters who always vote to convict, given that there are $e_{s_0} - 1$ voters who always vote to convict and the other $n - e_{s_0}$ voters who vote sincerely, the voter forms the posterior beliefs of guilty equal to $\Pr(G|I_g(n - e_{s_0} + 1, k - e_{s_0} + 1))$ and $\Pr(G|I_i(n - e_{s_0} + 1, k - e_{s_0} + 1))$ when she receives $s_j = g$ and $s_j = i$, respectively. Since $\Pr(G|I_g(n - e_{s_0} + 1, k - e_{s_0} + 1)) > \Pr(G|I_g(n - e_{s_0}, k - e_{s_0}))$ and $\Pr(G|I_i(n - e_{s_0} + 1, k - e_{s_0} + 1)) > \Pr(G|I_i(n - e_{s_0}, k - e_{s_0}))$, from inequality (1), we obtain

$$\Pr\left(G|I_g\left(n - e_{s_0} + 1, k - e_{s_0} + 1\right)\right) > \Pr\left(G|I_i\left(n - e_{s_0} + 1, k - e_{s_0} + 1\right)\right) > q.$$

Otherwise, it must be $\Pr(G|I_g(n - e_{s_0} + 1, k - e_{s_0} + 1)) > q > \Pr(G|I_i(n - e_{s_0} + 1, k - e_{s_0} + 1))$. But this violates that e_{s_0} is the smallest number of uninformative votes. Therefore, always voting to convict is a best response of the voter if she is pivotal. Thus, we have shown that the strategy profile $(d^s(n - e_{s_0}), 1(e_{s_0}))$ is a Nash equilibrium when inequality (1) holds.

When there exists no e_{s_0} such that inequality (1) holds, then the inequality $\Pr(G|I_g(n - e, k - e)) > \Pr(G|I_i(n - e, k - e)) > q$ holds for all $e \leq k - 1$. That is, when there are $e \leq (k - 1)$ voters who always vote to convict, it is the best response for a voter in the game of $(n - e, k - e)$ to always vote to convict if she is pivotal. Note that when there are k or more voters who always vote to convict, any vote for a voter is a best response since she is not pivotal. Therefore, always voting to convict is a best response for the voters when there exists no e_{s_0} such that inequality (1) holds.

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Proof of Lemma 3

Since $k^*_{g_0}$ is the statistical rule when $s_0 = g_0$, the strategy profile d^s is a Nash equilibrium if the following condition holds:

$$\begin{aligned}\Pr\left(G|I_g\right) &= \frac{1}{1+\frac{\Pr(I|g_0)}{\Pr(G|g_0)}\left(\frac{p}{1-p}\right)^{n-2k^*_{g_0}}} > q > \Pr\left(G|I_i\right)\\ &= \frac{1}{1+\frac{\Pr(I|g_0)}{\Pr(G|g_0)}\left(\frac{p}{1-p}\right)^{n-2k^*_{g_0}+2}}\\ \Leftrightarrow \frac{1}{1+\left(\frac{1-p_0}{p_0}\right)\left(\frac{p}{1-p}\right)^{n-2k^*_{g_0}}} &> \frac{1}{2}\\ > \frac{1}{1+\left(\frac{1-p_0}{p_0}\right)\left(\frac{p}{1-p}\right)^{n-2k^*_{g_0}+2}}&.\end{aligned}$$

By using $(1-p_0/p_0)(p/1-p)^{n-2k^*_{g_0}} < 1$ and $k^*_{g_0} = k^* - m = ((n+1)/2) - m$, we obtain $(1-p_0/p_0) < (1-p/p)^{2m-1}$. Similarly, by using $(1-p_0/p_0)(p/1-p)^{n-2k^*_{g_0}+2} > 1$, we obtain $(1-p_0/p_0) > (1-p/p)^{2m+1}$. Therefore,

$$\left(\frac{1-p}{p}\right)^{2m+1} < \left(\frac{1-p_0}{p_0}\right) < \left(\frac{1-p}{p}\right)^{2m-1}.$$

Let p_m be the solution of the equation $(1-p_m/p_m) = (1-p/p)^{2m-1}$. Then $p_m < p_0 < p_{m+1}$ if and only if $(1-p_{m+1}/p_{m+1}) < (1-p_0/p_0) < (1-p_m/p_m)$ and if and only if $(1-p/p)^{2m+1} \le (1-p_0/p_0) \le (1-p/p)^{2m-1}$.

