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Durable Goods Monopoly with Network Effects

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This paper examines the pricing behavior of a monopoly in a durable goods market with non-delayed network effects. Network externalities ensure that each consumer will have a strictly positive willingness to pay for holding the goods, which in turn implies that the trading process must end within a finite time. The monopolist faces a trade-off between the respective benefits of price discrimination and accelerating the trading process, and the effect of the non-delayed network effects is to increase the latter benefit; therefore, equilibrium prices must decrease over time. We show that there exists a unique stationary equilibrium, and that the Coase conjecture holds in equilibrium. Furthermore, when the network effects are sufficiently strong, the monopolist will flood the market immediately. We also provide examples in which equilibrium prices and states can be explicitly computed. The comparative statics results suggest that the stronger the network effects, the more quickly the whole market will be covered.

Keywords: durable goods monopoly, network externality, coase conjecture
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1 Introduction

Many durable goods industries exhibit some form of network effect that arises when a consumer's willingness to pay increases with the number of other consumers buying the same product. One type of network effect can

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have a delayed influence in the sense that the consumer's willingness to pay in the current period depends on aggregate consumption in the previous period.[1] This type of network effect appears when the quality of the good depends on some "learning by doing" or "word of mouth" process.[2] Bensaid and Lesne (1996) show that a monopolist may charge a sequence of increasing prices when network effects are delayed. This result provides a theoretical basis for the practice of introductory pricing: setting a low initial price to attract a critical mass of customers, then raising the price later on.

In this paper, we focus on another perhaps more prevalent type of network effect for which there is no delay, in the sense that the benefit derived from network externalities depends on the current size of the network.[3] The classic example is the telephone services industry. As soon as a consumer subscribes to such services, she can enjoy the benefit of sharing messages or information with other subscribers without any delay. This distinction between delayed and non-delayed network effects is important. As we shall show, the characteristics of network effects imply very different qualitative properties.

We consider a discrete-time, infinite-period model in which the flow utility per period a consumer derives from a unit of the durable good comes

[1]For example, when new software is released, early adapters who decide to buy it immediately must be prepared to pay two extra costs. First, they take the risk of suffering from numerous bugs being attached to the first version of the software. Second, they find few people to help them use the software. On the other hand, because the experiences of early consumers allow firms to improve their products or offer better services and assistance, consumers who buy the software in later periods can benefit from the network externalities generated by these early adapters.

[2]According to the terminology employed by Bensaid and Lesne (1996), the "learning by doing" externality means that the greater the number of initial buyers of a product, the better the quality of the updated product release. Hence, the product quality is higher for consumers who buy the product in later periods. On the other hand, the "word of mouth" externality means that the greater the number of initial buyers of a product, the greater the amount of information conveyed regarding its existence, quality and use. Hence, potential consumers' search costs are lowered, and their willingness to pay will increase.

[3]In practice, even though the first version of the software may have many bugs, most software companies now provide online updating services to immediately correct such bugs. Hence, this type of network effect can be classified as non-delayed. Another example is the digital camera industry, in which it is common for companies to provide immediate firmware-updating services to correct errors or add new features to their products.

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from two sources: (1) the intrinsic value she derives from consuming this good, which depends on her type,[4] and (2) the benefit generated from network effects by holding it. The network effects ensure that the consumer with the lowest intrinsic value can benefit from the positive network externalities, and her willingness to pay by holding the good is strictly above zero. Consequently, the monopolist can commit to not lowering its prices indefinitely. Moreover, the monopolist will at some point set a market clearing price to cover the market immediately, such market price being equal to the willingness of the consumer with the lowest intrinsic value to pay. Hence, the existence of network effects, even of a small magnitude, greatly shortens the entire trading process and ensures that trade ends within a finite time.

Furthermore, we find that as long as the network effects are non-delayed, equilibrium prices can only decrease.[5] Intuitively, if the monopolist charges a low price today and a higher price tomorrow, it can increase its profit by accelerating the trading process through charging tomorrow's high price today. Based on the skimming property,[6] buyers with a high intrinsic value who are willing to purchase (at the previous low price) in both periods will then accept this single high price, because buyers with a low intrinsic value have already accepted. Consequently, charging a sequence of increasing prices is suboptimal for the monopolist, and equilibrium prices cannot go up.[7] This

[4]Without loss of generality, we can order the intrinsic value of consumers such that low (high) consumers attach a high (low) intrinsic value to the good. In an ordinary durable goods monopoly model without network effects, the intrinsic value described here represents the willingness of the consumer to pay.

[5]From a technical viewpoint, our setup extends the typical durable goods monopoly model by allowing for more general value functions, this enabling us to characterize the equilibria in these environments. Theorem 1 below shows that the Coase conjecture still holds if the value function is upper semicontinuous.

[6]In Lemma 1, we show that for any subgame perfect equilibrium, high intrinsic value (low-type) consumers will purchase the good earlier than low intrinsic value consumers (high-type).

[7]However, the above argument breaks down if network effects are delayed. In this situation, the consumer's acceptance decision depends not only on her type, but also on the previous state (the size of the network in the previous period). When the monopolist raises the currently low price, i.e., charges tomorrow's high price today, consumers who would have bought tomorrow might refuse to purchase at this high price, because the benefit of the network effects they derive is now smaller due to their delay. As a result, equilibrium prices do not necessarily fall.

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price monotonicity allows us to characterize the stationary equilibria of the game. We also show that the Coase conjecture is satisfied in equilibrium. Moreover, because the monopolist faces a trade-off between the respective benefits of price discrimination and accelerating the trading process, and the effect of the non-delayed network externalities is to increase the benefit of the latter, the monopolist may want to secure the benefit of the network effects and flood the market immediately while these effects are sufficiently strong. In contrast to the results derived by Bensaid and Lesne (1996), our findings suggest that the monopolist will not use an introductory pricing strategy in a market with non-delayed network effects.

In addition to our general existence results, we also provide a class of examples in which equilibrium prices and states can be computed explicitly. We find that the stronger the network effects, the more quickly the market is covered. This suggests that markets that exhibit stronger network effects will become saturated within a shorter period, even without any competition.

Related Literature

In ordinary durable goods monopoly models, Stokey (1981), Bulow (1982) and Gul et al. (1986) formalize Coase's intuition and show that stationary equilibria (subgame perfect equilibria in which buyers use stationary strategies) are characterized by equilibrium prices decreasing over time (the price monotonicity property), and that the first offer by the monopolist will be very close to the price at which the least willing consumer is prepared to trade, as the period length converges to zero. On the other hand, most of the early literature on network effects was developed in a static framework. See, for example, the comprehensive surveys by Katz and Shapiro (1994) and Liebowitz and Margolis (1994). We depart from these early studies by considering a dynamic model in which network effects are spread over time.

One important issue on which the literature has focused is whether firms will adopt the practice of introductory pricing. The literature has identified at least three possible reasons[8] for the failure of price monotonicity.[9] First,

[8]Our analysis is based on the assumption of complete information. Incomplete information can be another cause of the failure of price monotonicity. See Cabral et al. (1999).

[9]The three reasons we point out are neither sufficient nor necessary for violating the price monotonicity property. Indeed, Bensaid and Lesne (1996) show that when the monopolist can commit to not selling its product in the future, which is equivalent to the assumption of finite periods, and network effects are delayed, equilibrium prices can rise. However, it is not

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when there is competition between networks, firms may offer a low introductory price to compete for new consumers to join their network. Once firms have locked consumers into their proprietary network, they may want to raise their prices, either because consumers need to pay switching costs if they change to another network or because their willingness to pay is high due to the network effects. Fudenberg and Tirole (2000) show that under some conditions, there exists an equilibrium whereby an incumbent firm will set a lower price in the presence of network effects with a view to preempting competition from a potential entrant. Cabral (2011) considers a more general dynamic model of competition between two proprietary networks. In this model, consumers die and are replaced at a constant hazard rate, and firms compete for new consumers to join their network by offering network entry prices. The author finds that because the idiosyncratic preferences of consumers will generate stochastic dynamics, prices may be higher or lower in the presence of network effects. Driskill (2007) and Laussel and Resende (2007) also address dynamic competition between proprietary networks. Their oligopolistic models are more general than our monopolistic specification, which allows us to obtain more explicit analytical results.

