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Quality Licensing in a Vertically Differentiated Oligopoly

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This paper investigates the optimal licensing contract for product innovation in a vertically differentiated oligopoly in which a high-quality firm competes with other low-quality firms. We find that the high-quality firm will never license its technology to all the low-quality firms if there are more than two low-quality firms, which is a result that has never been documented in the literature. We also show that fixed-fee licensing is more profitable for the licensor, if the technology gap is small, which is a result opposite to that in the cost-reducing licensing literature. We further show that under certain circumstances, fixed-fee licensing is less socially desirable than royalty licensing. Finally, if the licensor firm adopts a two-part tariff contract, then the optimal contract may consist of a pure royalty, a positive royalty and a positive fixed fee, or a pure fixed fee, varying with the technology gap between the two types of firms. These results hold even if the licensees are subject to capacity constraints and cannot raise their quality to the leucl of then of the licensor via licensing.

Keywords: technology licensing, vertical product differentiation, oligopolistic competition, social welfare
JEL classification: D45, L13, L15, L24

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1 Introduction

Over the past two decades, technology licensing has drawn considerable attention and has been studied extensively in the literature. Although the empirical evidence has shown that a large portion of innovations are actually product innovations,[1] the studies on product licensing are rare and they all assume that one high-quality firm competes with only one low-quality firm in the market (see, for example, Li and Song (2009); Chang and Peng (2013); Nabin, Nguyen, and Sgro (2013)). However, a duopolistic framework can only reveal the rivalry between the licensor firm and the licensee firm, while failing to capture that the rivalry between the licensee and non-licensee firms may alter the licensor firm's licensing decision and subsequent welfare.

It is commonly observed that one high-quality firm competes with many low-quality firms in the market. For example, SanDisk, the global market leader in flash data storage products, not only produces memory cards, but also licenses its advanced technology to other manufacturers in China, South Korea and Taiwan. Sony produces blue-ray DVDs and licenses the technology on producing high-quality blue-ray DVDs to its rivals in China, Japan, South Korea, and Taiwan. The common features of these two examples are: (1) the quality-leading firm competes with more than one low-quality rival in the market; (2) the quality-leading firm only licenses its technology to some of its low-quality rivals. However, in a duopolistic model, no firm produces low-quality products after licensing, which conflicts with observations in the real world. Therefore, this paper considers an oligopolistic model in which one quality-leading firm competes with more than one low-quality firm and investigates the licensing behavior of the licensor firm and the resulting social welfare.

The literature on the optimal licensing contract of an insider patentee is vast (see for example, Wang (1998); Kamien and Tauman (2002); Poddar and Sinha (2004); Chang, Hwang, and Peng (2013)). The standard result along this line indicates that royalty licensing is the optimal contract for an insider licensor, but a royalty contract generates lower consumer surplus and

[1] As shown in Cohen, Nelson, and Walsh (1997), 51.5% of patent applications in the U.S. manufacturing sector between 1991 and 1993 were product innovations and only 33% were process innovations. Arundel and Kabla (1998) also find that the percentage of innovations for which a patent application is made for product innovations is 35.9%, and it is 24.8% for process innovations among Europe's industrial firms.

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social welfare than a fixed-fee contract. The result holds even in the literature of product licensing in a duopoly (Li and Song, 2009; Chang and Peng, 2013). The reason is that a royalty rate incurs weaker competition between the licensor firm and the licensee firm, in turn resulting in higher profit and lower social welfare. In this paper we propose a contrary result and show that royalty licensing is more socially desirable under certain circumstances as more low-quality firms are licensed under a royalty contract.

This is the first paper in the literature to consider product licensing in an oligopolistic market. This oligopolistic market structure is important as it not only reveals the rivalry between the licensor firm and the licensee firms, but also generates the competition effect among the licensee and non-licensee firms. In this sense, our paper is more general than previous studies as a duopoly is only a special case of an oligopoly.

We find that the high-quality firm will never license its technology to all its low-quality rivals, regardless of the licensing contracts adopted by the high-quality firm and the technology gap between the firms. This result reflects the reality in which only some of the low-quality firms can acquire new technologies via licensing. In addition, the number of licenses offered by the high-quality firm increases as the technology gap decreases. We also find that fixed-fee licensing is superior (inferior) to royalty licensing for the high-quality firm, if the technology gap is small (large). This finding is in contrast to the result in Wang (2002), as he considers the optimal licensing contract in a horizontal product differentiation model and finds that a fixed-fee contract is superior to a royalty contract if product differentiation is large. We will show that under certain circumstances, royalty licensing is more socially desirable than fixed-fee licensing, because more low-quality firms are licensed under a royalty contract. The optimal two-part tariff licensing contract may consist of a pure royalty, a positive royalty and a positive fixed fee, or a pure fixed fee, varying with the technology gap between the two types of firms. This result is also different from that in Li and Song (2009) in which a two-part tariff contract only comprises a pure royalty. When the licensee firms are subject to absorptive capacity and can improve the quality not as much as the licensor via licensing, the optimal one-part and two-part tariff licensing schedules remain hold if the absorptive capacity is high; however licensing will not occur if the absorptive capacity is low.

The remainder of this paper is organized as follows. Section 2 introduces vertically differentiated oligopolistic licensing models under a royalty contract and a fixed-fee contract. Section 3 compares the profitability un-

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der the two licensing contracts and the resulting welfare levels. Section 4 examines the two-part tariff licensing contract. Section 5 investigates the case in which the licensee firms are subject to absorptive capacity. Section 6 concludes the paper.

2 Royalty and Fixed-Fee Licensing

Consider a Cournot model with one high-quality firm, Firm H, and three low-quality firms, from Firm 1 to Firm 3.[2] Firm H produces a product, q_H, with high quality, $s_H = 1$, while the low-quality firms produce a product, q_j for $j = 1, 2, 3$, with low quality, $s_L = \beta$ and $0 < \beta < 1$. We further assume that the firms compete in Cournot fashion in the market and the marginal production costs for the firms are nil. Following Motta (1993), it is assumed that consumers buy, at most, one unit of either product with the utility function specified as follows:

$$U = \begin{cases} \theta s_k - p_k & \text{if the consumer buys product } k, \text{ for } k = H, L; \\ 0 & \text{if the consumer buys neither product,} \end{cases} \quad (1)$$

where s_k and p_k denote the quality and the price of product k, respectively. Each consumer decides which product to purchase or not to buy at all in order to maximize his/her surplus. The parameter θ represents the consumers' willingness to pay for quality and is distributed uniformly on [0,1] with unit density. Moreover, the population of consumers is normalized to one.

Let $\theta_{\phi L}$ be the value at which the corresponding consumer is indifferent between buying nothing and buying the low-quality product, and θ_{LH} is the value at which the corresponding consumer is indifferent between buying the high-quality product and the low-quality one. Consumers whose willingness to pay for quality is within the range $\theta_{LH} \leq \theta \leq 1$ ($\theta_{\phi L} \leq \theta < \theta_{LH}$) will buy the high-quality (low-quality) product, whereas consumers whose willingness to pay for quality is within the range $\theta < \theta_{\phi L}$ will not buy either product. Given the above assumption, the inverse demand functions for

[2]This assumption is to simplify our analysis and make our results more obvious and clear. The detailed calculations for the case with n low-quality firms can be obtained upon request. In addition, if there are only two low-quality firms, the competition effect of the low-quality firms will not be strong enough. Thus, the results are mainly the same with the literature. The results under two low-quality firms can also be obtained upon request to the corresponding author.

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the high-quality and the low-quality products are derivable from the above utility function as:

$$p_H(Q_H, Q_L; \beta) = 1 - Q_H - \beta Q_L,$$
$$p_L(Q_H, Q_L; \beta) = \beta(1 - Q_H - Q_L),$$

where Q_H and Q_L represent the total outputs of the high-quality product and the low-quality product, respectively. Moreover, $Q_H = q_H$ and $Q_L = \sum_{j=1}^{3} q_i$ with no licensing. Therefore, the profits of the high-quality firm and the low-quality firms with no licensing are respectively specified as follows:

$$\Pi_H = \left(1 - q_H - \beta \sum_{j=1}^{3} q_j\right) q_H,$$
$$\Pi_j = \beta \left(1 - q_H - \sum_{j=1}^{3} q_j\right) q_j, \quad \text{for} \quad j = 1, 2, 3.$$

Through routine calculations, we can derive the equilibrium outputs and profits for the two kinds of firms as follows:

$$q_H^{NL} = \frac{4 - 3\beta}{8 - 3\beta}, q_j^{NL} = \frac{1}{8 - 3\beta},$$
$$\Pi_H^{NL} = \left(\frac{4 - 3\beta}{8 - 3\beta}\right)^2, \quad \text{and} \quad \Pi_j^{NL} = \beta \left(\frac{1}{8 - 3\beta}\right)^2,$$

where the variables with the superscript "NL" are associated with the no licensing regime.

We next proceed to consider the case in which Firm H (the licensor firm, hereafter) licenses its superior technology to the low-quality firms (hereafter, the licensee firms) via either a fixed-fee or a royalty contract. It is assumed that the licensee firms will have the same quality as the licensor firm if licensing occurs. The licensing game comprises three stages. In the first stage, the licensor firm determines the number of licensee firms and its optimal fixed fee or royalty under the corresponding licensing contract. In the second stage, the chosen licensee firms (firm) decide(s) whether to accept the offer. In the third stage, the firms compete in quantity terms in the market. The sub-game perfect Nash equilibrium is solved through backward induction

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by first considering the third-stage game (the output decisions), followed by the second-stage game (the licensee firms' decision), and then the first-stage game (the licensor firm's optimal licensing contract).

In what follows, we shall examine the equilibria under the fixed-fee licensing regime followed by the royalty licensing regime. We now proceed to the case that the licensor firm licenses its advanced technology to the licensee firms by using a fixed fee, F. Following the literature, we assume that the licensor firm is a dominant player in the licensing game, being able to extract the entire rent from licensing. Hence, the profits for the licensee firms remain the same before and after licensing, and all the rent from the licensing goes to the licensor firm.

We focus on the case in which the licensor firm licenses its technology to $m(m \leq 3)$ firms. As a result, the set of low-quality firms N can be partitioned into two subsets: the set of licensee firm(s) M and the set of non-licensee firm(s) $N\backslash M$. Let i and j denote the firm(s) in M and $N\backslash M$, respectively. Thus, we specify the objective functions for the firms as follows:

$$\Pi_H^F = \left(1 - q_H - \sum_{i \in M} q_i - \beta \sum_{j \in N\backslash M} q_j\right) q_H + mF, \tag{2}$$

$$\Pi_i^F = \left(1 - q_H - \sum_{i \in M} q_i - \beta \sum_{j \in N\backslash M} q_j\right) q_i - F, \tag{3}$$

$$\Pi_j^F = \beta \left(1 - q_H - \sum_{i \in M} q_i - \sum_{j \in N\backslash M} q_j\right) q_j, \tag{4}$$

where the variables with the superscript "F" are associated with the fixed-fee licensing regime, and F is the fixed fee charged by the licensor firm. By routine calculation, the equilibrium outputs of the firms in the third stage are as follows:

$$q_H^F = q_i^F = \frac{1 + (1-\beta)(3-m)}{5 + (1-\beta)(3-m)(m+1)}, \tag{5}$$

$$q_j^F = \frac{1}{5 + (1-\beta)(3-m)(m+1)}. \tag{6}$$

In the second stage, the licensee firms take the licensing contract offered by the licensor firm if their profits are no worse off after licensing. In

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the first stage, the licensor firm determines its optimal fixed fee, which is the difference in profits for a licensee firm with and with no licensing, i.e., $\Pi_i^F|_m - \Pi_j^F|_{m-1}$. Therefore, should licensing occur, we derive the fixed fee charged by the licensor firm in the first stage as follows:

$$F = \left(\frac{1+(1-\beta)(3-m)}{5+(1-\beta)(3-m)(m+1)}\right)^2 - \beta\left(\frac{1}{5+(1-\beta)(3-m+1)m}\right)^2. \tag{7}$$

By substituting (5) to (7) into (2) to (4), we derive the profits for the licensor firm, the licensee firm, and the non-licensee firm as follows:

$$\begin{aligned}
\Pi_H^F &= \left(\frac{1+(1-\beta)(3-m)}{5+(1-\beta)(3-m)(m+1)}\right)^2 \\
&\quad + m\left[\left(\frac{1+(1-\beta)(3-m)}{5+(1-\beta)(3-m)(m+1)}\right)^2 \right. \\
&\quad \left. -\beta\left(\frac{1}{5+(1-\beta)(3-m+1)m}\right)^2\right], \qquad (8)\\
\Pi_i^F &= \beta\left(\frac{1}{5+(1-\beta)(3-m+1)m}\right)^2, \\
\Pi_j^F &= \beta\left(\frac{1}{5+(1-\beta)(3-m)(m+1)}\right)^2.
\end{aligned}$$

We now examine whether the licensor firm has an incentive to license its technology to all the low-quality firms. By substituting $m = 0, 1, 2, 3$ into (8) respectively, the profits of the licensor firm with different numbers of licenses sold can be expressed as follows:

$$\begin{aligned}
\Pi_H^F\big|_{m=0} &= \left(\frac{4-3\beta}{8-3\beta}\right)^2 = \Pi_H^{NL}, \\
\Pi_H^F\big|_{m=1} &= \left(\frac{3-2\beta}{9-4\beta}\right)^2 + \left[\left(\frac{3-2\beta}{9-4\beta}\right)^2 - \beta\left(\frac{1}{8-3\beta}\right)^2\right],
\end{aligned}$$

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$$\Pi_H^F\big|_{m=2} = \left(\frac{2-\beta}{8-3\beta}\right)^2 + 2\left[\left(\frac{2-\beta}{8-3\beta}\right)^2 - \beta\left(\frac{1}{9-4\beta}\right)^2\right],$$

$$\Pi_H^F\big|_{m=3} = \left(\frac{1}{5}\right)^2 + 3\left[\left(\frac{1}{5}\right)^2 - \beta\left(\frac{1}{8-3\beta}\right)^2\right].$$