Proof of Proposition 4

Consider the voting game of public information with precision $p_0 \in (p_m, p_{m+1}]$. From lemma 3, we know that $e_{s_0} = 2|k - k^*_{s_0}(n)| = 2|k - ((n+1)/2 \pm m|$ for $s_0 = g_0, i_0$. From lemma 2, we know that the strategy profile $(d^s(n-e_{s_0}), 1(e_{s_0}))$ and $(d^s(n-e_{s_0}), 0(e_{s_0}))$ are the equilibrium for $s_0 = g_0$ and $s_0 = i_0$, respectively.

Proof of Proposition 5

There are three parts for the proof. We prove the properties of IE in (1), the properties of EE in (2), and that $\mathrm{Err}^P - \mathrm{Err} > 0$ when $p_0 \downarrow p$ for any

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$k < n$ in (3).

(1) IE is decreasing and piecewise linear in p_0. It has kinks at $p_0 = p_m$ and $\lim_{p_0 \downarrow p} \text{IE} = 0$.

For $p_0 \in (p_m, p_{m+1})$, from lemma 3, we have $k^*_{g_0} = (n + 1/2) - m$ and $k^*_{i_0} = (n + 1/2) + m$. Then we have

$$\begin{aligned}
\text{Err}_{g_0}\left(n, k^*_{g_0}(n)\right) &= \text{Err}_{g_0}\left(n, \frac{n+1}{2} - m\right) \\
&= q \Pr(I|g_0) \sum_{x \geq \left(\frac{(n+1)}{2} - 1\right)} \binom{n}{x} (1-p)^x p^{n-x} \\
&\quad + (1-q) \Pr(G|g_0) \sum_{x < \left(\frac{(n+1)}{2} - 1\right)} \binom{n}{x} (1-p)^{n-x} p^x \\
&= \frac{1}{2}\left[(1-p_0) \sum_{x \geq \left(\frac{(n+1)}{2} - m\right)} \binom{n}{x} (1-p)^x p^{n-x} \right. \\
&\quad \left. + p_0 \sum_{x < \left(\frac{(n+1)}{2} - m\right)} \binom{n}{x} (1-p)^{n-x} p^x \right] \\
&= \frac{1}{2}\left[\sum_{x \geq \left(\frac{(n+1)}{2} - m\right)} \binom{n}{x} (1-p)^x p^{n-x} \right. \\
&\quad \left. - p_0 \left(\sum_{\frac{(n+1)}{2} \geq x \geq \left(\frac{(n+1)}{2} - m\right)} \binom{n}{x} (1-p)^x p^{n-x} \right) \right],
\end{aligned}$$

which is decreasing and linear in p_0. Similarly, $\text{Err}_{i_0}(n, k^*_{i_0}(n))$ is also decreasing and linear in p_0. Therefore, IE decreases and linear in p_0 for $p_0 \in (p_m, p_{m+1})$.

Next, we claim that $\lim_{p_0 \uparrow p_m} \text{Err}_{s_0}(n, k^*_{s_0}(n)) = \lim_{p_0 \downarrow p_m} \text{Err}_{s_0}(n, k^*_{s_0}(n))$. We prove for the case of $s_0 = g_0$ and the argument for $s_0 = i_0$ is

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similar. When $p_0 = p_m - \varepsilon$, $k^*_{g_0}(n) = (n+1)/2 - (m-1)$, while when $p_0 = p_m + \varepsilon$, $k^*_{g_0}(n) = (n+1)/2 - m$. Note that the only composition of the votes that makes difference under the two different statistical rules is the one that there are $(n+1)/2 - m$ votes for conviction and the rest of $(n - ((n+1)/2 - m))$ votes for acquittal. Hence, the difference between $\lim_{p_0 \uparrow p_m} \mathrm{Err}_{s_0}(n, k^*_{s_0}(n))$ and $\lim_{p_0 \downarrow p_m} \mathrm{Err}_{s_0}(n, k^*_{s_0}(n))$ is that under the rule $(n+1)/2-(m-1)$, the composition acquits the defendant and incurs the cost of the type II error equal to $(1-q)\binom{n}{((n+1)/2)-m} p^{((n+1)/2)-m}(1-p)^{n-(((n+1)/2)-m)}$. While under the rule $(n+1)/2 - m$, it convicts the defendant and incurs the cost of the type I error equal to $q\binom{n}{((n+1)/2)-m}(1-p)^{((n+1)/2)-m} p^{n-(((n+1)/2)-m)}$. Then the ratio of the errors is equal to