Second, equilibrium properties depend on whether firms can commit to not selling their products in the future. A model of finite periods implicitly assumes that the monopolist has commitment power. This allows the monopolist to avoid lowering its prices in subsequent periods. When combined with network effects, the monopolist may be able to sustain higher prices in later periods because residual consumers are also more willing to pay due to the stronger network effects. In contrast to the finite-periods model proposed by Bensaid and Lesne (1996), we consider an infinite-period model to find that network effects will shorten the trading process and that the game will end within a finite time endogenously. Mason (2000) uses a continuous-time model to investigate a similar question. The author shows that the monopolist will set the equilibrium price at the same level as the marginal cost of the product and will earn zero profit. The continuous-time model corresponds to our model when the discount factor approaches 1. While Mason (2000) obtains the limiting results the same as ours, we depart from this study by addressing the issue of equilibrium-price dynamics.

Third, and more importantly, the characteristics of network effects will also affect the qualitative properties of the equilibrium pricing strategy. Ben-

clear whether this result still holds in a model with an infinite period.

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said and Lesne (1996) analyze a model of a durable good that exhibits delayed network externalities, and show that equilibrium prices can increase over time when the network effects are strong enough. In their model, a lower initial price, which increases the number of initial users (the size of the network in the first period), will raise the willingness of future consumers to pay (but not the willingness of current consumers to pay). This allows the monopolist to charge higher prices in later periods. In contrast, in a model with non-delayed network effects like ours, the skimming property implies that the monopolist will never charge a sequence of increasing prices. Gabszewicz and Garcia (2008) extend the setup of Bensaid and Lesne (1996) to a more general finite-periods model. The authors explicitly solve the optimal path of prices as a function of network intensity and horizon length, and find that prices increase over time. Furthermore, for very low network intensity or a very long horizon, the monopolist will offer the good at a price of zero in the initial period.

In contrast, Cabral et al. (1999) show that if there is incomplete information about demand or asymmetric information about costs, then it is possible that introductory pricing will occur in equilibrium. Hence, information asymmetry can be a reason for the failure of price monotonicity when network effects exist. Chien and Chu (2008) consider an environment in which a monopolist faces two heterogeneous groups of consumers and only the first group can purchase the good in both periods, i.e., they are early adapters of new products or technologies. The authors find that when there are consumption externalities, the monopolist has the option to build a customer base by setting a low initial price and raising the price later in the mass market. Hence, the equilibrium behavior of the firm explains the practice of introductory pricing. In our model, consumers are also heterogeneous; however, we impose no restriction on the timing of consumers' purchase decisions. Any such restriction has effects similar to those of the assumption of delayed network effects. Indeed, by setting a low initial price that increases the number of initial users, the monopolist raises the willingness of future consumers to pay for the good. This may allow the monopolist to charge a higher price in later periods.

The remainder of this paper is organized as follows. In Section 2, we describe the durable goods monopoly model in the presence of network effects. Section 3 shows the existence of a stationary equilibrium. Section 4 deals with a class of two-type examples that can be explicitly solved and describes some comparative static analysis. Section 5 concludes the paper.

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2 The Model

We consider a market for a good that is infinitely durable and exhibits non-delayed network effects. This means that the flow utility a user derives from consuming this durable good increases with total accumulated sales. Moreover, the consumer can acquire the benefit generated by the network effects without delay.

Specifically, there is a continuum of infinitely lived consumers indexed by $q \in I = [0, 1]$. All consumers are risk-neutral and have a common discount rate r. Each consumer demands at most one unit of the durable good.

The monopolist offers the durable good for sale at discrete moments in time spaced equally apart. Let $z > 0$ denote the time interval between successive trades. Hence, sales occur at times $t = 0, z, 2z, \cdots, nz, \cdots$, and neither the monopolist nor consumers are allowed to trade at any time $t \in (nz, (n+1)z)$. We refer to the time $t = nz$ as "period n". The timing of play within each period is as follows. Before trading, the monopolist sets a price p. Consumers who have not previously purchased then decide whether or not to buy. After a time interval z elapses, play repeats.

A history before trading on date n, denoted as H_n, includes all past prices $\{p_i\}_{i=0}^{n-1}$ and the sets of the number of consumers who own a durable good $\{Q_i\}_{i=0}^{n-1}$. In addition, let $H_n' = (H_n, p_n)$ be the history after the monopolist announces its price p_n.

$F(q, Q)$ is defined as a flow utility of services consumer q derives from a durable good when the number of consumers (after trading) who own a durable good is Q, where $F : [0, 1] \times \Omega \to \mathbf{R}_+$ and Ω is the Borel $\sigma-$algebra on I. Let $\hat{Q} : \mathbf{R}_+ \to \Omega$ be the function of consumer acceptances, such that the number of consumers who own a durable good is $\hat{Q}(t)$ at time t. Furthermore, let $\mathbf{Q}^n(z) = (\hat{Q}(nz), \hat{Q}((n+1)z), \hat{Q}((n+2)z), \cdots) \equiv (Q_n, Q_{n+1}, Q_{n+2}, \cdots)$ denote a sequence of consumer acceptances from period n onward, where the time interval between successive trades is z and $\mathbf{Q}^n : \mathbf{R}_+ \to \Omega^\infty$. The willingness of consumer q to pay to acquire one unit of the durable good at $t = 0$ is then

$$
\begin{aligned}
v\left(q, \mathbf{Q}^0(z)\right) &= \int_0^\infty F\left(q, \hat{Q}(z)\right) e^{-rz} dz \\
&= \left(\int_0^z F\left(q, Q_0\right) + \int_z^{2z} F\left(q, Q_1\right) + \cdots \right) e^{-rz} dz
\end{aligned}
$$

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$$= (1-\delta)\left(\frac{F(q, Q_0)}{r} + \delta\frac{F(q, Q_1)}{r} + \cdots\right)$$
$$= (1-\delta)\sum_{i=0}^{\infty}\delta^i \hat{f}(q, Q_i),$$

where $\delta \equiv e^{-rz}$ and $\hat{f}(q, Q) \equiv F(q, Q)/r$.

By rearranging the valuation of consumers, we can without loss of generality assume that $\hat{f}(q, Q)$ does not increase in q for all Q. In addition, based on the nature of the network effects, we assume that $\hat{f}(q, Q)$ does not decrease in Q for all q.[10]

Therefore, if the price is p in period n, by purchasing a unit of the durable good, consumer q can derive a net surplus of

$$\delta^n\left(v\left(q, \mathbf{Q}^n\right) - p\right).$$

The monopolist's strategy specifies the price to charge in each period as a function of histories H_n. Let $\sigma = \{\sigma^n(H_n)\}_{n=0}^{\infty}$ be a pure strategy for the monopolist. A consumer strategy specifies whether or not to buy in each period given the current price charged and the history of past prices and purchases H_n'. Let $\tau = \{\tau^n(H_n')\}_{n=0}^{\infty}$ denote a consumer strategy combination. The strategy profile (σ, τ) generates a path $\{p_n, Q_n\}_{n=0}^{\infty}$ of prices and sales over time, which in turn determines the payoffs to the players. Let $\pi(\sigma, \tau) = \sum_{n=0}^{\infty}\delta^n p_n(Q_n - Q_{n-1})$ be the net present value of profits generated by the strategy profile $\{\sigma, \tau\}$, and let $u_q(\sigma, \tau) = \delta^n(v(q, \mathbf{Q}^n) - p_n))$ denote the discounted net surplus derived by consumer q if she purchases in period n. We are interested in the stationary subgame perfect equilibria of this game.

We follow Gul et al. (1986) by assuming that the equilibrium actions of each agent are constant on histories in which the prices offered are the same and the sets of consumers who accept in each period differ at most by sets of measure zero. This assumption guarantees that a unilateral deviation by a consumer does not change the actions of other agents; only the monopolist's unilateral deviations can influence the course of the game.

The following lemma establishes the existence of a cutoff rule implying that along any equilibrium path, the remaining buyer valuations are a truncated sample of the original ones. Consequently, the single number q summarizes prior consumers' actions.

[10]Here, the intrinsic value of consumers will not change with respect to state Q.

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Lemma 1. The Skimming Property. In any subgame perfect equilibrium, after any history H_n and for any current price p, if the consumer q accepts the price p, then every consumer $q' < q$ will accept p.

proof. We have to show that if consumer q is willing to hold a durable good at a market price p_n, then consumer q' will choose to hold for all $q' < q$.