By comparing the above profit levels, we can find that:

$$\Pi_H^F\big|_{m=1} - \Pi_H^F\big|_{m=0} = \frac{(1-\beta)\left(72\beta^3 - 344\beta^2 + 471\beta - 144\right)}{(9-4\beta)^2(8-3\beta)^2} > 0 \quad \text{if } \beta > 0.427,$$

$$\Pi_H^F\big|_{m=2} - \Pi_H^F\big|_{m=0} = \frac{2(1-\beta)\left(48\beta^3 - 255\beta^2 + 404\beta - 162\right)}{(9-4\beta)^2(8-3\beta)^2} > 0 \quad \text{if } \beta > 0.6069,$$

$$\Pi_H^F\big|_{m=2} - \Pi_H^F\big|_{m=1} = \frac{(1-\beta)(5-\beta)\left(36 - 53\beta + 12\beta^2\right)}{(9-4\beta)^2(8-3\beta)^2} > 0 \quad \text{if } \beta > 0.8384,$$

$$\Pi_H^F\big|_{m=1} - \Pi_H^F\big|_{m=3} = \frac{2\,(1-\beta)\left(4032 - 6951\beta + 3656\beta^2 - 612\beta^3\right)}{25(8-3\beta)^2(9-4\beta)^2} > 0,$$

and

$$\Pi_H^F\big|_{m=2} - \Pi_H^F\big|_{m=3} = \frac{(1-\beta)\left(3564 - 5477\beta + 3162\beta^2 - 624\beta^3\right)}{25(8-3\beta)^2(9-4\beta)^2} > 0.$$

According to the above result, we can display the optimal fixed-fee licensing schedule for the licensor firm as follows:

$$\begin{cases} \Pi_H^F\big|_{m=0} \geq \Pi_H^F\big|_{m=1} > \Pi_H^F\big|_{m=2}, & m^* = 0 \quad \beta \in (0, 0.427], \\ \Pi_H^F\big|_{m=1} > \Pi_H^F\big|_{m=0} \geq \Pi_H^F\big|_{m=2}, & m^* = 1 \quad \text{if} \quad \beta \in (0.427, 0.6069], \\ \Pi_H^F\big|_{m=1} \geq \Pi_H^F\big|_{m=2} > \Pi_H^F\big|_{m=0}, & m^* = 1 \quad \beta \in (0.6069, 0.8384], \\ \Pi_H^F\big|_{m=2} > \Pi_H^F\big|_{m=1} > \Pi_H^F\big|_{m=0}, & m^* = 2 \quad \beta \in (0.8384, 1). \end{cases} \tag{9}$$

This result implies that the licensor firm will never license its technology to all the low-quality firms under an oligopolistic market. In other words, both high-quality and low-quality products exist in the market after licensing occurs. In addition, it is obvious that the number of licenses sold by the licensor firm increases as the technology gap decreases under the fixed-fee licensing regime. In sum, we build the first proposition.

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Proposition 1. In a vertically differentiated oligopolistic market, fixed-fee licensing occurs if the technology gap is small. In addition, the number of licenses sold by the licensor firm increases as the technology gap decreases. However, the licensor firm will never license to all its low-quality rivals.

The intuition for the above proposition is as follows. Licensing generates two effects for the licensor firm.[3] The first effect is called the *own operation profit effect* which is negative, as more firms produce the high-quality product with the same marginal cost if fixed-fee licensing occurs. This effect is small when β is large or m is small. The second effect is called the *licensing revenue effect* which is positive, as the licensor firm can extract from the licensee firms the entire rent accrued from the technology. This effect is small when β is large, but it is non-monotone with respect to m. When β is very small, the first effect is huge and dominates the second effect, the licensor firm has no incentive to license its technology. When β is median, the first effect becomes weaker and thus is dominated by the positive *licensing revenue effect*. Under such a circumstance, relative to no licensing, licensing to one firm incurs the highest profit for the licensor firm. When β is large, the first effect is very small and the second effect becomes more important. The licensor firm can increase its profit by licensing to two firms. However, licensing to all the three low-quality firms will never be an optimal strategy as it will make the high-quality market too competitive, generating less profit than that under licensing to one firm or two firms.

We now consider the case where the licensor firm licenses its technology via a royalty contract. All the settings here are the same as those in the fixed-fee licensing regime, except that the licensor firm now charges a royalty rate r instead of a fixed fee. We first specify the objective functions for the firms as follows:

$$\Pi_H^R = \left(1 - q_H - \sum_{i \in M} q_i - \beta \sum_{j \in N \setminus M} q_j\right) q_H + mrq_i, \tag{10}$$

$$\Pi_i^R = \left(1 - q_H - \sum_{i \in M} q_i - \beta \sum_{j \in N \setminus M} q_j - r\right) q_i, \tag{11}$$

[3]We thank an anonymous referee for suggesting us to adopt these two effects in explaining our results.

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$$\Pi_j^R = \beta \left(1 - q_H - \sum_{i \in M} q_i - \sum_{j \in N \setminus M} q_j \right) q_j, \tag{12}$$

where the variables with superscript "R" indicate that they are associated with the royalty licensing regime. By routine calculation, the equilibrium outputs of the firms in the third stage are derived as follows:

$$q_H^R = \frac{(1+mr)(1+(1-\beta)(3-m))}{5+(m+1)(1-\beta)(3-m)}, \tag{13}$$

$$q_i^R = \frac{1+(1-\beta)(3-m)-(2+(2-\beta)(3-m))r}{5+(m+1)(1-\beta)(3-m)}, \tag{14}$$

$$q_j^R = \frac{1+mr}{5+(m+1)(1-\beta)(3-m)}. \tag{15}$$

From the above equations, it is apparent that the outputs of the non-licensee (licensee) firms increase (decrease) with the royalty rate. This implies that the profits of the licensee (non-licensee) firms decrease (increases) as the royalty rate increases.

In the second stage, the licensee firms determine whether to accept the contract offered by the licensor firm. Again, the licensee firms will take the offer only if their profits are no worse off – that is, $\Pi_i^R|_m = \Pi_j^R|_{m-1}$. Given this constraint, we derive the upper bound of the royalty rate as follows:

$$\begin{aligned} \bar{r} = & \Big\{ [5+m(1-\beta)(4-m)][1+(1-\beta)(3-m)] \\ & - [5+(m+1)(1-\beta)(3-m)]\sqrt{\beta} \Big\} \Big/ \\ & \Big\{ [5+m(1-\beta)(4-m)][2+(2-\beta)(3-m)] \\ & + (m-1)[5+(m+1)(1-\beta)(3-m)]\sqrt{\beta} \Big\}. \end{aligned} \tag{16}$$

In the first stage, the licensor firm determines its optimal royalty rate subject to the participating constraint of the licensee firms. Thus, we have:

$$\begin{aligned} \underset{r}{Max} \quad & \Pi_H^R = \left(q_H^R\right)^2 + mrq_i^R, \\ \text{s.t.} \quad & 0 \le r \le \bar{r}. \end{aligned}$$

By differentiating the profit function with respect to r and setting it equal to zero, we derive the royalty rate, $r*$, as follows:

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$$r^* = \left\{\left(4 - m^2(1-\beta) + 3(4-3\beta)\right)\left(1 + (3-m)(1-\beta)\right)\right\} \Big/ \left\{2\left((1-\beta)\left(m^3 - m^2(4+\beta)\right) + (2+3(2-\beta))^2 + m\left(16\beta - 16 - 6\beta^2\right)\right)\right\}.$$

Note that the licensor firm's optimization is subject to the licensee firm's incentive constraint, which requires $r* \leq \bar{r}$. By comparing $r*$ to $\bar{r}$, it is found that the former is always higher than the latter. Therefore, the optimal royalty rate is equal to $\bar{r}$ and is necessarily a corner solution. By substituting (13) to (16) into (10), we derive the equilibrium profit of the licensor firm as a function of β and m, expressed as follows:

$$\begin{aligned}\Pi_H^R &= \left(q_H^R\right)^2 + m\bar{r}q_i^R \\ &= \left(\frac{(1+m\bar{r})\left(1+(1-\beta)(3-m)\right)}{5+(m+1)(1-\beta)(3-m)}\right)^2 \\ &\quad + m\bar{r}\left(\frac{1+(1-\beta)(3-m)-(2+(2-\beta)(3-m))\bar{r}}{5+(m+1)(1-\beta)(3-m)}\right). \quad (17)\end{aligned}$$

By substituting $m = 0, 1, 2, 3$ into (17) respectively, we can derive the profits of the licensor firm with different numbers of licenses sold as follows:

$$\Pi_H^R\big|_{m=0} = \left(\frac{4-3\beta}{8-3\beta}\right)^2 = \Pi_H^{NL}, \quad (18)$$

$$\Pi_H^R\big|_{m=1} = \left(576 - 1245\beta + 943\beta^2 - 304\beta^3 + 36\beta^4 + 48\beta^{\frac{3}{2}} - 50\beta^{\frac{5}{2}} + 12\beta^{\frac{7}{2}}\right) \Big/ \left(4(8-3\beta)^2(3-\beta)^2\right), \quad (19)$$

$$\Pi_H^R\big|_{m=2} = \left(324 - 704\beta + 497\beta^2 - 147\beta^3 + 16\beta^4 + 144\beta^{\frac{1}{2}} - 172\beta^{\frac{3}{2}} + 66\beta^{\frac{5}{2}} - 8\beta^{\frac{7}{2}}\right) \Big/ \left(\left(36 - 25\beta + 4\beta^2 + 8\beta^{\frac{1}{2}} - 3\beta^{\frac{3}{2}}\right)^2\right), \quad (20)$$

$$\Pi_H^R\big|_{m=3} = \frac{64 - 107\beta + 9\beta^2 + 80\beta^{\frac{1}{2}} - 30\beta^{\frac{3}{2}}}{4\left(8 + 5\beta^{\frac{1}{2}} - 3\beta\right)^2}. \quad (21)$$

By comparing the above profit levels, we derive that:[4]

[4]Please refer to Appendix 1 for the detailed derivations and proofs of these results.

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$$\Pi_H^R\big|_{m=1} - \Pi_H^R\big|_{m=0} > 0; \tag{22}$$

$$\Pi_H^R\big|_{m=1} - \Pi_H^R\big|_{m=3} > 0; \tag{23}$$

$$\Pi_H^R\big|_{m=2} - \Pi_H^R\big|_{m=1} > 0, \quad \text{if } \beta > 0.4022; \tag{24}$$

$$\Pi_H^R\big|_{m=2} - \Pi_H^R\big|_{m=0} > 0, \quad \text{if } \beta > 0.1099. \tag{25}$$

Equations (22) and (23) imply that no licensing and licensing to all the low-quality firms are the strategies dominated by licensing to only one low-quality firm. The economic explanation is as follows. Similar to that under fixed-fee licensing, royalty licensing also has two effects: the *own operation profit effect* and the *licensing revenue effect*. Nevertheless, in this regime, these two effects now are not only affected by β and m but also by the royalty rate, r. Under royalty licensing, the *own operation profit effect* increases with r whereas the *licensing revenue effect* is concave in r.[5] Thus, the licensor firm has an incentive to carry out royalty licensing to at least one licensee firm. Nevertheless, the optimal royalty rate is decreasing with m. The *own operation profit effect* under $m = 3$ is much smaller than that under $m = 1$. Therefore, licensing to three firms will not be an optimal strategy for the licensor firm under the royalty licensing contract. We further investigate whether licensing to one or two low-quality firms is optimal for the licensor firm under royalty licensing. From (24), the licensor firm will license its technology to one firm if $\beta < 0.4022$; otherwise, licensing to two firms is more profitable. The intuition of this result is similar to that under fixed-fee licensing. Thus, we can summarize the optimal royalty licensing schedule for the licensor firm as follows:

$$\begin{cases} \Pi_H^R\big|_{m=1} > \Pi_H^R\big|_{m=0} \geq \Pi_H^R\big|_{m=2}, m^* = 1 & \beta \in (0, 0.1099], \\ \Pi_H^R\big|_{m=1} \geq \Pi_H^R\big|_{m=2} > \Pi_H^R\big|_{m=0}, m^* = 1 & \text{if } \beta \in (0.1099, 0.4022], \\ \Pi_H^R\big|_{m=2} > \Pi_H^R\big|_{m=1} > \Pi_H^R\big|_{m=0}, m^* = 2 & \beta \in (0.4022, 1). \end{cases} \tag{26}$$

Given the above discussion, we construct the second proposition as follows.

Proposition 2. In a vertically differentiated oligopolistic market, the licensor firm always has an incentive to carry out royalty licensing. The larger

[5]This is because an increase in r raises the marginal cost of the licensee firms. As a result, the *own operation profit* of the licensor firm will also increase. On the other hand, the optimal royalty rate, $\bar{r}$, decreases as m increases. Therefore, the *licensing revenue effect*, $m\bar{r}q_i^R$, is concave in r.

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the technology gap is, the fewer the number of licenses that will be sold. However, the licensor will never license to all the low-quality rivals.

The result in Proposition 2 is in contrast to the cost-reducing licensing literature in which the insider licensor firm always has an incentive to license its technology to all its high-cost rivals under royalty licensing (see, for example, Kamien and Tauman (2002); and Chang, Hwang, and Peng (2013)). This is because in cost-reducing licensing models, the firms produce an identical product and the substitutability of the product is not changed by licensing. Thus, the licensor firm can always charge a royalty rate up to its innovation level to control the outputs of the licensees exactly back to the no-licensing level. By doing so, the *own operation profit effect* is not changed by licensing and the *licensing revenue effect* is always positive and increases with the number of licenses sold. However, in our model, product licensing decreases the product differentiation (or substitutability, equivalently) between the licensor firm and the licensee firms, resulting in a lower *own operation profit effect* for the former.

3 The Optimal One-Part Tariff Licensing Contract and Welfare

We now move to compare the profitability under the fixed-fee and royalty licensing regimes from the licensor firm's perspective. We do this comparison, because sometimes the licensor firm can only charge a one-part tariff owing to the monitoring costs of production being high. In this context, the licensor firm can barely adopt its suboptimal one-part licensing contract instead of undertaking the optimal two-part tariff licensing contract (i.e., a pre-unit royalty plus an upfront fee). We will also compare the resulting social welfare levels under royalty and fixed-fee regimes.

In the previous section, we have derived the optimal fixed-fee contract and the optimal royalty contract. By comparing (9) to (26), we derive the optimal one-part tariff licensing contract under different technology gaps as follows:[6]

[6]The detailed proofs for the results in this section are shown in Appendix 2.