$$\begin{aligned}
\frac{\mathrm{Err}_{g_0}(p_m+\varepsilon)}{\mathrm{Err}_{g_0}(p_m-\varepsilon)} &= \frac{\Pr(I|g_0)\, q\binom{n}{\frac{(n+1)}{2}-m}(1-p)^{\frac{(n+1)}{2}-m} p^{n-\left(\frac{(n+1)}{2}-m\right)}}{\Pr(G|g_0)\,(1-q)\binom{n}{\frac{(n+1)}{2}-m} p^{\frac{(n+1)}{2}-m}(1-p)^{n-(\frac{(n+1)}{2}-m)}} \\
&= \frac{(1-(p_m-\varepsilon))\, q p^{2m-1}}{(p_m+\varepsilon)\,(1-q)(1-p)^{2m-1}} \\
&= \frac{1-p_m}{p_m}\left(\frac{p}{1-p}\right)^{2m-1} \quad \text{as } \varepsilon \to 0 = 1,
\end{aligned}$$

where the last equality is from the definition of p_m. Thus, we obtain

$$\lim_{p_0 \uparrow p_m} \mathrm{Err}_{g_0}\left(n, k^*_{g_0}(n)\right) = \lim_{p_0 \downarrow p_m} \mathrm{Err}_{g_0}\left(n, k^*_{g_0}(n)\right).$$

Since $\lim_{p_0 \uparrow p_m} \mathrm{IE} = \lim_{p_0 \downarrow p_m} \mathrm{IE}$, IE is continuous at $p_0 = p_m$. Note that the slope of IE depends on m. Therefore, it has kinks at $p_0 = p_m$.

Finally, we claim that $\lim_{p_0 \downarrow p} \mathrm{IE} = 0$. From lemma 3, we know that $k^*_{g_0} = k^* - 1 = (n+1)/2 - 1$ and $k^*_{i_0} = k^* + 1 = (n+1)/2 + 1$ when $p_0 \downarrow p$. Then

$$\begin{aligned}
\lim_{p_0 \downarrow p} \mathrm{IE} = \lim_{p_0 \downarrow p} \frac{1}{2}&\left(\mathrm{Err}_{g_0}(n, (n+1)/2 - 1) + \mathrm{Err}_{i_0}(n, (n+1)/2 + 1)\right) \\
&- \mathrm{Err}(n, (n+1)/2).
\end{aligned}$$

The following claim implies that $\lim_{p_0 \downarrow p} \mathrm{IE} = 0$.

Claim 1: $\mathrm{Err}_{g_0}(n, (n+1)/2 - 1) = \mathrm{Err}(n, (n+1)/2) = \mathrm{Err}_{i_0}(n, (n+1)/2 + 1)$ as $p_0 \downarrow p$.

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As $p_0 \downarrow p$, we have

$$
\begin{aligned}
&\mathrm{Err}_{g_0}(n, (n+1)/2-1) \\
&\quad = q \Pr(I|g_0) \sum_{x \geq \left(\frac{(n+1)}{2}-1\right)} \binom{n}{x} (1-p)^x p^{n-x} \\
&\quad + (1-q) \Pr(G|g_0) \sum_{x < \left(\frac{(n+1)}{2}-1\right)} \binom{n}{x} (1-p)^{n-x} p^x \\
&\quad = \frac{1}{2} \left((1-p_0) \left(\sum_{x \geq \left(\frac{(n+1)}{2}-1\right)} \binom{n}{x} (1-p)^x p^{n-x} \right) \right. \\
&\quad \left. + p_0 \left(\sum_{x < \left(\frac{(n+1)}{2}-1\right)} \binom{n}{x} (1-p)^{n-x} p^x \right) \right) \\
&\quad = \frac{1}{2} \left((1-p) \left(\sum_{x \geq \left(\frac{(n+1)}{2}-1\right)} \binom{n}{x} (1-p)^x p^{n-x} \right) \right. \\
&\quad \left. + p \left(\sum_{x \geq \left(\frac{(n+1)}{2}+1\right)} \binom{n}{x} (1-p)^x p^{n-x} \right) \right) \\
&\quad = \frac{1}{2} \left(\sum_{x \geq \left(\frac{(n+1)}{2}+1\right)} \binom{n}{x} (1-p)^x p^{n-x} \right. \\
&\quad + (1-p) \binom{n}{\frac{(n+1)}{2}} (1-p)^{\frac{(n+1)}{2}} p^{n-\frac{(n+1)}{2}} \\
&\quad \left. + (1-p) \binom{n}{\frac{(n+1)}{2}-1} (1-p)^{\frac{(n+1)}{2}-1} p^{n-\frac{(n+1)}{2}+1} \right) \\
&\quad = \frac{1}{2} \left(\sum_{x \geq \left(\frac{(n+1)}{2}+1\right)} \binom{n}{x} (1-p)^x p^{n-x} \right.
\end{aligned}
$$

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$$+\binom{n}{\frac{(n+1)}{2}}(1-p)^{\frac{(n+1)}{2}}p^{n-\frac{(n+1)}{2}}(1-p+p)\Bigg)$$
$$=\frac{1}{2}\left(\sum_{x\geq\frac{(n+1)}{2}}\binom{n}{x}(1-p)^{x}p^{n-x}\right)=\mathrm{Err}(n,(n+1)/2).$$

The argument for $\mathrm{Err}_{i_0}(n,(n+1)/2+1)=\mathrm{Err}(n,(n+1)/2)$ is similar. Thus, we prove Claim 1.