Given any sequence $\{(p_n, Q_n)\}_{n=0}^{\infty}$, suppose that consumer q chooses to buy a durable good at p_n. We have the optimality condition

$$v\left(q, \mathbf{Q^n}\right) - p_n \geq \delta^m \left(v\left(q, \mathbf{Q}^{n+m}\right) - p_{n+m}\right), \quad \forall m \geq 1,$$

i.e.,

$$v\left(q, \mathbf{Q}^n\right) - \delta^m v\left(q, \mathbf{Q}^{n+m}\right) \geq p_n - \delta^m p_{n+m}, \quad \forall m \geq 1.$$

Substituting the expressions of $v(q, \mathbf{Q}^n)$ and $v(q, \mathbf{Q}^{n+m})$ into the above inequality, we obtain

$$(1-\delta)\sum_{i=0}^{\infty}\delta^i \hat{f}\left(q, Q_{n+i}\right) - \delta^m(1-\delta)\sum_{i=0}^{\infty}\delta^i \hat{f}\left(q, Q_{n+m+i}\right)$$
$$= (1-\delta)\sum_{i=0}^{m-1}\delta^i \hat{f}\left(q, Q_{n+i}\right) \geq p_n - \delta^m p_{n+m}, \quad \forall m \geq 1.$$

Hence, for $\hat{f}(q', Q) \geq \hat{f}(q, Q)$, the above inequality must hold for $\hat{f}(q', Q)$ for all $m \geq 1$. This is equivalent to the condition that consumer q' prefers to buy a unit of the durable good either during or before period n. □

In general, a buyer's accept/reject decision may depend not only on the current price p, but also on the history H_n. We define a stationary equilibrium to be a subgame perfect equilibrium in which (after histories that contain no simultaneous buyer deviations of a positive measure) the accept/reject decisions of all (remaining) buyers depend only on the current price.

The buyer's strategy in a stationary equilibrium can be described by an acceptance function $P(\cdot)$, where consumer q accepts price p if, and only if, $p \leq P(q)$.

For a given stationary equilibrium, after any history the monopolist must maximize the net present value of its expected profit, $R(\cdot)$. Suppose that before trading, the state in the market is Q and the monopolist chooses

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a price to bring the state from Q to q. Because buyer acceptances depend only on the price the monopolist charges henceforth, the monopolist's net present value $R(\cdot)$ depends on Q only, and must satisfy the dynamic programming equation

$$R(Q) = \max_{q \in [Q,1]} \{P(q)(q - Q) + \delta R(q)\}. \tag{1}$$

Lemma 2. In any stationary equilibrium, the equilibrium prices charged by the monopolist cannot go up.

proof. Given an equilibrium price $\{p_n\}$ and a state $\{Q_n\}$ sequence, suppose that to the contrary, there exists a period n such that $p_n < p_{n+1}$ and $Q_{n+1} - Q_n > 0$. From dynamic programming equation (1), we must therefore have

$$R\left(Q_{n-1}\right) = p_n\left(Q_n - Q_{n-1}\right) + \delta p_{n+1}\left(Q_{n+1} - Q_n\right) + \delta^2 R\left(Q_{n+1}\right).$$

However, by accelerating the trading process in period n, the monopolist can improve its profit. Specifically, by replacing the strategies $\{p_0, \cdots, p_{n-1}, p_n, p_{n+1}, \cdots\}$ with $\{p_0, \cdots, p_{n-1}, p_{n+1}, \cdots\}$, which is feasible (i.e., the consumer in the interval $[Q_{n-1}, Q_{n+1})$ will accept p_{n+1} because of the skimming property), the monopolist has a profit equal to

$$\begin{aligned} R'\left(Q_{n-1}\right) &= p_{n+1}\left(Q_{n+1} - Q_{n-1}\right) + \delta R\left(Q_{n+1}\right) \\ &= p_{n+1}\left(Q_n - Q_{n-1}\right) + p_{n+1}\left(Q_{n+1} - Q_n\right) + \delta R\left(Q_{n+1}\right) \\ &> p_n\left(Q_n - Q_{n-1}\right) + \delta\left(p_{n+1}\left(Q_{n+1} - Q_n\right)\right) + \delta^2 R\left(Q_{n+1}\right) \\ &= R\left(Q_{n-1}\right). \end{aligned}$$

This leads to a contradiction. Hence, equilibrium prices cannot go up. □

The conclusion that the equilibrium price sequence $\{p_n\}$ is non-increasing depends on the assumption of stationarity and the skimming property. If the network effects are delayed so that the utility (and hence the acceptance function P) of consumers depends on the previous states, then equilibrium prices do not necessarily decrease.

Let $T(Q)$ be the argmax correspondence in (1). By the generalized theorem of the maximum (see Ausubel and Deneckere (1993)) and the contraction mapping theorem, we know that there exists a unique continuous function $R(\cdot)$ satisfying (1), such that $T(\cdot)$ is nonempty and compact-valued.[11]

[11] We make an assumption implying that P is upper semicontinuous to enable us to apply the generalized theorem of the maximum.

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We follow Ausubel and Deneckere (1993) in specifying

$$t(Q) = \min T(Q) \tag{2}$$

to be the monopolist's equilibrium action. Therefore, $t(\cdot)$ is a left-continuous, non-decreasing function. Hence, there are at most countably many discontinuities in $t(\cdot)$.

For a given stationary equilibrium, consumer q maximizes her discounted net surplus u_q. Suppose that all consumers in $[0, Q]$ chose to hold durable goods at a market price $P(Q)$ in the previous period. In the current period n, the state in the market is Q before trading, and the monopolist should charge $P(t(Q))$ to bring the state to $t(Q)$ in equilibrium. Hence, $P(\cdot)$ must satisfy the following equation:

$$v\left(Q, \mathbf{Q}^n\right) - P(Q) = \delta\left(v\left(Q, \mathbf{Q}^{n+1}\right) - P(t(Q))\right), \tag{3}$$

where $\mathbf{Q}^n = (Q_n, Q_{n+1}, Q_{n+2} \cdots) = (Q, t(Q), t^2(Q), \cdots)$. The above equation implicitly defines consumer Q as the marginal consumer who chose to hold the durable good in the previous period.

3 Existence of Stationary Equilibria

A triplet $(R(\cdot), t(\cdot), P(\cdot))$ is defined as a stationary triplet if it satisfies (1), (2) and (3). From the expression of $v(q, \mathbf{Q}^n)$, we can simplify equation (3) to[12]

$$f(Q) - P(Q) = \delta(f(Q) - P(t(Q))), \tag{4}$$

where $f(q) \equiv \hat{f}(q, q)$. Note that the function f plays the same role as the value function in ordinary durable goods monopoly models.

[12] Simple calculation shows

$$\begin{aligned}
&v\left(Q, \mathbf{Q}^n\right) - P(Q) = \delta\left(v\left(Q, \mathbf{Q}^{n+1}\right) - P(t(Q))\right) \\
\Leftrightarrow &(1-\delta)\sum_{i=0}^{\infty}\delta^i \hat{f}\left(Q, Q_{n+i}\right) - P(Q) = \delta\left[(1-\delta)\sum_{i=0}^{\infty}\delta^i \hat{f}\left(Q, Q_{n+1+i}\right) - P(t(Q))\right] \\
\Leftrightarrow &\hat{f}(Q, Q) - P(Q) = \delta\left(\hat{f}(Q, Q) - P(t(Q))\right).
\end{aligned}$$

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Given a stationary triplet, we construct a stationary equilibrium as follows. The strategy of consumer q is

$$\tau_q^n\left(H_n'\right) = \begin{cases} 0, & \text{if} \quad p_n > P(q), \\ 1, & \text{if} \quad p_n \leq P(q), \end{cases}$$

for all n.

Along the equilibrium path, the monopolist plays the pure strategy $p = P(t(q))$ in each period, which depends on the current state q only. Off the equilibrium path, the monopolist's strategy depends not only on the current state, but also on the price in the previous period p_{-1}. The monopolist should now play a (possibly) mixed strategy such that the expected price, $\bar{p}$, satisfies

$$f(q) - p_{-1} \geq \delta(f(q) - \bar{p}), \quad \text{but}$$

$$f(q') - p_{-1} \leq \delta(f(q') - \bar{p}), \quad \text{for all} \quad q' \in (q, 1].$$

Such a mixed strategy justifies the decision of consumer q to purchase in the previous period and the decision of all $q' \in (q, 1]$ consumers to reject. Thus, we can generate a stationary equilibrium from a stationary triplet.

Note that equations (1), (2) and (4) are the same as equations characterizing a stationary equilibrium in ordinary durable goods models, except that $f(q)$ is not necessarily a non-increasing function. For example, supposing that $f(q) = \hat{f}(q, q)$ is differentiable, then $df/dq = \hat{f}_1(q, q) + \hat{f}_2(q, q)$, where $\hat{f}_i$ denotes the partial derivative of $\hat{f}$ with respect to the ith argument. By assumption, we have $\hat{f}_1 \leq 0$ and $\hat{f}_2 \geq 0$. Hence, depending on these two countervailing effects, f is not necessarily decreasing.