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$$\begin{cases} \Pi_H^R\big|_{m^*=1} > \Pi_H^F\big|_{m^*=0}, \ r^* = \bar{r}|_{m^*=1}, & m^* = 1 & \beta \in (0, 0.4022]; \\ \Pi_H^R\big|_{m^*=2} > \Pi_H^F\big|_{m^*=0}, \ r^* = \bar{r}|_{m^*=2}, & m^* = 2 & \beta \in (0.4022, 0.427]; \\ \Pi_H^R\big|_{m^*=2} \geq \Pi_H^F\big|_{m^*=1}, \ r^* = \bar{r}|_{m^*=2}, & m^* = 2 \quad \text{if} & \beta \in (0.427, 0.6611]; \\ \Pi_H^F\big|_{m^*=1} > \Pi_H^R\big|_{m^*=2}, \ F^* = F(\beta)|_{m^*=1}, & m^* = 1 & \beta \in (0.6611, 0.8384]; \\ \Pi_H^F\big|_{m^*=2} > \Pi_H^R\big|_{m^*=2}, \ F^* = F(\beta)|_{m^*=2}, & m^* = 2 & \beta \in (0.8384, 1), \end{cases}$$

where

$$\bar{r}|_{m^*=1} = \frac{(3-2\beta)(8-3\beta) - (9-4\beta)\sqrt{\beta}}{(8-3\beta)(6-2\beta)},$$

$$\bar{r}|_{m^*=2} = \frac{18 - 17\beta + 4\beta^2 - 8\beta^{\frac{1}{2}} + 3\beta^{\frac{3}{2}}}{36 - 25\beta + 4\beta^2 + 8\beta^{\frac{1}{2}} - 3\beta^{\frac{3}{2}}},$$

$$F(\beta)|_{m^*=1} = \left(\frac{3-2\beta}{9-4\beta}\right)^2 - \beta\left(\frac{1}{8-3\beta}\right)^2; \quad \text{and}$$

$$F(\beta)|_{m^*=2} = \left(\frac{2-\beta}{8-3\beta}\right)^2 - \beta\left(\frac{1}{9-4\beta}\right)^2.$$

The above result implies that it is more profitable for the licensor firm to adopt a royalty (fixed-fee) licensing contract when the technology gap is large (small). The intuition is as follows. While the *own operation profit effect* is higher under royalty licensing, the *licensing revenue effect* is higher under fixed-fee licensing.[7] When the technology gap is small, firms are very competitive in the market and the competition profit is not high. Thus, the licensor firm will use fixed-fee licensing to extract more licensing revenue. By contrast, if the technology gap is large, using fixed-fee contract will incur a great loss on its *own operation profit*. Thus, the licensor firm shall adopt royalty licensing. This result is in contrast to that in Li and Song (2009), who examine the optimal licensing contract in a vertically differentiated duopoly. They find that the licensor firm always transfers its superior technology to its rivals by means of a royalty contract. Our finding is also different from that in Wang (2002), who considers the optimal licensing contract in a horizontal product differentiation model and finds that a fixed fee is superior to a royalty if the product differentiation is large. In our model, the licensor firm has an incentive to adopt fixed-fee licensing when the technology gap is small, because the fixed-fee licensing contract makes the licensee firms more aggressive than the non-licensee firms. The licensor firm then extracts the

[7] This is because the licensee firms' marginal cost is lower under fixed-fee licensing.

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entire gain of the licensee firms from licensing via the fixed fee. In sum, we build the next proposition as follows.

Proposition 3. In a vertically differentiated oligopolistic market, the licensor firm finds it more profitable to license its advanced technology to its low-quality rivals via a royalty (fixed-fee) contract if the technology gap is large (small).

We next proceed to compare the social welfare under the two one-part tariff licensing regimes. Given the Motta type utility function in Equation (1), consumer surplus is derivable as follows:

$$\begin{aligned} CS &= \int_{\theta_{\phi \mathrm{L}}}^{\theta_{\mathrm{LH}}} (\theta\beta - p_L)\, d\theta + \int_{\theta_{LH}}^{1} (\theta - p_H)\, d\theta \\ &= \int_{1-Q_H-Q_L}^{1-Q_H} (\theta\beta - p_L)\, d\theta + \int_{1-Q_H}^{1} (\theta - p_H)\, d\theta \\ &= \frac{1}{2}\beta Q_L^2 + \frac{1}{2}Q_H^2 + \beta Q_H Q_L. \end{aligned}$$

In addition, the social welfare is defined as the sum of the consumer surplus and the firms' profits, which can be expressed as follows:

$$SW = CS + \Pi_H + \sum_{i \in M} \Pi_i + \sum_{j \in N \setminus M} \Pi_j.$$

If $\beta \in (0, 0.427]$, fixed-fee licensing will not occur. The social welfare under fixed-fee licensing is the same as that under no licensing, i.e., $SW^F|_{m^*=0} = SW^{NL}$. In contrast, one license is (two licenses are) sold under royalty licensing if $\beta \in (0, 0.4022]$($\beta \in (0.4022, 0.427]$). Therefore, social welfare under royalty licensing is necessarily higher than that under fixed-fee licensing as $SW^R|_{m^*=1} > SW^R|_{m^*=0} = SW^F|_{m^*=0} = SW^{NL}$ if $\beta \in (0, 0.4022]$ and $SW^R|_{m^*=2} > SW^R|_{m^*=0} = SW^F|_{m^*=0} = SW^{NL}$ if $\beta \in (0.4022, 0.427]$.[8]

We then compare the welfare levels when $\beta \in (0.427, 1]$. Under a fixed-fee contract, the licensor licenses to only one low-quality firm if $\beta \in (0.427, 0.8384]$, but it licenses to two low-quality firms if $\beta \in (0.8384, 1)$. Thus, the corresponding social welfare levels under fixed-fee licensing are as

[8]The proof for this result is shown in Appendix 2.

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follows:

$$SW^F = \begin{cases} SW^F\big|_{m^*=1} = \frac{4\left(9-8\beta+2\beta^2\right)}{(9-4\beta)^2} & \beta \in (0.427, 0.8384], \\ \text{if} & \\ SW^F\big|_{m^*=2} = \frac{3\left(20-15\beta+3\beta^2\right)}{2(8-3\beta)^2} & \beta \in (0.8384, 1). \end{cases} \tag{27}$$

Under a royalty contract, the licensor firm always licenses out the technology to two low-quality firms if $\beta \in (0.427, 1)$. Therefore, the welfare level under royalty licensing contract is as follows:

$$\begin{aligned} &SW^R\big|_{m^*=2} \\ &= \left(972 - 1209\beta + 588\beta^2 - 143\beta^3 + 16\beta^4 + 648\beta^{\frac{1}{2}} - 702\beta^{\frac{3}{2}}\right. \\ &\quad \left. + 238\beta^{\frac{5}{2}} - 24\beta^{\frac{7}{2}}\right) \Big/ \left(2\left(36 - 25\beta + 4\beta^2 + 8\beta^{\frac{1}{2}} - 3\beta^{\frac{3}{2}}\right)^2\right) \\ &\quad \text{if } \beta \in (0.427, 1). \end{aligned} \tag{28}$$

By comparing (27) to (28), we derive the social welfare rankings of the two one-part tariff licensing contracts as follows:

$$\begin{cases} SW^F\big|_{m^*=1} \geq SW^R\big|_{m^*=2} & \beta \in (0.427, 0.6479], \\ SW^F\big|_{m^*=1} < SW^R\big|_{m^*=2} & \text{if } \beta \in (0.6479, 0.8384], \\ SW^F\big|_{m^*=2} > SW^R\big|_{m^*=2} & \beta \in (0.8384, 1). \end{cases} \tag{29}$$

The above result implies that royalty licensing is more socially desirable than fixed-fee licensing if the technology difference is large (i.e., $\beta \in (0, 0.427]$) and at the median ranges (i.e., $\beta \in (0.6479, 0.8384]$). The intuition behind this finding is as follows. When $\beta \in (0, 0.427]$, licensing does not occur under fixed-fee licensing, but occurs under royalty licensing. When $\beta \in (0.6479, 0.8384]$, the licensor firm licenses to two low-quality firms under royalty licensing, whereas only one low-quality firm is licensed under fixed-fee licensing. Although a royalty incurs a distortion and is detrimental to social welfare, there are more firms producing the high-quality product under royalty licensing, which do enhance social welfare. Thus, we build the fourth proposition as follows.

Proposition 4. In a vertically differentiated oligopolistic market, product licensing by a royalty contract is more socially desirable than by a fixed-fee contract if the technology gap is large or median (i.e., $\beta \in (0, 0.427]$ or $\beta \in (0.6479, 0.8384]$).

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In the licensing literature with a duopolistic market, fixed-fee licensing is superior to royalty licensing in terms of social welfare as the effective marginal cost of the licensee firm is lower. This result is different from ours as they ignore that the number of licenses sold may vary under different licensing contracts when the market is of oligopoly.

4 Two-Part Tariff Licensing

The previous sections have assumed that the licensor firm can charge only a fixed fee or a royalty. In this section we allow the licensor firm to employ a two-part tariff licensing contract (a royalty plus a fixed fee).[9] A licensor firm always prefers two-part tariffs to either a royalty or a fixed fee when designing an optimal licensing contract. We investigate whether or not the optimal two-part tariff contract will degenerate into a royalty-ridden or a fixed-fee-ridden contract. In the two-part tariff regime, the profits for firms are as follows:

$$\Pi_H^T = \left(1 - q_H - \sum_{i \in M} q_i - \beta \sum_{j \in N \setminus M} q_j\right) q_H + m\left(r q_i + F\right), \tag{30}$$

$$\Pi_i^T = \left(1 - q_H - \sum_{i \in M} q_i - \beta \sum_{j \in N \setminus M} q_j - r\right) q_i - F,$$

$$\Pi_j^T = \beta \left(1 - q_H - \sum_{i \in M} q_i - \sum_{j \in N \setminus M} q_j\right) q_j,$$

where $F = \Pi_i^T|_m - \Pi_j^T|_{m-1}$ and the variables with superscript "T" are associated with the two-part tariff regime. Moreover, each licensee firm pays the same royalty rate and fixed fee to the licensor firm as they are all identical.

The equilibrium in the third stage is the same as that in the royalty licensing regime and is as follows:

$$q_H^T = \frac{(1 + mr)(1 + (1 - \beta)(3 - m))}{5 + (m + 1)(1 - \beta)(3 - m)}, \tag{31}$$

[9] We assume that the licensor firm employs a uniform two-part tariff licensing contract. The result is robust if the licensor firm adopts discriminatory two-part tariff licensing contracts as the low-quality firms are all identical. We are grateful to an anonymous referee for suggesting this discussion.

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$$q_i^T = \frac{1+(1-\beta)(3-m)-(2+(2-\beta)(3-m))r}{5+(m+1)(1-\beta)(3-m)}, \tag{32}$$

$$q_j^T = \frac{1+mr}{5+(m+1)(1-\beta)(3-m)}. \tag{33}$$

In the second stage, the participating constraint for Firm m is $\Pi_i^T|_m \geq \Pi_j^T|_{m-1}$. We now proceed to solve for the optimal royalty rate in the first-stage game:

$$\begin{aligned} \underset{r}{Max} \quad & \Pi_H^T = \left(q_H^T\right)^2 + m\left(rq_i^T + F\right) \\ \text{s.t.} \quad & \Pi_i^T\big|_m \geq \Pi_j^T\big|_{m-1}. \end{aligned} \tag{34}$$

By differentiating the above equation with respect to r and utilizing the constraint, we derive the optimal royalty rate $r^*(m,\beta)$, taking either an interior ($\tilde{r}$) or a corner solution ($\bar{r}$). Proceeding as before, we derive the optimal two-part tariff licensing contract as follows:[10]

$$\begin{cases} r^* = \bar{r}|_{m^*=1}, & F^* = 0, & m^* = 1 & \beta \in (0, 0.159]; \\ r^* = \tilde{r}|_{m^*=1}, & F^* = F(r^*)|_{m^*=1}, & m^* = 1 & \beta \in (0.159, 0.75]; \\ r^* = 0, & F^* = F(r^*)|_{m^*=1}, & m^* = 1 \quad \text{if} & \beta \in (0.75, 0.8331]; \\ r^* = \tilde{r}|_{m^*=2}, & F^* = F(r^*)|_{m^*=2}, & m^* = 2 & \beta \in (0.8331, 0.8578]; \\ r^* = 0, & F^* = F(r^*)|_{m^*=2}, & m^* = 2 & \beta \in (0.8578, 1), \end{cases}$$

where

$$\bar{r}|_{m^*=1} = \frac{(3-2\beta)(8-3\beta)-(9-4\beta)\sqrt{\beta}}{(8-3\beta)(6-2\beta)},$$

$$\tilde{r}|_{m^*=1} = \frac{3-4\beta}{6},$$

$$F(r^*)\big|_{m^*=1} = \left(\frac{3-2\beta-6r^*+2\beta r^*}{9-4\beta}\right)^2 - \beta\left(\frac{1}{8-3\beta}\right)^2; \quad \text{and}$$

$$\tilde{r}|_{m^*=2} = \frac{648-1514\beta+1187\beta^2-394\beta^3+48\beta^4}{1296-1672\beta+736\beta^2-110\beta^3},$$

$$F\left(r^*\right)\big|_{m^*=2} = \left(\frac{2-\beta-(4-\beta)r^*}{8-3\beta}\right)^2 - \beta\left(\frac{1+r^*}{9-4\beta}\right)^2.$$

[10]The detailed derivation is provided in Appendix 3.

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It is found that no licensing and licensing to all low-quality rivals are the strategies dominated by licensing to one or two firms. As the licensor firm can also use r to control the *own operation profit effect* and the *licensing revenue effect* under the two-part tariff licensing regime, the intuition is similar to that under the royalty licensing regime. Based on the above result, we establish the following proposition.

Proposition 5. In a vertically differentiated oligopolistic market, the optimal two-part tariff licensing contract may encompass a pure royalty, a positive royalty and a positive fixed fee, or a pure fixed fee, varying with the technology gap. In addition, the licensor firm will not sell its technology to all the low-quality firms.

The above result differs from Li and Song (2009) in which a two-part tariff contract will degenerate to a pure royalty. We find that the optimal two-part tariff contract may consist of a pure royalty, a positive royalty and a positive fixed fee, or a pure fixed fee, depending on the technology gap between the two types of firms. The main cause of the result is due to the property of the vertically differentiated oligopoly. Moreover, the licensor firm will never license to three low-quality firms. This is because if the licensor firm licenses its technology to all the three firms, the market would be too competitive, decreasing not only its own operation profit but also licensing revenue.