(2) EE is piece-wise linear in p_0. and has a discrete jump at the point $p_0=p_m$.

Note that from the definition of EE

$$\begin{aligned}\mathrm{EE}\equiv&\left\{\left[\Pr(g_0)\,\mathrm{Err}_{g_0}(n,k)+\Pr(i_0)\,\mathrm{Err}_{i_0}(n,k)\right]\right.\\&-\left[\Pr(g_0)\,\mathrm{Err}_{g_0}\left(n,k^*_{g_0}(n)\right)+\Pr(i_0)\,\mathrm{Err}_{i_0}\left(n,k^*_{i_0}(n)\right)\right]\Big\}\\&-\left\{\mathrm{Err}(n,k)-\mathrm{Err}\left(n,k^*(n)\right)\right\},\end{aligned}$$

since $k^*_{s_0}$ does not change for $p_0\in(p_m,p_{m+1})$ and $\mathrm{Err}_{g_0}(n,k)$, $\mathrm{Err}_{i_0}(n,k)$, $\mathrm{Err}_{g_0}(n,k^*_{g_0}(n))$ and $\mathrm{Err}_{i_0}(n,k^*_{i_0}(n))$ are all linear in p_0, EE is linear for $p_0\in(p_m,p_{m+1})$. Furthermore, as $p_0\downarrow p_m$,

$$\begin{aligned}\mathrm{EE}&=\left\{\Pr(g_0)\,\mathrm{Err}_{g_0}(n,k)+\Pr(i_0)\,\mathrm{Err}_{i_0}(n,k)-\mathrm{Err}(n,k)\right\}-\mathrm{IE}\\&=\left\{\Pr(g_0)\,\mathrm{Err}_{g_0}(n-2m,k-2m)\right.\\&\quad\left.+\Pr(i_0)\,\mathrm{Err}_{i_0}(n-2m,k)-\mathrm{Err}(n,k)\right\}-\mathrm{IE}.\end{aligned}$$

While as $p_0\uparrow p_m$,

$$\begin{aligned}\mathrm{EE}&=\left\{\Pr(g_0)\,\mathrm{Err}_{g_0}(n,k)+\Pr(i_0)\,\mathrm{Err}_{i_0}(n,k)-\mathrm{Err}(n,k)\right\}-\mathrm{IE}\\&=\left\{\Pr(g_0)\,\mathrm{Err}_{g_0}(n-2(m-1),k-2(m-1))\right.\\&\quad\left.+\Pr(i_0)\,\mathrm{Err}_{i_0}(n-2(m-1),k)-\mathrm{Err}(n,k)\right\}-\mathrm{IE}\end{aligned}$$

Note that since $\lim_{p_0\uparrow p_m}\mathrm{IE}=\lim_{p_0\downarrow p_m}\mathrm{IE}$ (as we have shown in (1)) and

$$\begin{aligned}\lim_{p_0\downarrow p_m}\mathrm{EE}-\lim_{p_0\uparrow p_m}\mathrm{EE}=&\frac{1}{2}\left(\mathrm{Err}_{g_0}(n-2m,k-2m)+\mathrm{Err}_{i_0}(n-2m,k)\right)\\&-\left(\mathrm{Err}_{g_0}(n-2(m-1),k-2(m-1))\right.\\&\left.+\mathrm{Err}_{i_0}(n-2(m-1),k)\right)>0,\end{aligned}$$

where the last inequality is from the observation that $\mathrm{Err}_{g_0}(n-2m,k-2m)>\mathrm{Err}_{g_0}(n-2(m-1),k-2(m-1))$ since the former has $2m$ pieces of

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private information lost while the latter has $2(m-1)$ pieces, and similarly $\mathrm{Err}_{i_0}(n-2m,k) > \mathrm{Err}_{i_0}(n-2(m-1),k)$.

(3) $\mathrm{EE} > 0$ when $p_0 \downarrow p$ for any $k < n$.