To ensure the existence of a stationary triplet $(R(q), t(q), P(q))$ supporting a stationary equilibrium, we need

Assumption 1: $f(1) > 0$, and there exists $L < \infty$ such that $f(q) - f(1) \leq L(1 - q)$ for all $q \in [0, 1]$.

Note that when the network effects are not negligible, even the consumer with the lowest intrinsic value can benefit from the network externalities. This implies that $f(1) = \hat{f}(1, 1) > 0$. Moreover, the condition $f(1) > 0$ ensures a lower bound, $f(1)(1 - Q)$, of the monopolist's profit when the state before trading is Q. On the other hand, the Lipschitz condition mandates that when the size of the residual demand is small enough, i.e. the state before trading is close to 1, the monopolist will prefer to charge

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the market clearing price $f(1)$ to cover the rest of the market immediately. This allows us to solve the game by backward induction.

Assumption 2: $f(q)$ is an upper semicontinuous function for all $q \in [0, 1]$.[13]

From the construction of P, if f is upper semicontinuous, so is P.[14] Hence, this condition allows us to apply the generalized theorem of the maximum.

Note that P plays a role as a demand function the monopolist faces. The shape of P is determined by f. When f is flat, increasing or decreasing on a domain, the corresponding P will be flat, increasing or decreasing. From the skimming property, the monopolist will never charge a price in an ascending segment of P in equilibrium.

Theorem 1. Suppose that Assumptions 1 and 2 hold. For any $\delta < 1$, there then exists a unique stationary triplet $(R(q), t(q), P(q))$ on $q \in [0, 1]$. Furthermore, there exists $N(\delta) < \infty$ such that the game ends in $N(\delta)$ periods.[15]

proof. See Appendix A.[16] □

Corollary 1. $P(0) \to f(1)$ as $\delta \to 1$.

proof. From Theorem 1, the game ends in a finite number of periods. For

[13]This assumption includes a case we will consider later: $\hat{f}$ is separable, i.e., $\hat{f}(q, Q) = g(q) + h(Q)$ where the function g, which captures the intrinsic value of consumer q, is non-increasing and left continuous, and the function h, which captures the benefit generated by network effects when the current state (after trading) is Q, is non-decreasing and right continuous.

[14]Note that in the final period, the monopolist faces a static (one period) problem. The existence of a solution t to this one period problem is guaranteed by the generalized Weierstrass theorem. In addition, because we construct P backward, i.e., $P(q) = (1-\delta)f(q) + \delta(1-\delta)f(t(q)) + \cdots + \delta^n f(1)$, and $t(q)$ is non-decreasing, P is upper semicontinuous if f is upper semicontinuous.

[15]There exists a finite integer N such that the game ends in N periods for all $\delta < 1$. See Theorem 4 of Ausubel et al. (2002).

[16]The proof of Theorem 1 closely follows the approach of Deneckere and Liang (2006).

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any $\delta < 1$, let $N = \max\{n : q_n \leq 0\}$. Therefore, as $\delta \to 1$, we have

$$\begin{aligned} P(0) &= (1-\delta) f(0) + \delta p_N \\ &= (1-\delta) f(0) + \delta(1-\delta) f(q_N) + \cdots + \delta^N (1-\delta) f q_0 \\ &\quad + \delta^N f(q_{-1}) \to f(1). \end{aligned}$$

□

In Theorem 1 and Corollary 1, we show that a stationary equilibrium exists in a durable goods monopoly model with non-delayed network effects, and that the Coase conjecture is satisfied in equilibrium. From a technical viewpoint, our setup extends a typical durable goods monopoly model (Gul et al., 1986) to a more general environment in which the value function is not necessarily decreasing. To ensure the existence of a stationary triplet, a weaker assumption that the value function is upper semicontinuous is sufficient.

Note that because the price $p = f(1) = \hat{f}(1, 1)$ is the market clearing price, the monopolist can charge this price and cover the whole market immediately. In this situation, the monopolist earns a profit equal to $f(1)$, which is the benefit it can acquire if it faced a unit mass of consumers with the lowest intrinsic value. Thus, even if the monopolist does not discriminate among consumers by price, it can secure at least this benefit. We thus expect that when the network effects are strong enough, the benefit generated by accelerating the trading process will dominate the benefit generated by price discrimination. Hence, the monopolist will seek to flood the market immediately.

Specifically, let $\hat{f}(q, Q, k)$ be a twice differentiable function in which we introduce a parameter k to measure the intensity of the network effects. Assume that $\hat{f}_{13} = 0$ and $\hat{f}_{23} > 0$, where $\hat{f}_{ij}$ is the partial derivative of $\hat{f}$ with respect to arguments i and j. Note that the assumption $\hat{f}_{13} = 0$ captures the situation whereby the network effects do not affect the marginal intrinsic value of consumers. In addition, $\hat{f}_{23} > 0$ captures the situation whereby the marginal benefit generated by the network effects is increasing in k. We then have

Proposition 1. Supposing that $\hat{f}(q, Q, k)$ is twice differentiable, $\hat{f}_{13} = 0$, $\hat{f}_{23} > 0$ and $f_{11} q + 2 f_1 < 0$, where $f(q, k) = \hat{f}(q, q, k)$, then there exists k^* such that whenever $k \geq k^*$, the monopolist will charge $p = \hat{f}(1, 1, k)$ and cover the whole market immediately.

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proof. Define the static monopoly profit as

$$\pi(k) = \max_{q \in [0,1]} \; f(q,k)q,$$

where consumers' willingness to pay is f. In addition,

$$q^m(k) = \arg \; \max f(q,k)q.$$

Note that the second-order condition $f_{11}q + 2f_1 < 0$ implies the uniqueness of the solution. In addition, by the implicit function theorem, $q^m(k)$ is differentiable with respect to k.

We therefore have

$$\pi(k) = f(q^m(k),k)q^m(k).$$

Furthermore, let

$$D(k) = f(1,k) - \pi(k)$$

be the difference between the market clearing profit and the static monopoly profit. Then

$$\begin{aligned}
\frac{\partial D(k)}{\partial k} &= \frac{\partial f(1,k)}{\partial k} - \frac{\partial \pi(k)}{\partial k} \\
&= \frac{\partial f(1,k)}{\partial k} - \frac{\partial f(q^m(k),k)}{\partial k} q^m(k) \\
&\geq \frac{\partial f(1,k)}{\partial k} - \frac{\partial f(q^m(k),k)}{\partial k} \\
&= \frac{\partial \hat{f}(1,1,k)}{\partial k} - \frac{\partial \hat{f}(q^m(k),q^m(k),k)}{\partial k} \\
&= \frac{\partial \hat{f}(1,1,k)}{\partial k} - \frac{\partial \hat{f}(1,q^m(k),k)}{\partial k} \\
&\geq 0, \;\; \text{(the equality holds if and only if } q^m(k) = 1),
\end{aligned}$$

where the second equality is from the envelope theorem, the third inequality is from $q^m(k) \leq 1$, the fifth equality is from $\hat{f}_{13} = 0$, and the last equality is from $\hat{f}_{23} > 0$.

When $q^m(k) < 1$, define $k^* = \inf\{k : f(1,k) = \pi(k)\}$. Note that this k^* is well-defined, as we know that $\pi(k) \geq f(1,k)$ for all k and $D'(k) > 0$. On the other hand, when $q^m(k) = 1$, we choose $k^* = \inf\{k : g^m(k) = 1\}$. Then, for all $k \geq k^*$, we have $f(1,k) \geq \pi(k)$. Hence, 1 is a maximizer for the static monopoly problem. Thus, the monopolist will choose to cover the market immediately when the network effects are sufficiently strong. □

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4 Two-Type Examples

In this section, we consider a class of two-type examples that can be solved completely. Hence, we can perform comparative static analysis to observe how the intensity of network effects affects equilibrium behavior.

Let

$$\hat{f}(q, Q) = \begin{cases} \bar{v} + h(Q; k), & q \in [0, \hat{q}], \\ \underline{v} + h(Q; k), & q \in (\hat{q}, 1], \end{cases}$$

i.e.,

$$f(q) = \begin{cases} \bar{v} + h(q; k), & q \in [0, \hat{q}], \\ \underline{v} + h(q; k), & q \in (\hat{q}, 1], \end{cases}$$

where k measures the intensity of network effects, $\bar{v} > \underline{v} \geq 0$, $h(Q; k)$ is non-decreasing and right continuous in state Q, $h(0; k) = 0$ and $h(1; k) > 0$.