5 The Absorptive Capacity of the Licensee Firms

In the previous sections, we have assumed that the licensee firms can absorb the new technology thoroughly and produce the high-quality product as the licensor firm does with licensing. However, in some industries, the licensee firms are subject to absorptive capacity and can improve the quality not as much as the licensor firm. In this Section, we shall investigate the effect of absorptive capacity on the optimal one-part and two-part tariff licensing schedules.[11]

Since the equilibrium with no licensing is the same as that in Section 2, we proceed to the licensing regimes. The assumptions are similar to those in Section 2, except that there are three types of products after licensing: the high-quality product ($s_H = 1$) produced by the licensor firm, mid-quality

[11] We are indebted to a referee for suggesting this extension.

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($s_M = t$) produced by the licensee firms and low-quality ($s_L = \beta$) produced by the non-licensee firms with $1 > t > \beta$. Given the utility function in Equation (1), we can write the inverse demand functions for these three types of products as follows:

$$\begin{aligned} P_H &= 1 - Q_H - tQ_M - \beta Q_L, \\ P_M &= t - tQ_H - tQ_M - \beta Q_L, \\ P_L &= \beta - \beta Q_H - \beta Q_M - \beta Q_L, \end{aligned}$$

where Q_H, Q_M, and Q_L represent the total outputs of the licensor firm, the licensee firms and the non-licensee firms, respectively. Given the above assumptions, the profits of the licensor firm, the licensee firms and the non-licensee firms with fixed-fee licensing are respectively specified as follows:

$$\begin{aligned} \Pi_H^F &= \left(1 - q_H - t\sum_{i\in M} q_i - \beta \sum_{j\in N\setminus M} q_j\right) q_H + mF, \\ \Pi_i^F &= \left(t - tq_H - t\sum_{i\in M} q_i - \beta \sum_{j\in N\setminus M} q_j\right) q_i - F, \\ \Pi_j^F &= \beta\left(1 - q_H - \sum_{i\in M} q_i - \sum_{j\in N\setminus M} q_j\right) q_j. \end{aligned}$$

On the other hand, the profits of the licensor firm, the licensee firms and the non-licensee firms with royalty licensing are respectively specified as follows:

$$\begin{aligned} \Pi_H^R &= \left(1 - q_H - t\sum_{i\in M} q_i - \beta \sum_{j\in N\setminus M} q_j\right) q_H + mrq_i, \\ \Pi_i^R &= \left(t - tq_H - t\sum_{i\in M} q_i - \beta \sum_{j\in N\setminus M} q_j - r\right) q_i, \\ \Pi_j^R &= \beta\left(1 - q_H - \sum_{i\in M} q_i - \sum_{j\in N\setminus M} q_j\right) q_j. \end{aligned}$$

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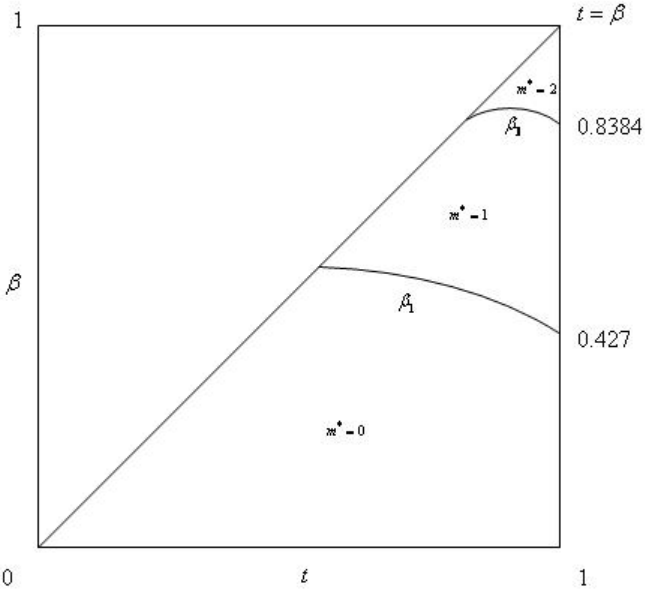


Figure 1: The Optimal Fixed-fee Licensing Schedule

Proceeding as that in Section 2, we obtain the optimal fixed-fee licensing schedule and the optimal royalty licensing schedule respectively as follows:[12]

$$\begin{cases} \Pi_H^F\big|_{m=0} \geq \Pi_H^F\big|_{m=1} > \Pi_H^F\big|_{m=2}, & m^* = 0 \quad \beta \in (0, \beta_1], \\ \Pi_H^F\big|_{m=1} > \Pi_H^F\big|_{m=0} \geq \Pi_H^F\big|_{m=2}, & m^* = 1 \quad \text{if} \quad \beta \in (\beta_1, \beta_2], \\ \Pi_H^F\big|_{m=1} \geq \Pi_H^F\big|_{m=2} > \Pi_H^F\big|_{m=0}, & m^* = 1 \quad \beta \in (\beta_2, \beta_3], \\ \Pi_H^F\big|_{m=2} > \Pi_H^F\big|_{m=1} > \Pi_H^F\big|_{m=0}, & m^* = 2 \quad \beta \in (\beta_3, 1). \end{cases} \tag{35}$$

$$\begin{cases} \Pi_H^R\big|_{m=0} \geq \Pi_H^R\big|_{m=1} > \Pi_H^R\big|_{m=2}, & m^* = 0 \quad \beta \in (0, \beta_4], \\ \Pi_H^R\big|_{m=1} > \Pi_H^R\big|_{m=0} \geq \Pi_H^R\big|_{m=2}, & m^* = 1 \quad \text{if} \quad \beta \in (\beta_4, \beta_5], \\ \Pi_H^R\big|_{m=1} \geq \Pi_H^R\big|_{m=2} > \Pi_H^R\big|_{m=0}, & m^* = 1 \quad \beta \in (\beta_5, \beta_6], \\ \Pi_H^R\big|_{m=2} > \Pi_H^R\big|_{m=1} > \Pi_H^R\big|_{m=0}, & m^* = 2 \quad \beta \in (\beta_6, 1). \end{cases} \tag{36}$$

We use Figures 1 and 2 to demonstrate these results. In Figure 1 and 2, we can find that the results restore to those in Section 2 if $t = 1$. It

[12]The derivations in this section are very tedious and the approaches are similar to those with no capacity constraint. As a result, we show the proofs in Appendix 4.

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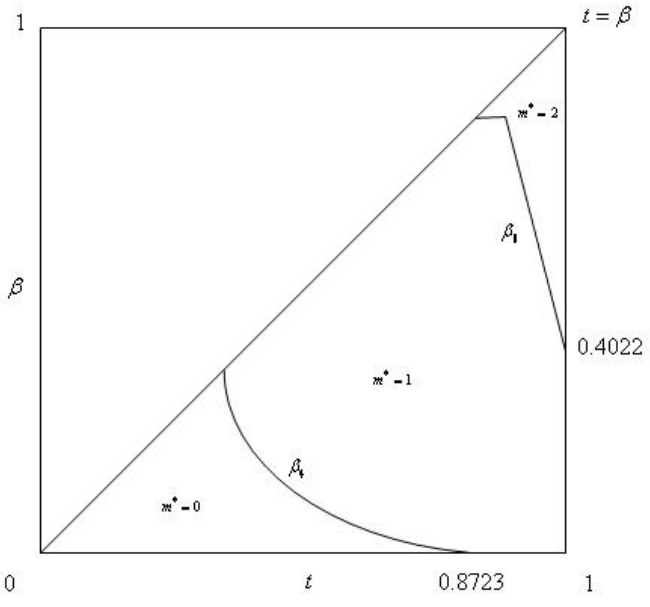


Figure 2: The Optimal Royalty Licensing Schedule

is obviously that licensing to all low-quality firms will never be an optimal licensing strategy for the licensor firm. In addition, if t is high, the number of the licenses sold increases as β increases. These results are in parallel to those in Propositions 1 and 2. Nevertheless, licensing will not take place if the absorptive capacity is low. The intuition is very straightforward. If the absorptive capacity is very low, the *licensing revenue effect* will be nil, thereby decreasing the incentive for the licensor firm to carry out licensing.

Furthermore, by comparing (34) with (36), we derive the optimal one-part tariff licensing schedule as follows:

$$\begin{cases} r^* = 0 \text{ and } F^* = 0, & m^* = 0 & \text{Region I;} \\ r^* = \bar{r}|_{m^*=1}, & m^* = 1 & \text{Region II;} \\ r^* = \bar{r}|_{m^*=2}, & m^* = 2 \quad \text{if } \beta \in & \text{Region III;} \\ F^* = F(\beta, t)|_{m^*=1}, & m^* = 1 & \text{Region IV;} \\ F^* = F(\beta, t)|_{m^*=2}, & m^* = 2 & \text{Region V.} \end{cases}$$

We use Figure 3 to illustrate this result. In Figure 3, it is shown that the licensor firm will find it more profitable to license out its product tech-

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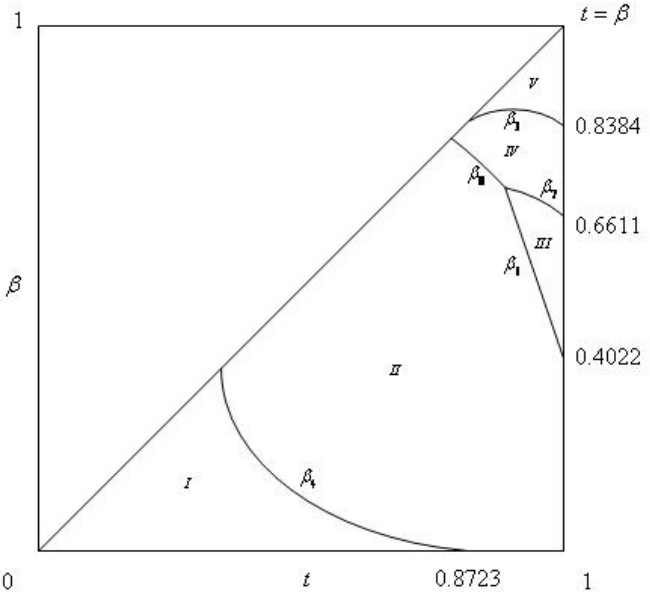


Figure 3: The Optimal One-part Tariff Licensing Schedule

nology to its low-quality rivals via a royalty contract if the technology gap is large (i.e., Regions II and III), but it will adopt fixed-fee licensing if the technology gap is small (i.e., Regions IV and V). In addition, the licensor firm will not license to three low-quality firms. These results are in line with that in Proposition 3. However, licensing will not occur when the absorptive capacity is low (Region I). This is owing to the low *licensing revenue effect*. In sum, we can establish the proposition as follows:

Proposition 6. If the absorptive capacity of the licensee firms is low, the licensor firm will not carry out one-part tariff licensing; if the absorptive capacity is high and the technology gap is large (small), the optimal one-part tariff for the licensor firm is royalty (fixed-fee) licensing. Moreover, the licensor firm will not license to all the low-quality firms.

We proceed to examine the optimal two-part tariff licensing schedule for the licensor firm. The profits of the licensor firm, the licensee firms and the non-licensee firms under two-part tariff licensing are respectively specified as follows:

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$$\Pi_H^T = \left(1 - q_H - t\sum_{i\in M} q_i - \beta \sum_{j\in N\setminus M} q_j\right) q_H + m\left(rq_i + F\right),$$

$$\Pi_i^T = \left(t - tq_H - t\sum_{i\in M} q_i - \beta \sum_{j\in N\setminus M} q_j - r\right) q_i - F,$$

$$\Pi_j^T = \beta\left(1 - q_H - \sum_{i\in M} q_i - \sum_{j\in N\setminus M} q_j\right) q_j,$$

where $F = \Pi_i^T|_m - \Pi_j^T|_{m-1}$.

Proceeding as that in Section 4, we can derive the optimal two-part tariff licensing schedule as follows:

$$\begin{cases} r^* = 0, & F^* = 0, & m^* = 0 & & \text{Region I,} \\ r^* = \bar{r}|_{m^*=1}, & F^* = 0, & m^* = 1 & & \text{Region II,} \\ r^* = \tilde{r}|_{m^*=1}, & F^* = F\left(r^*\right)|_{m^*=1}, & m^* = 1 & & \text{Region III,} \\ r^* = 0, & F^* = F\left(r^*\right)|_{m^*=1}, & m^* = 1, & \text{if } \beta \in & \text{Region IV,} \\ r^* = \tilde{r}|_{m^*=2}, & F^* = F\left(r^*\right)|_{m^*=2}, & m^* = 2 & & \text{Region V,} \\ r^* = 0, & F^* = F\left(r^*\right)|_{m^*=2}, & m^* = 2 & & \text{Region VI.} \end{cases}$$

It is apparent that the optimal two-part tariff licensing schedule can be a pure royalty, a pure fixed fee or a positive royalty plus a positive fixed fee, depending on the technology gap and the absorptive capacity. In addition, the licensor firm shall never license to three low-quality firms. This result is similar to that in Proposition 5. However, licensing will not occur if the absorptive capacity is low. We use Figure 4 to show the above result. In sum, we can establish the proposition as follows:

Proposition 7. If the absorptive capacity of the licensee firms is low, the licensor firm will not carry out two-part tariff licensing. If the absorptive capacity is high, the optimal two-part tariff licensing contract may encompass a pure royalty, a positive royalty and a positive fixed fee, or a pure fixed fee, varying the technology gaps. Moreover, the licensor firm will not license to all the low-quality firms.

6 Conclusions

The existing studies in the literature on quality licensing all focus on a duopolistic framework. This paper is the first one to investigate this is-

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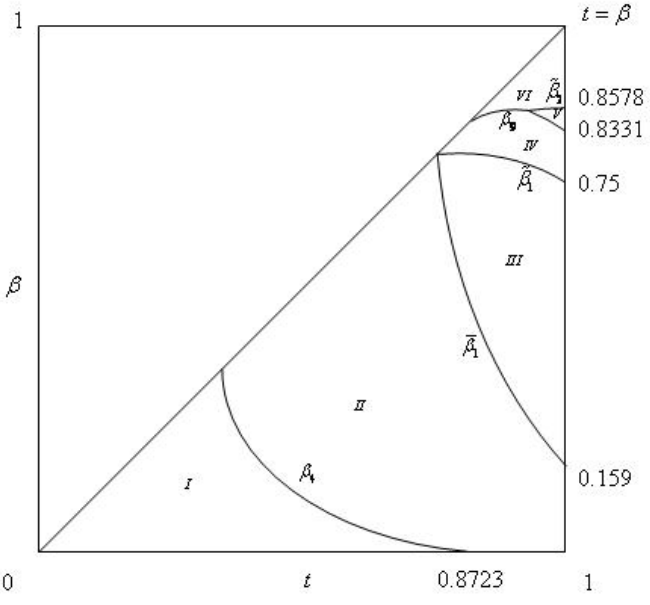


Figure 4: The Optimal Two-part Tariff Licensing Schedule

sue with an oligopolistic model. The oligopolistic licensing model is distinct from the duopolistic licensing model, as the former further incorporates the competition between the licensee and non-licensee firms.