Since $\mathrm{IE} \to 0$ as $p_0 \downarrow p$, $\mathrm{Err}^P - \mathrm{Err}(n,k) = \mathrm{EE}$. Therefore, we have

$$\begin{aligned}
\mathrm{EE} &= \Pr(g_0)\,\mathrm{Err}_{g_0}(n,k) + \Pr(i_0)\mathrm{Err}_{i_0}(n,k) - \mathrm{Err}(n,k) \\
&= \frac{1}{2}\left(\mathrm{Err}_{g_0}\left(n-e_{g_0},k-e_{g_0}\right) + \mathrm{Err}_{i_0}\left(n-e_{i_0},k-e_{i_0}\right)\right) \\
&\quad - \mathrm{Err}(n-e,k-e) \\
&= \frac{1}{2}\left(\mathrm{Err}_{g_0}(2(n-k)-1,(n-k)-1) + \mathrm{Err}_{i_0}(2(n-k)\right. \\
&\quad \left.+3,(n-k)+3)\right) - \mathrm{Err}(2(n-k)+1,(n-k)+1) \\
&= \frac{1}{2}(\mathrm{Err}(2(n-k)-1,(n-k)) + \mathrm{Err}(2(n-k) \\
&\quad + 3,(n-k)+2)) - \mathrm{Err}(2(n-k)+1,(n-k)+1),
\end{aligned}$$

where $e_{g_0} = 2(k-(n+1)/2+1)$, $e_{i_0} = 2(k-(n+1)/2-1)$ and $e = 2(k-(n+1)/2)$ are from lemma 3, and the last equality is from the claim in (1) Note that by replacing the variable $2(n-k)+1$ with n, we obtain

$$\begin{aligned}
\mathrm{EE} = \frac{1}{2}&(\mathrm{Err}(n+2,(n+3)/2) - \mathrm{Err}(n,(n+1)/2)) \\
&- (\mathrm{Err}(n,(n+1)/2) - \mathrm{Err}(n-2,(n-1)/2)).
\end{aligned}$$

Therefore, $\mathrm{EE} > 0$ if the following claim holds.

Claim 2: The function $\mathrm{Err}(n,(n+1)/2)$ satisfies the property of the increasing differences, i.e.,

$$\begin{aligned}
&\mathrm{Err}((n+1),((n+1)+1)/2) - \mathrm{Err}(n,(n+1)/2) > \\
&\quad \mathrm{Err}(n,(n+1)/2) - \mathrm{Err}((n-1),((n-1)+1)/2).
\end{aligned}$$

Let $B(p,n,k)$ denote the incomplete beta function, where $B(1,n,k) = B(n,k) \equiv ((x-1)!(y-1)!/(x+y-1)!)$ is the beta function, and $I_p(n,k) = (B(p,n,k)/B(n,k))$ denote the regularized beta function. Then

$$\begin{aligned}
\Pr(X \le k) &= \sum_{x=0}^{k} \binom{n}{x} p^x (1-p)^{n-x} \\
&= I_{1-p}(n-k,k+1),
\end{aligned}$$

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where the random variable X follows the binomial distribution. Note that (from the claim in (1))

$$\begin{aligned}\mathrm{Err}(n,(n+1)/2) &= \frac{1}{2}\sum_{x\geq\frac{(n+1)}{2}}\binom{n}{x}(1-p)^x p^{n-x}\\ &= \frac{1}{2}\sum_{x=0}^{k}\binom{n}{x}p^x(1-p)^{n-x}\\ &= \frac{1}{2}\Pr\left(X\leq\frac{(n-1)}{2}\right)\\ &= \frac{1}{2}I_{1-p}(k+1,k+1),\end{aligned}$$

where $k=(n-1)/2$. Hence, to prove that $\mathrm{Err}(n,(n+1)/2)$ satisfies the increasing differences is equivalent to show that

$$\begin{aligned}&I_{1-p}(k+2,k+2)-I_{1-p}(k+1,k+1) >\\ &\quad I_{1-p}(k+1,k+1)-I_{1-p}(k,k).\end{aligned}$$

Note that the regularized beta function has the following properties:

$$I_p(n+1,k)=I_p(n,k)-\frac{p^n(1-p)^k}{nB(n,k)}$$

and

$$I_p(n,k+1)=I_p(n,k)+\frac{p^n(1-p)^k}{kB(n,k)}.$$

Therefore,

$$\begin{aligned}&I_{1-p}(k+2,k+2)-I_{1-p}(k+1,k+1)\\ &\quad=\frac{(1-p)^{k+1}(p)^{k+1}}{(k+1)B(k+1,k+1)}-\frac{(1-p)^{k+1}(p)^{k+2}}{(k+1)B(k+1,k+2)}\\ &\quad=\frac{(1-p)^{k+1}(p)^{k+1}}{(k+1)}\left(\frac{1}{B(k+1,k+1)}-\frac{p}{B(k+1,k+2)}\right)\\ &\quad=\frac{(1-p)^{k+1}(p)^{k+1}}{(k+1)}\left(\frac{1}{B(k+1,k+1)}-\frac{2p}{B(k+1,k+1)}\right)\\ &\quad=\frac{(1-p)^{k+1}(p)^{k+1}}{(k+1)B(k+1,k+1)}(1-2p)<0.\end{aligned}$$