For simplicity, we assume that $h(q; k)$ is differentiable with respect to q on $[0, 1]$. This implies that $f(q)$ is differentiable on $[0, 1]$ except at $\hat{q}$. Thus, $f'(q) = h'(q; k) \geq 0$ for all $q \in [0, 1] \backslash \{\hat{q}\}$.

Because $f(\hat{q}) \geq f(q)$ for all $q \in [0, \hat{q}]$ and $f(1) \geq f(q)$ for all $q \in (\hat{q}, 1]$ when $f(1) \geq f(\hat{q})$, the monopolist will set a price equal to $f(1)$ and clear the market immediately. We therefore assume that $f(1) < f(\hat{q})$ in the following.

There are two cases to be considered separately.

Case 1: $f(1) \geq f(\hat{q})\hat{q}$.

Note that in this case, the game ends in the first period.

Lemma 3. $(P(q), t(q), R(q))$, defined as follows,

$$\begin{cases} P(q) = (1 - \delta) f(q) + \delta f(1), & q \in [0, 1], \\ t(q) = 1, & q \in [0, 1], \\ R(q) = f(1)(1 - q), & q \in [0, 1], \end{cases}$$

is the stationary triplet.

proof. See Appendix B. □

Case 2: $f(1) < f(\hat{q})\hat{q}$.

The construction of cutoff states $\{q_n\}$ in this case is similar to that in the ordinary two-type case. In particular, we shall show that these cutoff states are the actual states along the equilibrium path except for the initial state 0.

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Define $q_{-1} = 1$, $q_0 = \hat{q}$, and q_1 satisfies $(\hat{q} - q_1) f(\hat{q}) = (1 - q_1) f(1)$, i.e. q_1 is the state in which static monopoly profits at the monopoly prices $f(\hat{q})$ and $f(1)$ are equal.

Lemma 4. $q_1 = \inf\{x : \hat{R}(\hat{q}, x) \leq \hat{R}(1, x)\}$, where $\hat{R}(y, x) = P(y)(y - x) + \delta R(y)$, P and R is as defined in Lemma 3.

proof. See Appendix B. □

Lemma 4 shows that when the current state $Q > q_1$, the monopolist will bring the next state to 1.

Lemma 5. $(P(q), t(q), R(q))$, defined as follows,

$$\begin{cases} P(q) = (1 - \delta) f(q) + \delta f(1), & q \in (q_1, 1], \\ t(q) = 1, & q \in (q_1, 1], \\ R(q) = f(1)(1 - q), & q \in (q_1, 1], \end{cases}$$

$$\begin{cases} P(q_1) = (1 - \delta) f(q_1) + \delta P(t(q_1)), \\ t(q_1) = \hat{q}, \\ R(q_1) = P(t(q_1))(q_1 - t(q_1)) + \delta R(t(q_1)), \end{cases}$$

is the stationary triplet on $[q_1, 1]$.

proof. See Appendix B. □

Lemma 5 ensures that a final period occurs that allows us to perform backward induction.

Given q_{-1}, q_0, q_1, $p_{-1} \equiv f(1)$, $p_0 \equiv (1 - \delta) f(\hat{q}) + \delta f(1)$, and $p_1 \equiv P(q_1)$ as in Lemma 5, we define $\{q_n\}$ and $\{p_n\}$ for $n \geq 2$ recursively as follows:

$$q_n = \frac{q_{n-1}(p_{n-1} - \delta p_{n-3}) - q_{n-2}(p_{n-2} - \delta p_{n-3})}{p_{n-1} - p_{n-2}}, \tag{5}$$

$$p_n = (1 - \delta) f(q_n) + \delta p_{n-1}. \tag{6}$$

Given $\{q_n\}$ and $\{p_n\}$ as defined in equations (5) and (6), by letting $N = \max\{n : q_{n+1} \leq 0\}$, we can construct $(P(q), t(q), R(q))$ as follows,

$$P(q) = \begin{cases} (1 - \delta) f(q) + \delta p_{n-1}, & \text{for } q \in [0, q_N], \text{ if } n = N, \\ & \text{for } q \in (q_{n+1}, q_n], \text{ if } n < N, \end{cases}$$

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$$t(q) = \begin{cases} q_{n-1}, & \text{for } q \in [0, q_N], & \text{if } n = N, \\ & \text{for } q \in (q_{n+1}, q_n], & \text{if } 1 < n < N, \\ & \text{for } q \in (q_1, 1], & \text{if } n = 0, \end{cases}$$

$$R(q) = \begin{cases} p_{n-1}(q_{n-1} - q) + \delta R(q_{n-1}), & \text{for } q \in [0, q_N], & \text{if } n = N, \\ & \text{for } q \in (q_{n+1}, q_n], & \text{if } 1 < n < N, \\ & \text{for } q \in (q_1, 1], & \text{if } n = 0. \end{cases}$$

Theorem 2. $(P(q), t(q), R(q))$ as constructed above is the stationary triplet on $[0, 1]$.

proof. See Appendix B.[17] □

Corollary 2. $P(0) \to f(1)$ as $\delta \to 1$.

proof. It follows immediately from Theorem 2 and Corollary 1. □

From Theorem 2, we can use equations (5) and (6) to compute the equilibrium price $\{p_n\}$ and state $\{q_n\}$ sequences respectively.

Discussion:

When there are no network effects, equations (5) and (6) reduce to the sequences of $\{p_n\}$ and $\{q_n\}$ as in the ordinary durable goods monopoly model of two types. Specifically, we have[18]

$$\begin{aligned} p_n &= P(q_n) \\ &= (1-\delta) f(q_n) + \delta p_{n-1} \\ &= (1-\delta) f(q_n) + \delta(1-\delta) f(q_{n-1}) + \cdots + \delta^n (1-\delta) f q_0 \\ &\quad + \delta^{n+1} f(q_{-1}) \end{aligned}$$

[17] The proof of Theorem 2 closely follows the approach of Deneckere and Liang (2008).

[18] q_n is calculated by the equations below:

$$p_{n-1} - p_{n-2} = \left(1-\delta^n\right)\bar{v} + \delta^n \underline{v} - \left(1-\delta^{n-1}\right)\bar{v} + \delta^{n-1}\underline{v} = \delta^{n-1}(1-\delta)(\bar{v} - \underline{v}).$$

$$\begin{aligned} p_{n-1} - \delta p_{n-3} &= \left(1-\delta^n\right)\bar{v} + \delta^n \underline{v} - \delta\left[\left(1-\delta^{n-2}\right)\bar{v} + \delta^{n-2}\underline{v}\right] \\ &= (1-\delta)\left[\left(1+\delta^{n-2}\right)\bar{v} - \delta^{n-2}\underline{v}\right]. \end{aligned}$$

$$p_{n-2} - \delta p_{n-3} = (1-\delta) f\left(q_{n-2}\right) = (1-\delta)\bar{v}.$$

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$$= \left(1-\delta^{n+1}\right)\bar{v} + \delta^{n+1}\underline{v},$$

$$q_n = \frac{q_{n-1}\left[\left(1+\delta^{n-2}\right)\bar{v} - \delta^{n-2}\underline{v}\right] - q_{n-2}\bar{v}}{\delta^{n-1}\left(\bar{v}-\underline{v}\right)}.$$

From the formula above, it is clear that p_n is increasing in n and $p_n \to f(1) = \underline{v}$ as $\delta \to 1$. Thus, equilibrium prices decrease over time and the Coase conjecture is satisfied.

When network effects exist, the two examples discussed below demonstrate how the intensity of network effects k affects the equilibrium price p_n and state q_n.

Case 1.

$$h(q;k) = \begin{cases} 0, & q \in [0, q^*), \\ k, & q \in [q^*, 1], \end{cases} \quad \text{where} \quad q^* \le \hat{q}.$$

Case 2.

$$h(q;k) = kq.$$

Case 1 captures the situation in which network effects are generated uniformly when the size of the network exceeds a threshold level q^*. Case 2 captures the situation in which network effects increase linearly with the size of the network. Note that these two cases satisfy the assumptions that $h(q;k)$ is non-decreasing, right continuous and differentiable except at a finite number of points.