Our main findings are as follows. First, if there are more than two low-quality firms, the high-quality firm will never license its advanced technology to all of its low-quality rivals. Moreover, the number of licensee firms decreases as the technology gap between the high-quality firm and the low-quality firms increases. Second, a royalty contract is more (less) profitable than a fixed-fee contract, if the technology gap is large (small). Third, a royalty contract is more socially desirable than a fixed-fee contract if the technology gap is large and median. Fourth, the optimal two-part tariff licensing contract consists of a pure royalty, a positive royalty and a positive fixed fee, or a pure fixed fee, varying with the technology gap between the firms. Fifth, the above results remain robust if the absorptive capacity of the licensee firms is high; however, if the absorptive capacity is low, licensing will not occur.

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Appendix 1: This Appendix Is to Prove the Optimal Royalty Licensing Contract in Section 2.

By subtracting (18) from (19) in the main text, we derive that:

$$\Pi_H^R\big|_{m=1} - \Pi_H^R\big|_{m=0} = \frac{3\beta - 21\beta^2 + 8\beta^3 + 48\beta^{\frac{3}{2}} - 50\beta^{\frac{5}{2}} + 12\beta^{\frac{7}{2}}}{4(3-\beta)^2(8-3\beta)^2} > 0,$$

which is the result of Equation (22) in the main text.

By subtracting (21) from (19), we derive that:

$$\begin{aligned}\Pi_H^R\big|_{m=1} - \Pi_H^R\big|_{m=3} = &\Big(20928\beta - 22197\beta^2 + 2413\beta^3 + 4856\beta^4 \\ &- 2043\beta^5 + 243\beta^6 - 31248\beta^{\frac{3}{2}} + 49366\beta^{\frac{5}{2}} \\ &- 29110\beta^{\frac{7}{2}} + 7494\beta^{\frac{9}{2}} - 702\beta^{\frac{11}{2}}\Big)\Big/4(3-\beta)^2 \\ &(8-3\beta)^2\left(8 + 5\beta^{\frac{1}{2}} - 3\beta\right)^2 > 0,\end{aligned}$$

which is the result of Equation (23) in the main text.

By subtracting (19) from (20), it is derived that:

$$\begin{aligned}\Pi_H^R\big|_{m=2} - \Pi_H^R\big|_{m=1} = &\Big(66096\beta - 94776\beta^2 + 36075\beta^3 + 8718\beta^4 \\ &- 9785\beta^5 + 2480\beta^6 - 208\beta^7 - 143424\beta^{\frac{3}{2}} \\ &+ 322344\beta^{\frac{5}{2}} - 301862\beta^{\frac{7}{2}} + 150648\beta^{\frac{9}{2}} \\ &- 42190\beta^{\frac{11}{2}} + 6268\beta^{\frac{13}{2}} - 384\beta^{\frac{15}{2}}\Big)\Big/4(3-\beta)^2 \\ &(8-3\beta)^2\left(36 - 25\beta + 4\beta^2 + 8\beta^{\frac{1}{2}} - 3\beta^{\frac{3}{2}}\right)^2 > 0, \\ &\text{if} \quad \beta > 0.4022,\end{aligned}$$

which is the result of Equation (24) in the main text.

By subtracting (18) from (20) in the main text, we derive that:

$$\begin{aligned}\Pi_H^R\big|_{m=2} - \Pi_H^R\big|_{m=0} = &4\Big(-432\beta + 337\beta^2 - 40\beta^3 - 18\beta^4 + 3\beta^5 + 1440\beta^{\frac{3}{2}} \\ &- 2404\beta^{\frac{5}{2}} + 1459\beta^{\frac{7}{2}} - 381\beta^{\frac{9}{2}} + 36\beta^{\frac{11}{2}}\Big)\Big/ \\ &\left(36 - 25\beta + 4\beta^2 + 8\beta^{\frac{1}{2}} - 3\beta^{\frac{3}{2}}\right)^2(8-3\beta)^2 > 0, \\ &\text{if} \quad \beta > 0.1099,\end{aligned}$$

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which is the result of Equation (25) in the main text.

According to the above results, we can find the optimal royalty licensing schedule as follows:

$$\begin{cases} \Pi_H^R\big|_{m=1} > \Pi_H^R\big|_{m=0} \geq \Pi_H^R\big|_{m=2}, \ m^* = 1 & \beta \in (0, 0.1099], \\ \Pi_H^R\big|_{m=1} \geq \Pi_H^R\big|_{m=2} > \Pi_H^R\big|_{m=0}, \ m^* = 1 & \text{if } \beta \in (0.1099, 0.4022], \\ \Pi_H^R\big|_{m=2} > \Pi_H^R\big|_{m=1} > \Pi_H^R\big|_{m=0}, \ m^* = 2 & \beta \in (0.4022, 1), \end{cases}$$

which is the result of equation (26) in the main text.

Appendix 2: The Optimal One-part Tariff Licensing Contract and the Welfare

This appendix is to present the proofs of the optimal one-part tariff licensing contract and the welfare implications in Section 3.

We first show the proof of Proposition 3. By comparing (9) to (26), we derive the optimal one-part tariff licensing contract between fixed fee and royalty under difference technology gaps as follows:

$$\begin{cases} \Pi_H^R\big|_{m^*=1} > \Pi_H^F\big|_{m^*=0}, \ r^* = \bar{r}|_{m^*=1}, & m^* = 1 & \beta \in (0, 0.4022]; \\ \Pi_H^R\big|_{m^*=2} > \Pi_H^F\big|_{m^*=0}, \ r^* = \bar{r}|_{m^*=2}, & m^* = 2 & \beta \in (0.4022, 0.427]; \\ \Pi_H^R\big|_{m^*=2} \geq \Pi_H^F\big|_{m^*=1}, \ r^* = \bar{r}|_{m^*=2}, & m^* = 2 \quad \text{if} & \beta \in (0.427, 0.6611]; \\ \Pi_H^F\big|_{m^*=1} > \Pi_H^R\big|_{m^*=2}, \ F^* = F(\beta)|_{m^*=1}, & m^* = 1 & \beta \in (0.6611, 0.8384]; \\ \Pi_H^F\big|_{m^*=2} > \Pi_H^R\big|_{m^*=2}, \ F^* = F(\beta)|_{m^*=2}, & m^* = 2 & \beta \in (0.8384, 1), \end{cases}$$

where $\Pi_H^F|_{m^*=0} = \Pi_H^R|_{m^*=0} = \Pi_H^{NL}$,

$$\bar{r}|_{m^*=1} = \frac{(3-2\beta)(8-3\beta) - (9-4\beta)\sqrt{\beta}}{(8-3\beta)(6-2\beta)},$$

$$\bar{r}|_{m^*=2} = \frac{18 - 17\beta + 4\beta^2 - 8\beta^{\frac{1}{2}} + 3\beta^{\frac{3}{2}}}{36 - 25\beta + 4\beta^2 + 8\beta^{\frac{1}{2}} - 3\beta^{\frac{3}{2}}},$$

$$F(\beta)|_{m^*=1} = \left(\frac{3-2\beta}{9-4\beta}\right)^2 - \beta\left(\frac{1}{8-3\beta}\right)^2; \text{ and}$$

$$F(\beta)|_{m^*=2} = \left(\frac{2-\beta}{8-3\beta}\right)^2 - \beta\left(\frac{1}{9-4\beta}\right)^2.$$

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$$\begin{aligned}\Pi_H^R\big|_{m^*=2} - \Pi_H^F\big|_{m^*=1} = &\Big(186624 - 1186992\beta + 2482044\beta^2 - 2651119\beta^3 \\ &+ 1667488\beta^4 - 646741\beta^5 + 152760\beta^6 - 20216\beta^7 \\ &+ 1152\beta^8 + 82944\beta^{\frac{1}{2}} + 23616\beta^{\frac{3}{2}} - 314520\beta^{\frac{5}{2}} \\ &+ 380506\beta^{\frac{7}{2}} - 210594\beta^{\frac{9}{2}} + 61832\beta^{\frac{11}{2}} - 9360\beta^{\frac{13}{2}} \\ &+ 576\beta^{\frac{15}{2}}\Big)\Big/\Big((9-4\beta)^2(8-3\beta)^2\Big(36 - 25\beta \\ &+ 4\beta^2 + 8\beta^{\frac{1}{2}} - 3\beta^{\frac{3}{2}}\Big)^2\Big) > 0, \quad \text{if} \quad \beta < 0.6611.\end{aligned}$$

and

$$\begin{aligned}\Pi_H^F\big|_{m^*=2} - \Pi_H^R\big|_{m^*=2} = &-2\Big(209952 - 1084752\beta + 2093572\beta^2 - 2126076\beta^3 \\ &+ 1282577\beta^4 - 478550\beta^5 + 108929\beta^6 + 13920\beta^7 \\ &+ 768\beta^8 + 93312\beta^{\frac{1}{2}} - 192528\beta^{\frac{3}{2}} + 166100\beta^{\frac{5}{2}} \\ &- 76870\beta^{\frac{7}{2}} + 20160\beta^{\frac{9}{2}} - 2842\beta^{\frac{11}{2}} \\ &+ 168\beta^{\frac{13}{2}}\Big)\Big/\Big((9-4\beta)^2(8-3\beta)^2\Big(36 - 25\beta \\ &+ 4\beta^2 + 8\beta^{\frac{1}{2}} - 3\beta^{\frac{3}{2}}\Big)^2\Big) > 0, \quad \text{if} \quad \beta \in (0.8384, 1).\end{aligned}$$

We then prove the result in Proposition 4. If $\beta \in (0, 0.427]$, licensing will not occur under fixed-fee licensing. Therefore, the welfare level is the same as that with no licensing. By contrast, under royalty licensing, the licensor firm will license to one low-quality firm if $\beta \in (0, 0.4022]$ and two low-quality firms if $\beta \in (0.4022, 0.427]$. The corresponding welfare levels are shown as follows:

$$SW^F\big|_{m^*=0} = SW^R\big|_{m^*=0} = SW^N = \frac{3\left(3\beta^2 + 16 - 11\beta\right)}{2(8-3\beta)^2},$$

$$SW^R = \begin{cases} SW^R\big|_{m^*=1} & \beta \in (0, 0.4022], \\ \text{if} & \\ SW^R\big|_{m^*=2} & \beta \in (0.4022, 0.427], \end{cases}$$

where

$$SW^R\big|_{m^*=1} = \Big[1728 - 2329\beta + 1275\beta^2 - 336\beta^3 + 36\beta^4 + 48\beta^{\frac{5}{2}}$$

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$$- 182\beta^{\frac{3}{2}} + 144\beta^{\frac{1}{2}}\Big]\Big/\Big[8(3-\beta)^2(8-3\beta)^2\Big],$$

$$SW^R\big|_{m^*=2} = \Big[972 - 1209\beta + 588\beta^2 - 143\beta^3 + 16\beta^4 + 648\beta^{\frac{1}{2}} - 702\beta^{\frac{3}{2}} + 238\beta^{\frac{5}{2}} - 24\beta^{\frac{7}{2}}\Big]\Big/\Big[2\Big(36 - 25\beta + 4\beta^2 + 8\beta^{\frac{1}{2}} - 3\beta^{\frac{3}{2}}\Big)^2\Big].$$

Given $\beta \in (0, 0.4022]$, by comparing $SW^F|_{m^*=0}$ to $SW^R|_{m^*=1}$ and $SW^F|_{m^*=0}$ to $SW^R|_{m^*=2}$ when $\beta \in (0.4022, 0.427]$.

Given $\beta \in (0, 0.4022]$, by comparing $SW^F|_{m^*=0}$ to $SW^R|_{m^*=1}$, we have:

$$SW^R\big|_{m^*=1} - SW^F\big|_{m^*=0} = \frac{11\beta - 33\beta^2 + 12\beta^3 + 144\beta^{\frac{1}{2}} - 182\beta^{\frac{3}{2}} + 48\beta^{\frac{5}{2}}}{8(8-3\beta)^2(3-\beta)^2} > 0.$$

Given $\beta \in (0.4022, 0.427]$, by comparing $SW^F|_{m^*=0}$ to $SW^R|_{m^*=2}$, we have:

$$SW^R\big|_{m^*=2} - SW^F\big|_{m^*=0} = 2\Big(516\beta - 1515\beta^2 + 1270\beta^3 - 419\beta^4 + 48\beta^5 + 3456\beta^{\frac{1}{2}} - 6864\beta^{\frac{3}{2}} + 4744\beta^{\frac{5}{2}} - 1380\beta^{\frac{7}{2}} + 144\beta^{\frac{9}{2}}\Big)\Big/\Big((8-3\beta)^2\Big(36 - 25\beta + 4\beta + 8\beta^{\frac{1}{2}} - 3\beta^{\frac{3}{2}}\Big)^2\Big) > 0.$$

The above result follows that royalty licensing is more socially desirable than fixed-fee licensing if $\beta \in (0, 0.427]$. If $\beta \in (0.427, 1)$, the licensor firm will license to one or two low-quality firms under fixed-fee licensing; however, it will license to two firms under royalty licensing. Therefore, the welfare levels under fixed-fee licensing and royalty licensing contracts are respectively as follows:

$$SW^F = \begin{cases} SW^F\big|_{m^*=1} = \frac{4(9-8\beta+2\beta^2)}{(9-4\beta)^2} & \beta \in (0.427, 0.8384], \\ \text{if} & \\ SW^F\big|_{m^*=2} = \frac{3(20-15\beta+3\beta^2)}{2(8-3\beta)^2} & \beta \in (0.8384, 1). \end{cases}$$

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This is Equation (27) in the main text. Besides,

$$SW^R\big|_{m^*=2} = \Big(972 - 1209\beta + 588\beta^2 - 143\beta^3 + 16\beta^4 + 648\beta^{\frac{1}{2}} - 702\beta^{\frac{3}{2}} + 238\beta^{\frac{5}{2}} - 24\beta^{\frac{7}{2}}\Big) \Big/ \Big(2\left(36 - 25\beta + 4\beta^2 + 8\beta^{\frac{1}{2}} - 3\beta^{\frac{3}{2}}\right)^2\Big) \quad \text{if} \quad \beta \in (0.427, 1).$$

This is Equation (28) in the main text.