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Then

$$
\begin{aligned}
&I_{1-p}(k+2,k+2) - I_{1-p}(k+1,k+1) \\
&\quad - \left(I_{1-p}(k+1,k+1) - I_{1-p}(k,k)\right) \\
&\quad = \frac{(1-p)^{k+1}(p)^{k+1}}{(k+1)B(k+1,k+1)}(1-2p) - \frac{(1-p)^k(p)^k}{(k)B(k,k)}(1-2p) \\
&\quad = (1-2p)p^k(1-p)^k\left[p(1-p)\frac{(2k+1)!}{(k+1)(k!)^2} - \frac{(2k-1)!}{k((k-1)!)^2}\right] \\
&\quad = (1-2p)p^k(1-p)^k\left[p(1-p)\frac{(2k+1)2k(2k-1)!}{(k+1)k^2((k-1)!)^2}\right. \\
&\qquad \left. - \frac{(2k-1)!}{k((k-1)!)^2}\right] \\
&\quad = (1-2p)p^k(1-p)^k\left(\frac{(2k-1)!}{k((k-1)!)^2}\right) \\
&\qquad \left[p(1-p)\left(\frac{2(2k+1)}{k+1}\right) - 1\right] > 0,
\end{aligned}
$$

where the second equality is from the definition of beta function, and the last inequality is from $p(1-p)(2(2k+1)/k+1)-1 < -p(1-p)(2/k+1)/ < /0$. Thus, $I_{1-p}(k+1,k+1)$ as well as $\mathrm{Err}(n,(n+1)/2$ satisfy the property of the increasing differences and Claim 2 is proved.

Furthermore, since

$$
\begin{aligned}
&\mathrm{Err}(n,(n+1)/2) - \mathrm{Err}(n-2,(n-1)/2) \\
&\quad = \frac{1}{2}\left(I_{1-p}(k+1,k+1) - I_{1-p}(k,k)\right) \\
&\quad = \frac{1}{2}\frac{(1-p)^k(p)^k}{kB(k,k)}(1-2p) \to 0 \ \text{ as } \ k = \frac{n-1}{2} \to \infty,
\end{aligned}
$$

we obtain that $\mathrm{Err}^P - \mathrm{Err}$, which is equal to *the difference* of the difference of errors, approaches 0. Thus, the loss of efficiency due to public information is negligible when the size of jury approaches infinity.

Proof of Proposition 6

When $k = n$, i.e., the unanimous rule, since $e = 2(k - k^*) = 2(n - (n+1)/2)) = n-1$, the error without public information is $\mathrm{Err}(n,n) = \mathrm{Err}(1,1) = (1-p)/2$. It is the error incurred by a single voter who votes

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sincerely. On the other hand, when public information with precision $p_0 \downarrow p$ is available, since $k^*_{g_0}(n) = (n+1)/2 - 1$ and $k^*_{i_0}(n) = (n+1)/2 + 1$, which imply $e_{g_0} = n$, and $e_{i_0} = n - 3$, the error with public information is

$$\begin{aligned}\mathrm{Err}^P &= \frac{1}{2}\mathrm{Err}_{g_0}(n,k) + \frac{1}{2}\mathrm{Err}_{i_0}(n,k)\\ &= \frac{1}{2}\mathrm{Err}_{g_0}(n,n) + \frac{1}{2}\mathrm{Err}_{i_0}(3,3)\\ &= \frac{1}{2}\left(\frac{1}{2}(1-p_0) + \mathrm{Err}(3,2)\right) < \frac{1}{2}(1-p) = \mathrm{Err},\end{aligned}$$

where $\mathrm{Err}_{g_0}(n,n)$ is equal to the error that all voters vote to convict, which is the cost of type I error i.e., $\mathrm{Err}_{g_0}(n,n) = (1-p_0)/2$, and $\mathrm{Err}_{i_0}(3,3) = \mathrm{Err}(3,2)$ when $p_0 \downarrow p$ from Claim 1 in proposition 5. Since $\mathrm{Err}(3,2)$ is smaller than $(1-p)/2$, public information thus reduces the error under the unanimous rule.