For Case 1, let $n_0(k)$ be the period in which $q^* \in (q_{n_0(k)}, q_{n_0(k)-1}]$. Following some computation, we have

$$\begin{aligned} p_n &= (1-\delta) f(q_n) + \delta(1-\delta) f(q_{n-1}) + \cdots + \delta^n (1-\delta) f(q_0) \\ &\quad + \delta^{n+1} f(q_{-1}), \\ &= \begin{cases} \left(1-\delta^{n+1}\right)\bar{v} + \delta^{n+1}\underline{v} + k, & \text{if } n < n_0, \\ \left(1-\delta^{n+1}\right)\bar{v} + \delta^{n+1}\underline{v} + \delta^{n-n_0+1}k, & \text{if } n \ge n_0, \end{cases} \\ q_n &= \begin{cases} \frac{q_{n-1}\left[\left(1+\delta^{n-1}\right)\bar{v} - \delta^{n-1}\underline{v}\right] - q_{n-2}\bar{v} + \delta(q_{n-1}-q_{n-2})k}{\delta^{n-1}\left(\bar{v}-\underline{v}\right)}, & \text{if } n \le n_0, \\ \frac{q_{n-1}\left[\left(1+\delta^{n-1}\right)\bar{v} - \delta^{n-1}\underline{v}\right] - q_{n-2}\bar{v} - \delta q_{n-2}k}{\delta^{n-1}\left(\bar{v}-\underline{v}\right) - \delta k}, & \text{if } n = n_0+1, \\ \frac{q_{n-1}\left[\left(1+\delta^{n-1}\right)\bar{v} - \delta^{n-1}\underline{v}\right] - q_{n-2}\bar{v} - \delta^{(n-n_0-1)}q_{n-1}k}{\delta^{n-1}\left(\bar{v}-\underline{v}\right) - \delta^{(n-n_0-1)}k}, & \text{if } n \ge n_0+2. \end{cases} \end{aligned}$$

Proposition 2. In Case 1, at points where $n_0(k)$ is differentiable,[19] we obtain

[19] Because $n_0(k)$ is a step function of k, it is differentiable, and $\partial n_0(k)/\partial k = 0$ almost everywhere. At these points, a small change in k does not cause a (discrete) change in $n_0(k)$.

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(i) p_n is increasing in k.

(ii) $p_n - p_{n-1}$ is non-increasing in k and n;

(iii) q_1 is decreasing in k;

(iv) $q_n - q_{n-1}$ is non-decreasing in k and n;

(v) Combining (iii) and (iv), we find that q_n (and hence, $N(k)$, the number of periods in which the whole market is covered) is non-increasing in k.

proof. See Appendix C. □

We have already seen that the existence of network effects provides incentives for the monopolist to accelerate the trading process. In particular, the market will always be covered in finite periods, even though this is not necessarily the case in an ordinary durable goods monopoly model without network effects. The property (v) in Proposition 2 makes this point clear, and shows that the stronger the network effects, the more quickly the whole market will be covered.

For Case 2, we have

$$\begin{aligned} p_n &= (1-\delta) f(q_n) + \delta(1-\delta) f(q_{n-1}) + \cdots + \delta^n (1-\delta) f q_0 \\ &\quad + \delta^{n+1} f(q_{-1}) \\ &= \left(1-\delta^{n+1}\right)\bar{v} + \delta^{n+1}\underline{v} + k\Big[(1-\delta)q_n + \delta(1-\delta)q_{n-1} + \cdots \\ &\quad + \delta^n(1-\delta)q_0 + \delta^{n+1}\Big], \\ q_n &= \frac{q_{n-1}\left[\left(1+\delta^{n-1}\right)\bar{v} - \delta^{n-1}\underline{v}\right] - q_{n-2}\bar{v} + k\left(Aq_{n-1} + q_{n-1}q_{n-2} - q_{n-2}^2\right)}{\delta^{n-1}\left(\bar{v}-\underline{v}\right) + kA}, \end{aligned}$$

where $A = q_{n-1} - (1-\delta)(\sum_{i=2}^{n} \delta^{i-2} q_{n-i}) - \delta^{n-1}$.

Because q_n is defined recursively, it is difficult to obtain comparative static results as in Case 1. On the other hand, in contrast to Proposition 2 (i), it is not necessary that the equilibrium price $p_n = (1-\delta) f(q_n) + \delta p_{n-1}$ is increasing in k. The reason for this is that if the equilibrium state q_n decreases in k (i.e., $\partial q_n / \partial k < 0$), then $\partial f(q_n)/\partial k = (q_n + k\partial q_n/\partial k)$ will be negative when k is large. Therefore, p_n can decrease in k when k is large. Proposition 3 shows this situation for p_1.

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Proposition 3. In Case 2,

$$q_1 = \frac{\bar{v}\hat{q} - \underline{v} - \left(1 - \hat{q}^2\right)k}{\bar{v} - \underline{v} - \left(1 - \hat{q}\right)k},$$

which is decreasing in k. Furthermore, p_1 is decreasing in k if, and only if, $k \geq (\bar{v} - \underline{v})/((1 - \hat{q})(1 + \bar{v}))$.

proof. See Appendix C. □

Proposition 2 and Proposition 3 suggest that the equilibrium state q_n is non-increasing in k. However, they are ambiguous about the direction of the equilibrium price p_n with respect to a change in k.

5 Conclusion

We propose a model developed to capture the pricing behavior of a monopoly in a durable goods market that exhibits non-delayed network effects. Because the monopolist trades off between the respective benefits of the price discrimination and accelerating the trading process, and the effect of the network externalities is to increase the benefit of the latter, equilibrium prices must decrease over time. This result allows us to show the existence of a unique stationary equilibrium. In equilibrium, the first offer made by the monopolist will be close to the price at which the consumer with the lowest intrinsic value is willing to trade when the length of successive time periods converges to zero. Hence, our theory suggests that in durable goods markets with non-delayed network effects, the monopolist may not adopt an introductory pricing strategy. Furthermore, when the network effects are strong, the monopolist will seek to flood the market immediately.

Appendix A

Proof of Theorem 1.

The theorem is proved through the Lemmas below.

Define $\pi(y; q) = f(y)(y - q)$.[20]

[20] For a one period game, f is the demand function the monopolist faces.

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Lemma A.1 Suppose Assumption 1 holds. There exists a $\bar{q} < 1$ such that for all $q > \bar{q}$, the unique maximizer of $\pi(y; q)$ is $y = 1$.

proof. Define $\bar{q} = 1 - (f(1)/L)$, i.e., $f(1) = L(1 - \bar{q})$.

From Assumption 1, we have

$$\begin{aligned}
\pi(y; q) &= f(y)(y - q) \\
&\leq (f(1) + L(1 - y))(y - q) \\
&= f(1)(1 - q) - f(1)(1 - y) + L(1 - y)(y - q) \\
&= f(1)(1 - q) - (1 - y)(f(1) - L(y - q)) \\
&\leq f(1)(1 - q) - (1 - y)(f(1) - L(1 - q)) \\
&\leq f(1)(1 - q) - (1 - y)(f(1) - L(1 - \bar{q})) \\
&= \pi(1; q),
\end{aligned}$$

for all $q > \bar{q}$. □

Define $S(q) = \arg\max \pi(y; q)$ and let $q_1 = \inf\{q : 1 \in S(q)\}$.

Note that Lemma A.1 implies that the set $\{q : 1 \in S(q)\}$ is nonempty. So q_1 is well-defined.

Lemma A.2 Suppose that Assumption 1 holds. Then, in every stationary equilibrium, after any history and the state $q > q_1$, the monopolist makes an offer $f(1)$, and all remaining consumers accept.

proof. Since in equilibrium consumers can never expect an offer $p < f(1)$, they accept this price immediately. This implies that the monopolist's equilibrium continuation payoff is bounded below by $\pi(1; q)$. So from the definition of q_1, the monopolist will offer $f(1)$ when the state $q > q_1$ and all remaining consumers accept. □

Define $(P(q), t(q), R(q))$ on $[q_1, 1]$ as follows

$$\begin{aligned}
P(q) &= (1 - \delta) f(q) + \delta(1 - \delta) f(t(q)) + \delta^2 f(1), \\
t(q) &= \min \arg \max_{y \geq q} \pi(y; q), \\
R(q) &= \max_{y \geq q} \pi(y; q).
\end{aligned}$$

Lemma A.3 $(P(q), t(q), R(q))$ on $[q_1, 1]$ defined above is the stationary triplet on $[q_1, 1]$.

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proof. First, we show that $R(q_1) = \pi(1; q_1) \equiv f(1)(1 - q_1)$. From the definition of q_1, we must have $R(q_1) \geq \pi(1; q_1)$. Based on the generalized theorem of the maximum, R is a continuous function. Let $\{q_m\}$ be a sequence in $(q_1, 1]$ such that $q_m \to q_1$, then we have $R(q_m) = \pi(1; q_m)$ for all m and

$$R(q_1) = \lim_{m\to\infty} R(q_m) = \lim_{m\to\infty} \pi(1; q_m) = \pi(1; q_1).$$

So we have $1 \in \arg\max \pi_{y\geq q_1}(y; q)$ for $q \in [q_1, 1]$.