Given $\beta \in (0.427, 0.8384]$, by comparing $SW^F|_{m^*=1}$ to $SW^R|_{m^*=2}$, we have:

$$SW^F\big|_{m^*=1} - SW^R\big|_{m^*=2} = \Big(14580 - 40023\beta + 43892\beta^2 - 23625\beta^3 + 6216\beta^4 - 640\beta^5 - 11016\beta^{\frac{1}{2}} + 22302\beta^{\frac{3}{2}} - 16142\beta^{\frac{5}{2}} + 5032\beta^{\frac{7}{2}} - 576\beta^{\frac{9}{2}}\Big) \Big/ \left(2(9 - 4\beta)^2\left(36 - 25\beta + 4\beta + 8\beta^{\frac{1}{2}} - 3\beta^{\frac{3}{2}}\right)^2\right).$$

The sign of this equation is positive if $\beta \in (0.427, 0.6479]$, but it is negative if $\beta \in (0.6479, 0.8384]$.

Given $\beta \in (0.8384, 1]$, by comparing and $SW^F|_{m^*=2}$ to $SW^R|_{m^*=2}$, we have:

$$SW^F\big|_{m^*=2} - SW^R\big|_{m^*=2} = 4\Big(1944 - 4806\beta + 4659\beta^2 - 2219\beta^3 + 520\beta^4 - 48\beta^5 - 864\beta^{\frac{1}{2}} + 1644\beta^{\frac{3}{2}} - 1127\beta^{\frac{5}{2}} + 333\beta^{\frac{7}{2}} - 36\beta^{\frac{9}{2}}\Big) \Big/ \left((8 - 3\beta)^2\left(36 - 25\beta + 4\beta + 8\beta^{\frac{1}{2}} - 3\beta^{\frac{3}{2}}\right)^2\right) > 0.$$

This result implies that fixed-fee licensing is superior to royalty licensing if $\beta \in (0.8384, 1]$.

The above results can be summarized as follows:

$$\begin{cases} SW^F\big|_{m^*=1} \geq SW^R\big|_{m^*=2} & \beta \in (0.427, 0.6479]; \\ SW^F\big|_{m^*=1} < SW^R\big|_{m^*=2} & \text{if} \quad \beta \in (0.6479, 0.8384]; \\ SW^F\big|_{m^*=2} > SW^R\big|_{m^*=2} & \beta \in (0.8384, 1). \end{cases}$$

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This is the result of Equation (29) in the main text. The above results imply that royalty licensing is in superior to fixed-fee licensing in terms of welfare if $\beta \in (0, 0.427]$ and $\beta \in (0.6479, 0.8384]$. Otherwise, the reverse is true.

Appendix 3: The optimal two-part tariff licensing contract

This appendix is to show the proofs of the results in Section 4 and Proposition 5. We first examine whether the licensor firm has an incentive to license its technology to all the low-quality firms under two-part tariff licensing. If the licensor firm licenses its advanced technology to all the low-quality firms (i.e., $m = 3$), by substituting (31), (32) and (33) into (34) and then differentiating (34) with respect to r, subject to the participating constraint of the licensee firms, the optimal royalty rate can be derived as follows:

$$r^* = \begin{cases} \bar{r}|_{m=3} = \frac{8-5\beta^{\frac{1}{2}}-3\beta}{2\left(8+5\beta^{\frac{1}{2}}-3\beta\right)} & \beta \in (0, 0.7466], \\ \tilde{r}|_{m=3} = \frac{192-244\beta+27\beta^2}{2\left(192-44\beta+27\beta^2\right)} & \text{if} \quad \beta \in (0.7466, 0.8708], \\ 0 & \beta \in (0.8708, 1]. \end{cases}$$

Therefore, the resulting profits of the licensor firm are derivable as follows:

$$\Pi_H^T\big|_{m=3} = \begin{cases} \frac{64-107\beta+9\beta^2+80\beta^{\frac{1}{2}}-30\beta^{\frac{3}{2}}}{4\left(8+5\beta^{\frac{1}{2}}-3\beta\right)^2} & \beta \in (0, 0.7466], \\ \frac{192-188\beta+27\beta^2}{4\left(192-44\beta+27\beta^2\right)} & \text{if} \quad \beta \in (0.7466, 0.8708], \\ \frac{256-267\beta+36\beta^2}{12(8-3\beta)^2} & \beta \in (0.8708, 1]. \end{cases}$$

By contrast, if the licensor firm licenses its technology to one or two low-quality firms, then the optimal royalty rates and its profits are derived as follows:

$$r^* = \begin{cases} \bar{r}|_{m=1} = \frac{24-25\beta+6\beta^2-(9-4\beta)\sqrt{\beta}}{2(8-3\beta)(3-\beta)} & \beta \in (0, 0.159], \\ \tilde{r}|_{m=1} = \frac{1}{2} - \frac{2}{3}\beta & \text{if} \quad \beta \in (0.159, 0.75], \\ 0 & \beta \in (0.75, 1]. \end{cases}$$

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$$\Pi_H^T\big|_{m=1} = \begin{cases} \frac{576-1245\beta+943\beta^2-304\beta^3+36\beta^4+48\beta^{\frac{3}{2}}-50\beta^{\frac{5}{2}}+12\beta^{\frac{7}{2}}}{(2(8-3\beta)(3-\beta))^2} & \beta \in (0, 0.159], \\ \frac{192-284\beta+123\beta^2-18\beta^3}{12(8-3\beta)^2} & \text{if} \quad \beta \in (0.159, 0.75], \\ \frac{1152-2481\beta+1898\beta^2-616\beta^3+72\beta^4}{(8-3\beta)^2(9-4\beta)^2} & \beta \in (0.75, 1]. \end{cases}$$

$$r^* = \begin{cases} \bar{r}|_{m=2} = \frac{8-17\beta+4\beta^2-8\beta^{\frac{1}{2}}+3\beta^{\frac{3}{2}}}{36-25\beta+4\beta^2+8\beta^{\frac{1}{2}}-3\beta^{\frac{3}{2}}} & \beta \in (0, 0.6094], \\ \tilde{r}|_{m=2} = \frac{648-1514\beta+1187\beta^2-394\beta^3+48\beta^4}{2\left(648-836\beta+368\beta^2-55\beta^3\right)} & \text{if} \quad \beta \in (0.6094, 0.8578], \\ 0 & \beta \in (0.8578, 1]. \end{cases}$$

$$\Pi_H^T\big|_{m=2} = \begin{cases} \frac{324-704\beta+497\beta^2-147\beta^3+16\beta^4+144\beta^{\frac{1}{2}}-172\beta^{\frac{3}{2}}+66\beta^{\frac{5}{2}}-8\beta^{\frac{7}{2}}}{\left(36-25\beta+4\beta^2+8\beta^{\frac{1}{2}}-3\beta^{\frac{3}{2}}\right)^2} & \beta \in (0, 0.6094], \\ \frac{324-652\beta+465\beta^2-142\beta^3+16\beta^4}{2\left(648-836\beta+368\beta^2-55\beta^3\right)} & \text{if} \quad \beta \in (0.6094, 0.8 \\ \frac{972-1964\beta+1395\beta^2-426\beta^3+48\beta^4}{(8-3\beta)^2(9-4\beta)^2} & \beta \in (0.8578, 1]. \end{cases}$$

By using Maple to compare $\Pi_H^T|_{m=1}$ and $\Pi_H^T|_{m=2}$ to $\Pi_H^T|_{m=3}$, it is found that:[13]

$$\Pi_H^T\big|_{m=1} - \Pi_H^T\big|_{m=3} > 0 \text{ and } \Pi_H^T\big|_{m=2} - \Pi_H^T\big|_{m=3} > 0.$$

It implies that the licensor firm has no incentive to license its advanced technology to all of the low-quality firms. This is part of the results in Proposition 5.

In addition, by comparing $\Pi_H^T|_{m=0}(= \Pi_H^{NL})$, $\Pi_H^T|_{m=1}$ and $\Pi_H^T|_{m=2}$, it can be derived that

$$\begin{aligned} &\Pi_H^T\big|_{m=1} - \Pi_H^T\big|_{m=0} > 0, \\ &\Pi_H^T\big|_{m=2} - \Pi_H^T\big|_{m=0} > 0, \text{ if } \beta > 0.1099; \\ &\Pi_H^T\big|_{m=1} - \Pi_H^T\big|_{m=2} \geq 0, \text{ if } \beta \leq 0.8331. \end{aligned}$$

That is to say, the licensor firm will license to only one low-quality firm if $\beta \in (0, 0.8331]$ while it will license to two firms if $\beta \in (0.8331, 1)$.

[13]The equations are very complicated. We use figures drawn by Maple to identify the results.

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As a result, the optimal two-part tariff licensing contract is summarized as follows:

$$\begin{cases} r^* = \bar{r}|_{m^*=1}, & F^* = 0, & m^* = 1 & \beta \in (0, 0.159]; \\ r^* = \tilde{r}|_{m^*=1}, & F^* = F(r^*)|_{m^*=1}, & m^* = 1 & \beta \in (0.159, 0.75]; \\ r^* = 0, & F^* = F(r^*)|_{m^*=1}, & m^* = 1 & \text{if } \beta \in (0.75, 0.8331]; \\ r^* = \tilde{r}|_{m^*=2}, & F^* = F(r^*)|_{m^*=2}, & m^* = 2 & \beta \in (0.8331, 0.8578]; \\ r^* = 0, & F^* = F(r^*)|_{m^*=2}, & m^* = 2 & \beta \in (0.8578, 1), \end{cases}$$

It is obvious that the optimal two-part tariff licensing contract may encompass a pure royalty, a positive royalty and a positive fixed fee, or a pure fixed fee, varying with the technology gap between the two types of firms. Besides, no licensing is always less profitable than licensing for the licensor firm. This is the main result in Proposition 5.

7 Appendix 4: The extension on absorptive capacity

This appendix is to show the results in Section 5. Note that if licensing does not occur, the equilibrium profits for the firms are the same as those in our basic model which are as follows:

$$\Pi_H^{NL} = \Pi_H^F|_{m=0} = \Pi_H^R|_{m=0} = \left(\frac{4-3\beta}{8-3\beta}\right)^2,$$
$$\Pi_1^{NL} = \Pi_2^{NL} = \Pi_3^{NL} = \beta\left(\frac{1}{8-3\beta}\right)^2.$$

With licensing, there are three types of products: the high-quality product ($s_H = 1$) produced by the licensor firm, mid-quality ($s_M = t$) produced by the licensee firms and low-quality ($s_L = \beta$) produced by the non-licensee firms with $1 > t > \beta$. Therefore, the inverse demand functions for the three types of products are as follows:

$$P_H = 1 - q_H - t\sum_{i\in M} q_i - \beta\sum_{j\in N\setminus M} q_j,$$
$$P_M = t - tq_H - t\sum_{i\in M} q_i - \beta\sum_{j\in N\setminus M} q_j,$$
$$P_L = \beta - \beta q_H - \beta\sum_{i\in M} q_i - \beta\sum_{j\in N\setminus M} q_j,$$

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where q_H, q_i and q_j represent the outputs of the licensor firm, the licensee firms and the non-licensee firms, respectively. The profits of the three types of firms with fixed-fee licensing are respectively specified as follows:

$$\Pi_H^F = \left(1 - q_H - t\sum_{i\in M} q_i - \beta \sum_{j\in N\setminus M} q_j\right) q_H + mF,$$
$$\Pi_i^F = \left(t - tq_H - t\sum_{i\in M} q_i - \beta \sum_{j\in N\setminus M} q_j\right) q_i - F,$$
$$\Pi_j^F = \beta\left(1 - q_H - \sum_{i\in M} q_i - \sum_{j\in N\setminus M} q_j\right) q_j,$$

where $F = \Pi_i^F|_m - \Pi_j^F|_{m-1}$.

By routine calculation, we derive the profits for the licensor firm under different number of licenses sold as follows:

$$\Pi_H^F\big|_{m=0} = \left(\frac{4-3\beta}{8-3\beta}\right)^2,$$
$$\Pi_H^F\big|_{m=1} = \left(\frac{6t-3t^2-2\beta}{12t-3t^2-4\beta}\right)^2 + F|_{m=1},$$
$$\Pi_H^F\big|_{m=2} = \left(\frac{6t-4t^2-2\beta+t\beta}{12t-4t^2-4\beta+t\beta}\right)^2 + 2F|_{m=2}, \text{ and}$$
$$\Pi_H^F\big|_{m=3} = \left(\frac{4-3t}{8-3t}\right)^2 + 3F|_{m=3},$$

where

$$F|_{m=1} = t\left(\frac{3t-2\beta}{12t-3t^2-4\beta}\right)^2 - \beta\left(\frac{1}{8-3\beta}\right)^2,$$
$$F|_{m=2} = t\left(\frac{2t-\beta}{12t-4t^2-4\beta+t\beta}\right)^2 - \frac{t^2\beta}{\left(12t-3t^2-4\beta\right)^2},$$
$$F|_{m=3} = t\left(\frac{1}{8-3t}\right)^2 - \frac{t^2\beta}{\left(12t-4t^2-4\beta+t\beta\right)^2}.$$

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By using Maple to compare the above four profit levels, we can find that:[14]

$$\begin{aligned}
&\Pi_H^F\big|_{m=1} - \Pi_H^F\big|_{m=3} = \\
&\quad -2(t-\beta)\Big(5695488t^5 - 3437568t^6 + 884736t^7 - 82944t^8 - 384t^5\beta \\
&\quad + 1403712\beta t^6 - 936704t^3\beta^3 - 1419552t^5\beta^2 - 501912t^7\beta \\
&\quad - 32768t^2\beta^4 - 97056t^4\beta^3 + 78984t^6\beta^2 + 74196t^7\beta^2 + 49680t^8\beta \\
&\quad - 11340t^8\beta^2 + 316656t^5\beta^3 - 98064\beta^3t^6 + 231840t^3\beta^4 + 15192t^3\beta^5 \\
&\quad + 29322t^5\beta^4 + 8991t^7\beta^3 - 36000t^2\beta^5 - 137016t^4\beta^4 - 2646t^4\beta^5 \\
&\quad - 2106t^6\beta^4 + 28800t\beta^5 + 648t^9\beta + 162t^5\beta^5 + 788480\beta^3t^2 \\
&\quad - 4737024t^4\beta + 2719744t^4\beta^2 - 143360t\beta^4 - 3317760t^4 - 8192\beta^4 \\
&\quad - 1916928t^2\beta^2 + 294912t\beta^3 + 4423680t^3\beta - 129024t^3\beta^2\Big)\Big/ \\
&\quad \left(\left(12t - 4\beta - 3t^2\right)^2\left(12t - 4t^2 - 4\beta + t\beta\right)^2(8-3t)^2(8-3\beta)^2\right) > 0.
\end{aligned}$$

This follows that licensing to all low-quality firms is less profitable than licensing to one low-quality firm. In addition, we have:

$$\begin{aligned}
\Pi_H^F\big|_{m=1} - \Pi_H^F\big|_{m=0} &= -(t-\beta)\left(36t\beta^3 - 108\beta^3 + 348t\beta^2 - 180t^2\beta^2\right. \\
&\quad + 176\beta^2 - 624t\beta - 72t^2\beta + 225t^3\beta - 432t^3 \\
&\quad \left.+576t^2\right)\Big/\left(12t - 4\beta - 3t^2\right)^2(8-3\beta)^2 > 0 \\
&\quad \text{if } \beta > \beta_1(t) \\
\Pi_H^F\big|_{m=2} - \Pi_H^F\big|_{m=0} &= -2(t-\beta)\left(1728t^7\beta - 6384t^2\beta^4 - 6342t^4\beta^3\right. \\
&\quad - 1404t^6\beta^2 - 864\beta^5 - 13824\beta t^6 + 8736t\beta^4 \\
&\quad + 30768t^3\beta^3 + 17502t^5\beta^2 - 26784\beta^3t^2 - 54928\beta^2t^4 \\
&\quad - 118272t^3\beta + 68096t^2\beta^2 + 82176t^4\beta + 73728t^4 \\
&\quad - 92160t^5 + 32256t^6 - 3456t^7 + 1536\beta^4 - 9t^4\beta^4 \\
&\quad - 16896t\beta^3 + 7904t^3\beta^2 + 14560t^5\beta + 432t\beta^5 \\
&\quad \left.+720t^3\beta^4 + 306t^5\beta^3\right)\Big/\left(12t - 4\beta - 4t^2 - t\beta\right)^2
\end{aligned}$$

[14]The derivations and comparisons are very complex. Thus, we use Maple assisting us to draw the figures and show the results.