Proof of Proposition 7

To investigate the asymptotic information efficiency, we consider the jury of M folds replication of the original jury, i.e., the jury of nM voters with the voting rule kM, where M is an integer. There are $n_1 M$ non-strategic and $n_2 M$ strategic voters with $r_1 = n_1/n$ and $r_2 = n_2/n$ representing the shares of non-strategic and strategic voters, respectively. Let $R = (k/n) \in [1/2, 1]$ denote the voting rule in terms of the ratio of the votes for conviction to the total votes and let $\varepsilon_{s_0} \equiv (e_{g_0}/nM)$ for $s_0 = \emptyset, i_0, g_0$ denote the share of uninformative votes. Note that from lemma 1, since the statistical rule increases at half the speed of the size of the jury, we obtain that $k^*_{s_0}(nM) = k^*_{s_0}(n) + (n(M-1)/2)$ and $R^* \equiv (k^*_{s_0}(nM)/nM) \to (1/2)$ as $M \to \infty$ for $s_0 = \emptyset, i_0, g_0$.[32] Thus, when M is large, the statistical rule is close to $(1/2)$, i.e., the simple majority rule, regardless of whether or not public information is available.

We first analyze the case where public information is absent. Recall that when $(1/2) \le R < 1$, since $\varepsilon = (2|kM - k^*(nM)|/nM) \simeq [2|kM - (1/2)(nM)|/nM] = 2R - 1 < 1$, we have $\mathrm{Err}(1, (k/n)) = \mathrm{Err}(nM, kM) = \mathrm{Err}((1-\varepsilon)nM, (k - n\varepsilon)M) \simeq \mathrm{Err}(2(n-k)M, (n-k)M)$, which approaches 0 as $M \to \infty$ for any $p \in (1/2, 1)$ by the law of large numbers.

[32] Note that $k^*_{s_0}(n) = k^*(n) \pm m < \infty$ when the precision of the public information $p_0 \in (p_m, p_{m+1}]$.

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That is, when the voting rule k is non-unanimous, a $(2k-n)/n = 2R-1$ share of voters will vote to convict regardless of their private signals, and the equilibrium error is equal to the error that there are $2(n-k)M$ voters who vote sincerely under the statistical rule $(k^*(2(n-k)M)/2(n-k)M) \rightarrow ((n-k)M/2(n-k)M) = (1/2)$. Hence, when the state is that the defendant is innocent (guilty), by the law of large numbers, there is a p share of the voters vote to acquit (convict), and the jury makes the correct decisions in equilibrium, i.e., the asymptotic efficiency is reached.

When there are n_1M non-strategic and n_2M strategic voters, if the share of strategic voters is large, i.e., $\varepsilon \simeq [2|kM - (1/2)(nM)|/nM] = (2R-1) < r_2$, or $r_1 < 2(1-R)$, then $\mathrm{Err}(nM, kM) \simeq \mathrm{Err}(2(n-k)M, (n-k)M)$, which approaches 0 as $M \rightarrow \infty$ as $M \rightarrow \infty$. On the other hand, if $r_2 \leq (2R-1)$ or equivalently $r_1 \geq 2(1-R)$, all strategic voters vote to convict in equilibrium and the error is equal to the one of the jury consisting of n_1M sincere voters with the super majority rule: $(R - r_2/r_1) > (1/2)$. Therefore, $\mathrm{Err}(nM, kM) \rightarrow \Pr(G)(1-q) = (1/2)(1-q) \neq 0$, if $p < (k - n_2/n_1) = (R - r_2/r_1)$. Thus, when the share of the non-strategic voters is large and the precision of the private information is low, the jury always acquits the defendant and the asymptotic efficiency cannot be reached..

In sum, when public information is absent, the error is:

$$\mathrm{Err} = \begin{cases} \frac{1}{2}(1-q) \text{ when } r_1 \geq 2(1-R) \text{ and } p < \frac{k-r_2}{r_1}, \\ 0 \text{ when } r_1 < 2(1-R). \end{cases}$$

Next we investigate the case where public information is available. Since all the votes from the non-strategic voters replicate the public signals, when $s_0 = i_0$, all non-strategic voters vote to acquit, we obtain that $\mathrm{Err}_{i_0}(nM, kM) = \mathrm{Err}_{i_0}((n-n_1)M, kM)$. In addition, when $s_0 = g_0$, since all non-strategic voters vote to convict, $\mathrm{Err}_{g_0}(nM, kM) = \mathrm{Err}_{g_0}((n-n_1)M, (k-n_1)M)$. There are three possible cases.