Secondly, we claim that $t(q) > q$ for all $q \in [q_1, 1)$. Otherwise, we have

$$\begin{aligned} R(q) &= P(q)(q - q) + \delta R(q) \\ &= \delta R(q), \end{aligned}$$

which implies that $R(q) = 0$. However, we know that $R(q) = f(1)(1 - q) > 0$. This leads to a contradiction.

Then we have

$$\begin{aligned} &\max_{y\geq q_1} P(y)(y - q_1) + \delta R(y) \\ &\quad = \left[(1 - \delta) f(t(q_1)) + \delta(1 - \delta) f\left(t^2(q_1)\right) + \delta^2 f(1)\right](t(q_1) - q_1) \\ &\qquad + \delta R(t(q_1)) \\ &\quad = (1 - \delta) f(1)(1 - q_1) + \delta f(1)(t(q_1) - q_1) + \delta f(t(q_1))(t(q_1) - q_1) \\ &\quad = (1 - \delta) f(1)(1 - q_1) + \delta f(1)(t(q_1) - q_1) + \delta f(1)(1 - t(q_1)) \\ &\quad = f(1)(1 - q_1) \\ &\quad = R(q_1), \end{aligned}$$

where the third equality is from $f(t(q_1))(t(q_1) - q_1) = f(1)(1 - q_1)$.

Hence, equation (1) is satisfied at q_1. Furthermore, for all $q \in (q_1, 1]$, we have

$$\begin{aligned} &\max_{y\geq q} P(y)(y - q) + \delta R(y) \\ &\quad = \max_{y\geq q} \left[(1 - \delta) f(y) + \delta(1 - \delta) f(t(y)) + \delta^2 f(1)\right](y - q) \\ &\qquad + \delta f(1)(1 - y) \\ &\quad = \max_{y\geq q} (1 - \delta) f(y)(y - q) + \delta f(1)(1 - q) \\ &\quad = (1 - \delta) f(1)(1 - q) + \delta(f(1)(1 - q)) \end{aligned}$$

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$$
\begin{aligned}
&= f(1)(1-q) \\
&= \max_{y \geq q} \; f(y)(y-q) \\
&= R(q),
\end{aligned}
$$

where the first equality is from $t(y) = 1$. So equation (1) is satisfied on $[q_1, 1]$.

By the definition of q_1, we have $t(q) = 1$ for all $q > q_1$ and

$$
\begin{aligned}
P(q) &= (1-\delta) f(q) + \delta(1-\delta) f(t(q)) + \delta^2 f(1) \\
&= (1-\delta) f(q) + \delta f(1),
\end{aligned}
$$

which implies

$$
f(q) - P(q) = \delta(f(q) - f(1)) = \delta(f(q) - P(t(q))).
$$

Also, for $q = q_1$, we have

$$
\begin{aligned}
f(q_1) - P(q_1) &= \left[f(q_1) - (1-\delta) f(q_1) + \delta(1-\delta) f(t(q_1)) + \delta^2 f(1) \right] \\
&= \delta(f(q_1) - [(1-\delta) f(t(q_1)) + \delta f(1)]) \\
&= \delta(f(q_1) - P(t(q_1))).
\end{aligned}
$$

Thus, equation (3) is satisfied.

Therefore, $(P(q), t(q), R(q))$ is the stationary triplet on $[q_1, 1]$. □

Proof of the Theorem: Given a stationary triplet on the interval $[q_n, 1]$, we extend it to a stationary triplet on a larger interval $[q_{n+1}, 1]$ uniquely. For $q \in [0, 1]$, let R_1 be the monopolist's profit when constrained to select a state in $[q_n, 1]$, and T_1[21] be the corresponding argmax correspondence:

$$
\begin{cases}
R_1(q) = \max\limits_{y \in [q_n, 1]} \; P(y)(y-q) + \delta R(y), \\
T_1(q) = \arg \max\limits_{y \in [q_n, 1]} \; P(y)(y-q) + \delta R(y).
\end{cases}
$$

In addition, we extend $P(q)$ to the entire interval $[0, 1]$ by setting $P_1(q) = (1-\delta) f(q) + \delta P(t_1(q))$, where $t_1(q) = \min T_1(q)$. Next, for $q \in [0, q_n]$,

[21] By the generalized theorem of the maximum, see Ausubel and Deneckere (1993), $T_1(q)$ is nonempty and compact valued.

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let R_2 be the monopolist's profit when constrained to select a state in $[q, q_n]$ (using the extended acceptance function):

$$\begin{cases} R_2(q) = \max\limits_{y\in[q,q_n]} \; P_1(y)(y-q) + \delta R_1(y), \\ T_2(q) = \arg \max\limits_{y\in[q,q_n]} \; P_1(y)(y-q) + \delta R_1(y). \end{cases}$$

We also define $P_2(q) = (1-\delta) f(q) + \delta P_1(t_2(q))$ on $[0, 1]$, where $t_2(q) = \min T_2(q)$. Then we define $q_{n+1} = \sup\{q \geq 0 : R_1(q) \leq R_2(q)\}$ whenever the latter set is nonempty, and $q_{n+1} = 0$ otherwise.

Claim: $q_{n+1} < q_n$.

By the generalized theorem of the maximum R_1 and R_2 are continuous functions. If $q_n = q_{n+1}$, by the definition of q_{n+1}, we have $R_2(q_n) \leq R_1(q_n)$. However, this contradicts $R_2(q_n) = \delta R_1(q_n) < R_1(q_n)$. So we must have $q_{n+1} < q_n$.

Claim: $(P(q), t(q), R(q))$ defined below is the stationary triplet on $[q_{n+1}, 1]$.

$$\begin{cases} R(q) = R_1(q), \\ t(q) = t_1(q), \quad \text{if } q > q_{n+1}, \\ \qquad\; t_2(q), \quad \text{if } q = q_{n+1}. \\ P(q) = P_1(q), \quad \text{if } q > q_{n+1}, \\ \qquad\; P_2(q), \quad \text{if } q = q_{n+1}. \end{cases}$$

First, we need to show that $R_1(q) = \max_{y\in[q,1]} P(y)(y-q) + \delta R_1(y)$ for $q \in [q_{n+1}, 1]$. Note that $R(q) = R_1(q)$ for $q \in [q_n, 1]$. If the maximizer $y \in [q, q_n)$, we would have $R_1(q) = R_2(q)$, contradicting the definition of q_{n+1}. Hence, the maximizer $y \in [q_n, 1]$, and we have

$$\begin{aligned} R_1(q) &\equiv \max_{y\in[q_n,1]} \; P(y)(y-q) + \delta R(y) \\ &= \max_{y\in[q,1]} \; P(y)(y-q) + \delta R(y) \\ &= \max_{y\in[q,1]} \; P(y)(y-q) + \delta R_1(y). \end{aligned}$$

By the construction of $P(q)$ on $[q_{n+1}, 1]$, we have

$$\begin{aligned} f(q) - P(q) &= f(q) - P_1(q) \\ &= f(q) - ((1-\delta) f(q) + \delta P(t_1(q))) \\ &= \delta(f(q) - P(t(q))) \end{aligned}$$

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for $q \in (q_{n+1}, 1]$. Note that $P_1(q) = P(q))$ for $q \in [q_n, 1]$. In addition,

$$\begin{aligned} f(q_{n+1}) - P(q_{n+1}) &= f(q_{n+1}) - P_2(q_{n+1}) \\ &= f(q_{n+1}) - ((1-\delta) f(q_{n+1}) + \delta P_1(t_2(q_{n+1}))) \\ &= \delta(f(q_{n+1}) - P_1(t_2(q_{n+1}))) \\ &= \delta(f(q_{n+1}) - P(t(q_{n+1}))). \end{aligned}$$

Hence, $(P(q), t(q), R(q))$ defined below is the stationary triplet on $[q_{n+1}, 1]$.

Claim: There exists $N < \infty$ such that $q_N \leq 0$.