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$$\begin{aligned}
&\left(12t-4\beta-3t^2\right)^2(8-3\beta)^2>0, \quad \text{if} \quad \beta>\beta_2(t).\\
\Pi_H^F\big|_{m=1}-\Pi_H^F\big|_{m=2}=&-(t-\beta)\left(66816t^5-13824t^6-28592t^5\beta+8352\beta t^6\right.\\
&-7072t^3\beta^3+588t^5\beta^2+144t^7\beta-2224t^2\beta^4-7236t^4\beta^3\\
&-1872t^6\beta^2+1629t^5\beta^3+3228t^3\beta^4+36t^3\beta^5-396t^2\beta^5\\
&-450t^4\beta^4+576t\beta^5+18304\beta^3t^2-41088t^4\beta+36736t^4\beta^2\\
&-2944t\beta^4-64512t^4-256\beta^4-41728t^2\beta^2+6912t\beta^3\\
&\left.+91392t^3\beta-22144t^3\beta^2\right)/\left(\left(12t-4\beta-3t^2\right)^2\right.\\
&\left.\left(12t-4t^2-4\beta+t\beta\right)^2(8-3\beta)^2\right)>0, \quad \text{if} \quad \beta<\beta_3(t).
\end{aligned}$$

Therefore, we have the following result:

$$\begin{cases}
\Pi_H^F\big|_{m=0}\geq\Pi_H^F\big|_{m=1}>\Pi_H^F\big|_{m=2}, & m^*=0 \quad \beta\in(0,\beta_1],\\
\Pi_H^F\big|_{m=1}>\Pi_H^F\big|_{m=0}\geq\Pi_H^F\big|_{m=2}, & m^*=1 \quad \text{if} \quad \beta\in(\beta_1,\beta_2],\\
\Pi_H^F\big|_{m=1}\geq\Pi_H^F\big|_{m=2}>\Pi_H^F\big|_{m=0}, & m^*=1 \quad \beta\in(\beta_2,\beta_3],\\
\Pi_H^F\big|_{m=2}>\Pi_H^F\big|_{m=1}>\Pi_H^F\big|_{m=0}, & m^*=2 \quad \beta\in(\beta_3,1).
\end{cases}$$

According to the above result, we can use Maple to depict the figure as shown in Figure 1 in the main text.

On the other hand, the profits of the licensor firm, the licensee firms and the non-licensee firms with royalty licensing are respectively specified as follows:

$$\begin{aligned}
\Pi_H^R&=\left(1-q_H-t\sum_{i\in M}q_i-\beta\sum_{j\in N\setminus M}q_j\right)q_H+r\sum_{i\in M}q_i,\\
\Pi_i^R&=\left(t-tq_H-t\sum_{i\in M}q_i-\beta\sum_{j\in N\setminus M}q_j-r\right)q_i,\\
\Pi_j^R&=\beta\left(1-q_H-\sum_{i\in M}q_i-\sum_{j\in N\setminus M}q_j\right)q_j.
\end{aligned}$$

By using Maple for routine calculations, we derive the profits for the licensor firm under different number of licenses sold as follows:

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$$\Pi_H^R\big|_{m=1} = \left(\frac{6t - 2\beta - 3t^2 + 3t\bar{r}|_{m=1} - 2\beta\bar{r}|_{m=1}}{12t - 3t^2 - 4\beta}\right)^2 + \bar{r}|_{m=1}\left(\frac{3t - 2\beta - 6\bar{r}|_{m=1} + 2\beta\bar{r}|_{m=1}}{12t - 3t^2 - 4\beta}\right),$$

where

$$\bar{r}|_{m=1} = \frac{6t\beta^2 - 16\beta t - 9\beta t^2 + 24t^2 - \left(12t - 4\beta - 3t^2\right)\sqrt{t\beta}}{2t(8 - 3\beta)(3 - \beta)}.$$

$$\Pi_H^R\big|_{m=2} = \left(\frac{6t - 4t^2 - 2\beta + t\beta + 4t\ \bar{r}|_{m=2} - 2\beta\ \bar{r}|_{m=2}}{12t - 4t^2 - 4\beta + t\beta}\right)^2 + 2\ \bar{r}|_{m=2}\left(\frac{3t - 2\beta - 6\ \bar{r}|_{m=2} + 2\beta\ \bar{r}|_{m=2}}{12t - 3t^2 - 4\beta}\right),$$

where

$$\bar{r}|_{m=2} = \left(24t^{\frac{5}{2}} - 6t^{\frac{7}{2}} - 12\beta^{\frac{1}{2}}t^2 + 4\beta^{\frac{3}{2}}t + 4\beta^{\frac{1}{2}}t^3 - \beta^{\frac{3}{2}}t^2 + 3t^{\frac{5}{2}}\beta + 4t^{\frac{1}{2}}\beta^2 - 20t^{\frac{3}{2}}\beta\right) \Big/ \left(48t^{\frac{3}{2}} - 20\beta^{\frac{1}{2}}t^2 + 6\beta^{\frac{3}{2}}t + 4\beta^{\frac{1}{2}}t^3 - \beta^{\frac{3}{2}}t^2 + 24\beta^{\frac{1}{2}}t - 8\beta^{\frac{3}{2}} + 3t^{\frac{5}{2}}\beta + 4t^{\frac{1}{2}}\beta^2 - 12t^{\frac{3}{2}}\beta - 12t^{\frac{5}{2}} - 16t^{\frac{1}{2}}\beta\right).$$

$$\Pi_H^R\big|_{m=3} = \left(\frac{4 - 3t - 3\bar{r}|_{m=3}}{8 - 3t}\right)^2 + 3\bar{r}|_{m=3}\left(\frac{t - 2\bar{r}|_{m=3}}{t(8 - 3t)}\right),$$

where

$$\bar{r}|_{m=3} = \frac{12t^2 - 8\beta^{\frac{1}{2}}t^{\frac{3}{2}} + 3\beta^{\frac{1}{2}}t^{\frac{5}{2}} - 4t\beta - 4t^3 + t^2\beta}{2\left(12t - 4t^2 + 16\beta^{\frac{1}{2}}t^{\frac{1}{2}} - 14\beta^{\frac{1}{2}}t^{\frac{3}{2}} + 3\beta^{\frac{1}{2}}t^{\frac{5}{2}} - 4\beta + t\beta\right)}.$$

Note that all the royalty rates are of corner solution (i.e., the participating constraint of the licensee firms, $\Pi_i^R\big|_m = \Pi_j^R\Big|_{m-1}$, is binding).

By using Maple to compare the above four profit levels, we can find that:

$$\Pi_H^R\big|_{m=1} - \Pi_H^R\big|_{m=3} = \left(\frac{6t - 2\beta - 3t^2 + 3t\bar{r}|_{m=1} - 2\beta\bar{r}|_{m=1}}{12t - 3t^2 - 4\beta}\right)^2 + \bar{r}|_{m=1}\left(\frac{3t - 2\beta - 6\bar{r}|_{m=1} + 2\beta\bar{r}|_{m=1}}{12t - 3t^2 - 4\beta}\right) - \left(\frac{4 - 3t - 3\bar{r}|_{m=3}}{8 - 3t}\right)^2 - 3\bar{r}|_{m=3}\left(\frac{t - 2\bar{r}|_{m=3}}{t(8 - 3t)}\right) > 0.$$

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It is obvious that from the licensor firm's perspective, licensing to all low-quality firms is less profitable than licensing to one low-quality firm. In addition, we have:

$$\begin{aligned}\Pi_H^R\big|_{m=1} - \Pi_H^R\big|_{m=0} = \Bigg[\beta\Big(&48t(t\beta)^{\frac{1}{2}} + 24\beta - 24t - 39t\beta + 12t\beta^2 + 27t^2 \\ &- 4\beta^2 - 32(t\beta)^{\frac{1}{2}}\beta - 18t\beta(t\beta)^{\frac{1}{2}} + 12(t\beta)^{\frac{1}{2}}\beta^2 \\ &- 6t^2\beta\Big)\Bigg] \Big/ \Big(4t(8-3\beta)^2(3-\beta)^2\Big) > 0,\end{aligned}$$

if $\beta > \beta_4(t)$.

$$\begin{aligned}\Pi_H^R\big|_{m=2} - \Pi_H^R\big|_{m=0} = &\left(\frac{6t - 4t^2 - 2\beta + t\beta + 4t\bar{r}|_{m=2} - 2\beta\bar{r}|_{m=2}}{12t - 4t^2 - 4\beta + t\beta}\right)^2 \\ &+ 2\bar{r}|_{m=2}\left(\frac{3t - 2\beta - 6\bar{r}|_{m=2} + 2\beta\bar{r}|_{m=2}}{12t - 3t^2 - 4\beta}\right) \\ &- \left(\frac{4-3\beta}{8-3\beta}\right)^2 > 0, \quad \text{if} \quad \beta > \beta_5(t).\end{aligned}$$

$$\begin{aligned}\Pi_H^R\big|_{m=1} - \Pi_H^R\big|_{m=2} = &\left(\frac{6t - 2\beta - 3t^2 + 3t\bar{r}|_{m=1} - 2\beta\bar{r}t|_{m=1}}{12t - 3t^2 - 4\beta}\right)^2 \\ &+ \bar{r}|_{m=1}\left(\frac{3t - 2\beta - 6\bar{r}|_{m=1} + 2\beta\bar{r}|_{m=1}}{12t - 3t^2 - 4\beta}\right) \\ &- \left(\frac{6t - 4t^2 - 2\beta + t\beta + 4t\bar{r}|_{m=2} - 2\beta\bar{r}|_{m=2}}{12t - 4t^2 - 4\beta + t\beta}\right)^2 \\ &- 2\bar{r}|_{m=2}\left(\frac{3t - 2\beta - 6\bar{r}|_{m=2} + 2\beta\bar{r}|_{m=2}}{12t - 3t^2 - 4\beta}\right) > 0, \\ &\text{if} \quad \beta < \beta_6(t).\end{aligned}$$

In sum, we can come up with the following result and depict Figure 2 in the main text:

$$\begin{cases} \Pi_H^R\big|_{m=0} \geq \Pi_H^R\big|_{m=1} > \Pi_H^R\big|_{m=2}, \ m^* = 0 & \beta \in (0, \beta_4], \\ \Pi_H^R\big|_{m=1} > \Pi_H^R\big|_{m=0} \geq \Pi_H^R\big|_{m=2}, \ m^* = 1 & \text{if} \quad \beta \in (\beta_4, \beta_5], \\ \Pi_H^R\big|_{m=1} \geq \Pi_H^R\big|_{m=2} > \Pi_H^R\big|_{m=0}, \ m^* = 1 & \beta \in (\beta_5, \beta_6], \\ \Pi_H^R\big|_{m=2} > \Pi_H^R\big|_{m=1} > \Pi_H^R\big|_{m=0}, \ m^* = 2 & \beta \in (\beta_6, 1). \end{cases}$$

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By making use of Maple to compare the optimal fixed-fee licensing schedule and the optimal royalty licensing schedule, we derive the optimal one-part tariff licensing schedule as follows and portray Figure 3 in the main text.

$$\begin{cases} r^* = 0 \text{ and } F^* = 0, & m^* = 0 & \text{Region I;} \\ r^* = \bar{r}|_{m^*=1}, & m^* = 1 & \text{Region II;} \\ r^* = \bar{r}|_{m^*=2}, & m^* = 2 \text{ if } \beta \in & \text{Region III;} \\ F^* = F(\beta, t)|_{m^*=1}, & m^* = 1 & \text{Region IV;} \\ F^* = F(\beta, t)|_{m^*=2}, & m^* = 2 & \text{Region V .} \end{cases}$$

Note that by comparing $\Pi_H^F\big|_{m^*=1}$ to $\Pi_H^R\big|_{m^*=1}$ and $\Pi_H^R\big|_{m^*=2}$, we derive that

$$\begin{aligned} \Pi_H^R\big|_{m^*=2} - \Pi_H^F\big|_{m^*=1} = & \left(\frac{6t - 4t^2 - 2\beta + t\beta + 4t\bar{r}|_{m=2} - 2\beta\bar{r}|_{m=2}}{12t - 4t^2 - 4\beta + t\beta}\right)^2 \\ & + 2\bar{r}|_{m=2}\left(\frac{3t - 2\beta - 6\bar{r}|_{m=2} + 2\beta\bar{r}|_{m=2}}{12t - 3t^2 - 4\beta}\right) \\ & - \left(\left(\frac{6t - 3t^2 - 2\beta}{12t - 3t^2 - 4\beta}\right)^2 + t\left(\frac{3t - 2\beta}{12t - 3t^2 - 4\beta}\right)^2\right. \\ & \left. - \beta\left(\frac{1}{8 - 3\beta}\right)^2\right) > 0, \quad \text{if} \quad \beta < \beta_7(t). \end{aligned}$$

$$\begin{aligned} \Pi_H^R\big|_{m^*=1} - \Pi_H^F\big|_{m^*=1} = & \left(\frac{6t - 2\beta - 3t^2 + 3t\bar{r}|_{m=1} - 2\beta\bar{r}|_{m=1}}{12t - 3t^2 - 4\beta}\right)^2 \\ & + \bar{r}|_{m=1}\left(\frac{3t - 2\beta - 6\bar{r}|_{m=1} + 2\beta\bar{r}|_{m=1}}{12t - 3t^2 - 4\beta}\right) \\ & - \left(\left(\frac{6t - 3t^2 - 2\beta}{12t - 3t^2 - 4\beta}\right)^2 + t\left(\frac{3t - 2\beta}{12t - 3t^2 - 4\beta}\right)^2\right. \\ & \left. - \beta\left(\frac{1}{8 - 3\beta}\right)^2\right) > 0, \quad \text{if} \quad \beta < \beta_8(t). \end{aligned}$$

We then investigate the optimal two-part tariff licensing contract. The profits of the licensor firm, the licensee firms and the non-licensee firms with two-part tariff licensing are respectively specified as follows:

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$$\Pi_H^T = \left(1 - q_H - t\sum_{i\in M} q_i - \beta \sum_{j\in N\setminus M} q_j\right) q_H + r\sum_{i\in M} q_i + mF,$$

$$\Pi_i^T = \left(t - tq_H - t\sum_{i\in M} q_i - \beta \sum_{j\in N\setminus M} q_j - r\right) q_i - F,$$

$$\Pi_j^T = \beta\left(1 - q_H - \sum_{i\in M} q_i - \sum_{j\in N\setminus M} q_j\right) q_j,$$

where $F = \Pi_i^T|_m - \Pi_j^T|_{m-1}$.