(1) For the case of $r_1 \geq R$, the decisions of the jury replicate the public signals and the jury incurs an error equal to the expected cost that the public signals are wrong, i.e., $\mathrm{Err}^P = (1/2)(1-p_0)$.

(2) For the case of $R > r_1 > (1-R)$, when $s_0 = i_0$, since $r_1 > (1-R)$, the votes (for acquittal) from non-strategic voters alone are large enough to

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acquit the defendant. Therefore, we have

$$\begin{aligned}\mathrm{Err}_{i_0}\left(1, \frac{R}{1-r_1}\right) &\equiv \mathrm{Err}_{i_0}\left((n-n_1)M, kM\right)\\ &= \Pr\left(G|s_0 = i_0\right)(1-q)\\ &= (1-p_0)(1-q) > 0.\end{aligned}$$

When $s_0 = g_0$, since $\varepsilon_{g_0} = 2(R - r_1 - (1 - r_1/2)) = 2R - 1 - r_1 < r_2$, we have

$$\begin{aligned}\mathrm{Err}_{g_0}\left(1, \frac{R-r_1}{1-r_1}\right) &\equiv \mathrm{Err}_{g_0}\left((n-n_1)M, (k-n_1)M\right)\\ &= \mathrm{Err}_{g_0}\left((n-n_1)M - \varepsilon_{g_0}nM, (k-n_1)M\right.\\ &\quad \left.-\varepsilon_{g_0}nM\right)\\ &= \mathrm{Err}_{g_0}(2(1-R)nM, (1-R)nM)\\ &= \mathrm{Err}_{g_0}\left(1, \frac{1}{2}\right) \to 0 \text{ as } M \to \infty.\end{aligned}$$

Therefore, we obtain that $\mathrm{Err}^P = (1/2)\mathrm{Err}_{i_0}(1, (R/1-r_1)) + (1/2)\mathrm{Err}_{g_0}(1, (R-r_1/1-r_1)) = (1/2)(1-p_0)(1-q)$.

(3) For the case of $r_1 \leq (1-R)$, note that $\varepsilon_{g_0} = 2(R-r_1-(1-r_1/2)) < \varepsilon_{i_0} = 2(R-(1-r_1/2)) < r_2$ since $2(R-(1-r_1/2)) < 2(R-(R/2)) < R \leq r_2$. That is, the share of strategic voters is large enough to rectify the situation that the voting rule $(R/1-r_1)$ when $s_0 = i_0$ and $(R-r_1/1-r_1)$ when $s_0 = g_0$ are not equal to the statistical rule $R^* = (1/2)$. Therefore, $\mathrm{Err}_{i_0}(1, (R/1-r_1)) = \mathrm{Err}_{g_0}(1, (R-r_1/1-r_1\backslash)) \to 0$ as $M \to \infty$.

In sum, the asymptotic equilibrium error with public information is as follows:

$$\begin{aligned}\mathrm{Err}^P &= \frac{1}{2}\mathrm{Err}_{g_0}\left(1, \frac{R-r_1}{1-r_1}\right) + \frac{1}{2}\mathrm{Err}_{i_0}\left(1, \frac{R}{1-r_1}\right)\\ &= \begin{cases} \frac{1}{2}(1-p_0) & \text{when } r_1 \geq R,\\ \frac{1}{2}(1-p_0)(1-q) & \text{when } R > r_1 > (1-R),\\ 0 & \text{when } r_1 \leq (1-R).\end{cases}\end{aligned}$$

By comparing Err with Err^P, we thus obtain the results in the proposition.

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本文探討公開訊息 (public information) 如何影響投票行爲及結果。其影響可以分解成訊息效果 (the information effect) 和均衡效果 (the equilibrium effect)。公開訊息一方面提供額外的訊息, 讓投票者做出較好的決。另一方面, 由於策略性的投票, 公開訊息會影響投票者的均衡行爲。我們發現, 對任何非全體決的投票規則 (non-unanimous voting rules), 當公開訊息的準確度 (the precision of public information) 低時, 訊息效果可以忽略, 但是均衡效果會產生負面的影響, 因此, 公開訊息反而會讓投票結果的錯誤增加。相反的, 在全體決下 (the unanimous voting rule), 均衡效果的影響是正面的, 因此, 公開訊息會改善投票的結果。我們進一步探討非策略性投票者 (non-strategic voters) 對投票結果的影響, 當存在這些投票者時, 公開訊息實質上 (*de facto*) 改變了現存的投票規則, 如果這個改變讓實質的規則更接近最適的投票規則, 公開訊息就會改善投票的結果。

關鍵詞: 策略性的投票, 公開訊息, Condorcet jury theorem
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