By utilizing Maple, we can derive the profits for the licensor firm under different number of licenses sold as follows:

$$\Pi_H^T\big|_{m=0} = \Pi_H^{NL} = \left(\frac{4-3\beta}{8-3\beta}\right)^2,$$

$$\Pi_H^T\big|_{m=1} = \left(\frac{6t - 2\beta - 3t^2 + 3tr^* - 2\beta r^*}{12t - 3t^2 - 4\beta}\right)^2 + r^*\left(\frac{3t - 2\beta - 6r^* + 2\beta r^*}{12t - 3t^2 - 4\beta}\right) + F\left(r^*\right)\big|_{m=1},$$

where

$$r^* = \begin{cases} \bar{r}|_{m=1} = \frac{6t\beta^2 - 16\beta t - 9\beta t^2 + 24t^2 - \left(12t - 4\beta - 3t^2\right)\sqrt{t\beta}}{2t(8-3\beta)(3-\beta)} & \beta \in (0, \bar{\beta}_1], \\ \tilde{r}|_{m=1} = \frac{\left(12t - 8\beta - 9t^2 + 4\beta t\right)}{2(12 - 2\beta + 2\beta t - 9t)} & \text{if } \beta \in (\bar{\beta}_1, \tilde{\beta}_1], \\ 0 & \beta \in (\tilde{\beta}_1, 1], \end{cases}$$

$$F\left(r^*\right)\big|_{m=1} = t\left(\frac{3t - 2\beta - 6r^* + 2\beta r^*}{12t - 3t^2 - 4\beta}\right)^2 - \beta\left(\frac{1}{8-3\beta}\right)^2,$$

and $\bar{\beta}_1$ is derived by setting $\bar{r}|_{m=1} = \tilde{r}|_{m=1}$; $\tilde{\beta}_1$ is derived by setting $\tilde{r}|_{m=1} = 0$.

$$\Pi_H^T\big|_{m=2} = \left(\frac{6t - 4t^2 - 2\beta + t\beta + 4tr^* - 2\beta r^*}{12t - 4t^2 - 4\beta + t\beta}\right)^2 + 2r^*\left(\frac{3t - 2\beta - 6r^* + 2\beta r^*}{12t - 3t^2 - 4\beta}\right) + 2\,F\left(r^*\right)\big|_{m=2},$$

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where

$$r^* = \begin{cases} \bar{r}|_{m=2} & \beta \in (0, \bar{\beta}_2], \\ \tilde{r}|_{m=2} \quad \text{if} & \beta \in (\bar{\beta}_2, \tilde{\beta}_2], \\ 0 & \beta \in (\tilde{\beta}_2, 1], \end{cases}$$

$$F\left(r^*\right)\big|_{m=2} = t\left(\frac{2t - 4r^* - \beta(1 - r^*)}{12t - 4t^2 - 4\beta + t\beta}\right)^2 - \beta\left(\frac{t + 2r^* - tr^*}{12t - 3t^2 - 4\beta}\right)^2,$$

and

$$\begin{aligned}
\bar{r}|_{m=2} = &\left(24t^{\frac{5}{2}} - 6t^{\frac{7}{2}} - 12\beta^{\frac{1}{2}}t^2 + 4\beta^{\frac{3}{2}}t + 4\beta^{\frac{1}{2}}t^3 - \beta^{\frac{3}{2}}t^2 + 3t^{\frac{5}{2}}\beta\right. \\
&+ 4t^{\frac{1}{2}}\beta^2 - 20t^{\frac{3}{2}}\beta\Big)\Big/\Big(48t^{\frac{3}{2}} - 20\beta^{\frac{1}{2}}t^2 + 6\beta^{\frac{3}{2}}t + 4\beta^{\frac{1}{2}}t^3 - \beta^{\frac{3}{2}}t^2 \\
&\left.+ 24\beta^{\frac{1}{2}}t - 8\beta^{\frac{3}{2}} + 3t^{\frac{5}{2}}\beta + 4t^{\frac{1}{2}}\beta^2 - 12t^{\frac{3}{2}}\beta - 12t^{\frac{5}{2}} - 16t^{\frac{1}{2}}\beta\right), \\
\tilde{r}|_{m=2} = &-\left(-2016t^6 - 4608t^4 + 5760t^5 - 2\beta^3t^4 - 1104\beta^2t^4 - 230t^6\beta\right. \\
&- 9216t^4\beta + 8256t^3\beta + 2704t^5\beta - 1088\beta^3t^2 - 5120t^2\beta^2 \\
&+ 80t\beta^4 + 61\beta^2t^5 + 1344t\beta^3 + 140\beta^3t^3 + 4976t^3\beta^2 \\
&\\
&-128\beta^4 + 216t^7\Big) / 2\left(1664t\beta^2 - 4800t^2\beta + 4608t^3 - 216t^6\right. \\
&- 5760t^4 + 2016t^5 - 192\beta^3 + \beta^3t^4 + 346\beta^2t^4 + 52t^6\beta \\
&- 128t^4\beta + 4416t^3\beta - 376t^5\beta + 356\beta^3t^2 - 640t^2\beta^2 - \\
&\left.32t\beta^4 - 26\beta^2t^5 - 160t\beta^3 - 60\beta^3t^3 - 976t^3\beta^2 + 32\beta^4\right),
\end{aligned}$$

and $\bar{\beta}_2$ is derived by setting $\bar{r}|_{m=2} = \tilde{r}|_{m=2}$, $\tilde{\beta}_2$ is derived by setting $\tilde{r}|_{m=2} = 0$.

$$\Pi_H^T\big|_{m=3} = \left(\frac{4 - 3t - 3r^*}{8 - 3t}\right)^2 + 3r^*\left(\frac{t - 2r^*}{t(8 - 3t)}\right) + 3\ F\left(r^*\right)\big|_{m=3},$$

where

$$r^* = \begin{cases} \bar{r}|_{m=3} = \frac{12t^2 - 8\beta^{\frac{1}{2}}t^{\frac{3}{2}} + 3\beta^{\frac{1}{2}}t^{\frac{5}{2}} - 4t\beta - 4t^3 + t^2\beta}{2\left(12t - 4t^2 + 16\beta^{\frac{1}{2}}t^{\frac{1}{2}} - 14\beta^{\frac{1}{2}}t^{\frac{3}{2}} + 3\beta^{\frac{1}{2}}t^{\frac{5}{2}} - 4\beta + t\beta\right)} & \beta \in \left(0, \bar{\beta}_3\right], \\ \tilde{r}|_{m=3} & \text{if} \quad \beta \in \left(\bar{\beta}_3, \tilde{\beta}_3\right], \\ 0 & \beta \in \left(\tilde{\beta}_3, 1\right], \end{cases}$$

$$F\left(r^*\right)\big|_{m=3} = t\left(\frac{t - 2r^*}{t(8 - 3t)}\right)^2 - \beta\left(\frac{t + 4r^* - 2tr^*}{12t - 4t^2 - 4\beta + t\beta}\right)^2,$$

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and

$$\begin{aligned}\tilde{r}|_{m=3} = t\Big(&1664t\beta - 1728t^2 - 192\beta^2 - 2176\beta t^2 + 240t\beta^2 + 2448t^3 \\ &+ 864t^3\beta - 84t^2beta^2 - 108t^4\beta + 9t^3\beta^2 - 1056t^4 + 144t^5\Big)\Big/ \\ &2\Big(128t\beta - 1728t^2 - 192\beta^2 + 256\beta t^2 + 240t\beta^2 + 2448t^3 - 568t^3\beta \\ &- 84t^2\beta^2 + 264t^4\beta + 9t^3\beta^2 - 1056t^4 + 144t^5 - 36t^5\beta\Big),\end{aligned}$$

and $\bar{\beta}_3$ is derive by setting $\bar{r}|_{m=3} = \tilde{r}|_{m=3}$, $\tilde{\beta}_3$ is derived by setting $\tilde{r}|_{m=3} = 0$.

By utilizing Maple to compare the above four profit levels, we can find that:

$$\begin{aligned}&\Pi_H^T\big|_{m=1} - \Pi_H^T\big|_{m=0} > 0, \quad \text{if} \quad \beta > \beta_4(t); \\ &\Pi_H^T\big|_{m=2} - \Pi_H^T\big|_{m=0} > 0, \quad \text{if} \quad \beta > \beta_5(t); \\ &\Pi_H^T\big|_{m=1} - \Pi_H^T\big|_{m=3} > 0; \quad \Pi_H^T\big|_{m=2} - \Pi_H^T\big|_{m=3} > 0; \quad \text{and} \\ &\Pi_H^T\big|_{m=1} - \Pi_H^T\big|_{m=2} > 0, \quad \text{if} \quad \beta < \beta_9(t).\end{aligned}$$

Thus, we can derive the optimal two-part tariff licensing schedule as follows:

$$\begin{cases} r^* = 0, & F^* = 0, & m^* = 0 & & \text{Region I}, \\ r^* = \bar{r}|_{m^*=1}, & F^* = 0, & m^* = 1 & & \text{Region II}, \\ r^* = \tilde{r}|_{m^*=1}, & F^* = F(r^*)|_{m^*=1}, & m^* = 1 & & \text{Region III}, \\ r^* = 0, & F^* = F(r^*)|_{m^*=1}, & m^* = 1, & \text{if } \beta \in & \text{Region IV}, \\ r^* = \tilde{r}|_{m^*=2}, & F^* = F(r^*)|_{m^*=2}, & m^* = 2 & & \text{Region V}, \\ r^* = 0, & F^* = F(r^*)|_{m^*=2}, & m^* = 2 & & \text{Region VI}. \end{cases}$$

Based on the above result, we portray Figure 4 in the main text

References

Arundel, Anthony and Isabelle Kabla (1998), "What Percentage of Innovations are Patented? Empirical Estimates for European Firms," *Research Policy*, 27, 127–141.

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Chang, Ray-Yun, Hong Hwang, and Cheng-Hau Peng (2013), "Technology Licensing, R&D and Welfare," *Economics Letters*, 118, 396–399.

Chang, Ray-Yun and Cheng-Hau Peng (2013), "Cost Asymmetry and Vertical Product Licensing," *Asia-Pacific Journal of Accounting and Economics*, 20, 270–280.

Cohen, Wesley M., Richard R. Nelson, and John P. Walsh (1997), "Appropriability Conditions and Why Firms Patent and Why They Do Not in the American Manufacturing Sector," Working Paper, Carnegie Mellon University.

Kamien, Morton I. and Yair Tauman (2002), "Patent Licensing: the Inside Story," *Manchester School*, 70, 7–15.

Li, Changying and Juan Song (2009), "Technology Licensing in a Vertically Differentiated Duopoly," *Japan and the World Economy*, 21, 183–190.

Motta, Massimo (1993), "Endogenous Quality Choice: Price vs. Quantity Competition," *Journal of Industrial Economics*, 41, 113–131.

Nabin, Munirul H., Xuan Nguyen, and Pasquale M. Sgro (2013), "Technology Transfer, Quality Standards, and North-South Trade," *Review of International Economics*, 21, 783–796.

Poddar, Sougata and Uday Bhanu Sinha (2004), "On Patent Licensing in Spatial Competition," *Economic Record*, 80, 208–218.

Wang, X. Henry (1998), "Fees versus Royalty Licensing in a Cournot Duopoly Model," *Economics Letters*, 60, 55–62.

——— (2002), "Fee versus Royalty Licensing in a Differentiated Cournot Duopoly," *Journal of Economics and Business*, 54, 253–266.

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垂直異質寡占市場之品質授權

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本文建構一垂直產品差異性的寡佔模型, 探討擁有品質創新的高品質廠商面對多家低品質廠商競爭下之最適授權契約。研究結果顯示, 當市場上存在兩家以上的低品質廠商時, 高品質廠商僅會將其品質創新授權給部分的低品質廠商。當產品品質差異較小 (大) 時, 固定權利金授權下高品質廠商的總利潤將會高 (低) 於單位權利金授權。此外, 我們也發現固定權利金授權下的社會福利水準有可能低於單位權利金授權下的水準。這些結果與文獻中成本下降之最適授權契約相比, 有很大的不同。如果授權廠商採兩部訂價法之授權契約, 依產品品質差異的程度大小, 其最適授權契約可能爲純粹單位權利金、正的單位權利金加上正的固定權利金或純粹固定權利金。值得一提的是, 即使在被授權廠商受限於吸收能力而無法將其產品品質提升至授權之品質水準, 以上這些結果仍然成立。

關鍵詞: 技術授權, 垂直品質差異, 寡占競爭, 社會福利
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