Note that $t(q_{n+1}) \leq q_n$. Furthermore, $R(q') \leq R(q)$ for $q' > q$, since

$$\begin{aligned} R(q) &\geq P(t(q'))(t(q') - q) + \delta R(t(q')) \\ &= P(t(q'))(t(q') - q') + \delta R(t(q')) + P(t(q'))(q' - q) \\ &\geq R(q'). \end{aligned}$$

Let $\bar{v} \equiv \sup\{f(q) : q \in [0, 1]\}$. Since

$$\begin{aligned} R(q_{n+1}) &= P(t(q_{n+1}))(t(q_{n+1})) - q_{n+1}) + \delta R(t(q_{n+1})) \\ &\leq \bar{v}(t(q_{n+1})) - q_{n+1}) + \delta R(t(q_{n+1})) \\ &\leq \bar{v}(q_n - q_{n+1}) + \delta R(t(q_{n+1})) \\ &\leq \bar{v}(q_n - q_{n+1}) + \delta R(q_{n+1}), \end{aligned}$$

we have

$$\begin{aligned} q_n - q_{n+1} &\geq \frac{(1-\delta) R(q_{n+1})}{\bar{v}} \\ &\geq \frac{(1-\delta) R(q_1)}{\bar{v}} \\ &= \frac{(1-\delta) f(1)(1-q_1)}{\bar{v}} \equiv \epsilon > 0. \end{aligned}$$

Thus, we have $q_1 - q_{n+1} > n\epsilon$, i.e., when $n > (q_1/\epsilon)$, $q_{n+1} \leq 0$. □

Appendix B

Proof of Lemma 3. We have to show that $(P(q), t(q), R(q))$ satisfies equations (1) to (3). From the definition of $P(\cdot), t(\cdot), R(\cdot)$ in the statement

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of this lemma, we have

$$\begin{aligned}\max_{y\geq q} \; P(y)(y-q)+\delta R(y) &= P(t(q))(t(q)-q)+\delta R(t(q))\\ &= P(1)(1-q)+\delta R(1)\\ &= f(1)(1-q).\end{aligned}$$

Thus, equation (1) is satisfied.

Since

$$\begin{aligned}f(q)-P(q) &= f(q)-((1-\delta)f(q)+\delta f(1))\\ &= \delta(f(q)-f(1))\\ &= \delta(f(q)-P(1))\\ &= \delta(f(q)-P(t(q))),\end{aligned}$$

equation (3) is satisfied.

Let $\hat{R}(y;x) \equiv P(y)(y-x)+\delta R(y)$. Then, we have $\hat{R}$ which is differentiable with respect to y on $[0, 1]$ except at points $y = \hat{q}, 1$.

Since

$$\begin{aligned}\frac{\partial \hat{R}(y;x)}{\partial y} &= P'(y)(y-x)+P(y)+\delta R'(y)\\ &= (1-\delta)f'(y)(y-x)+(1-\delta)f(y)+\delta f(1)-\delta f(1)\\ &= (1-\delta)(f(q)+f'(q)(y-x))\\ &> 0, \quad \forall y \neq \hat{q}, 1,\end{aligned}$$

at these points the unique maximizer is 1. In addition, we have

$$\begin{aligned}\hat{R}(\hat{q};x) &= P(\hat{q})(\hat{q}-x)+\delta R(\hat{q})\\ &= ((1-\delta)f(\hat{q})+\delta f(1))(\hat{q}-x)+\delta(1-\hat{q})f(1)\\ &= (1-\delta)f(\hat{q})(\hat{q}-x)+\delta(1-x)f(1),\end{aligned}$$

and

$$\begin{aligned}\hat{R}(1;x) &= P(1)(1-x)+\delta R(1)\\ &= f(1)(1-x).\end{aligned}$$

Hence,

$$\hat{R}(1;x)-\hat{R}(\hat{q};x) = f(1)(1-x)-(1-\delta)f(\hat{q})(\hat{q}-x)+\delta(1-x)f(1)$$

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$$= (1-\delta)[f(1)(1-x) - f(\hat{q})(\hat{q}-x)]$$
$$= (1-\delta)[f(1) - f(\hat{q})\hat{q} + (f(\hat{q}) - f(1))x] > 0,$$

where, from the assumption, the first term of the last equality is greater than 0, and the second term $f(\hat{q}) - f(1)$ is strictly greater than 0.

Thus, $y = 1$ is the unique maximizer of the monopolist's problem, i.e., $t(q) = 1$ for all $q \in [0, 1]$. Therefore, equation (2) is satisfied. □

Proof of Lemma 4. First, we show that $\hat{R}(\hat{q}, q_1) = \hat{R}(1, q_1)$.

$$\begin{aligned}\hat{R}(\hat{q}, q_1) &= P(\hat{q})(\hat{q} - q_1) + \delta R(\hat{q}) \\ &= [(1-\delta)f(\hat{q}) + \delta f(1)](\hat{q} - q_1) + \delta(1-\hat{q})f(1) \\ &= (1-\delta)f(\hat{q})(\hat{q} - q_1) + \delta f(1)(1 - q_1) \\ &= f(1)(1 - q_1) \\ &= \hat{R}(1, q_1).\end{aligned}$$

Secondly, we show that $\hat{R}(\hat{q}, x) > \hat{R}(1, x)$ for all $x < q_1$, which implies $q_1 = \inf\{x : \hat{R}(\hat{q}, x) \leq \hat{R}(1, x)\}$.

Define

$$\begin{aligned}h(x) &\equiv \hat{R}(\hat{q}, x) - \hat{R}(1, x) \\ &= -(1-\delta)[f(1) - f(\hat{q})\hat{q} + (f(\hat{q}) - f(1))x],\end{aligned}$$

where the second equality has been previously derived. Then

$$\frac{\partial h(x)}{\partial x} = -(1-\delta)(f(\hat{q}) - f(1)) < 0.$$

Since we have shown that $\hat{R}(\hat{q}, q_1) = \hat{R}(1, q_1)$, we obtain that $\hat{R}(\hat{q}, x) > \hat{R}(1, x)$ for all $x < q_1$. □

Proof of Lemma 5. For $q \in (q_1, 1]$, the proof is the same as that for Lemma 3.

For $q = q_1$, note that

$$\begin{aligned}R(q_1) &= \max_{y \in [q_1, 1]} P(y)(y - q_1) + \delta R(y) \\ &= P(\hat{q})(\hat{q} - q_1) + \delta R(\hat{q}) \\ &= \hat{R}(\hat{q}, q_1) \\ &= \hat{R}(1, q_1) \\ &> \hat{R}(y, q_1),\end{aligned}$$

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for all $y \in (q_1, 1)$, where the last inequality is due to the fact derived previously that $\hat{R}(1, q_1) > \hat{R}(y, q_1)$ for all $y \in [q_1, 1]\backslash\hat{q}$. Thus, when the state is at q_1, there are two maximizers for the dynamic programming problem: 1 and $\hat{q}$. Therefore, $t(q_1) = \hat{q}$. In addition, we have

$$\begin{aligned} f(q_1) - P(q_1) &= f(q_1) - [(1-\delta) f(q_1) + \delta P(t(q_1))] \\ &= \delta(f(q_1) - P(t(q_1)). \end{aligned}$$

Hence, at $q = q_1$ equation (1) to (3) are satisfied. □

Proof of Theorem 2.

The theorem is proved through the Lemmas below.

Lemma B.1 The sequences $\{q_n\}$ and $\{p_n\}$ satisfy

$$R(q_n) = \hat{R}(q_{n-1}, q_n) = \hat{R}(q_{n-2}, q_n), \tag{7}$$

where $\hat{R}(y, x) = P(y)(y - x) + \delta R(y)$.

proof. Proof by induction.

From the proof of Lemma 4, when $n = 1$, equation (10) holds.

Suppose that the equation (10) holds for $\{q_n\}$ and $\{p_n\}$ defined by equations (5) and (6), respectively. We need to show that (10) also holds for $\{q_{n+1}\}$ and $\{p_{n+1}\}$.

Note first that from the induction hypothesis, we have

$$\begin{aligned} R(q_n) - R(q_{n-1}) &= \hat{R}(q_{n-2}, q_n) - \hat{R}(q_{n-2}, q_{n-1}) \\ &= P(q_{n-2})(q_{n-1} - q_n). \end{aligned}$$

Therefore,

$$\begin{aligned} \hat{R}(q_n, q_{n+1}) &= P(q_n)(q_n - q_{n+1}) + \delta R(q_n) \\ &= p_n(q_n - q_{n+1}) + \delta[R(q_{n-1}) + P(q_{n-2})(q_{n-1} - q_n)] \\ &= p_{n-1}(q_n - q_{n+1}) + p_n(q_n - q_{n+1}) - p_{n-1}(q_n - q_{n+1}) \\ &\quad + \delta R(q_{n-1}) + \delta p_{n-2}(q_{n-1} - q_n) \\ &= p_{n-1}(q_{n-1} - q_{n+1}) + \delta R(q_{n-1}) - \{p_{n-1}(q_n - q_{n-1}) \\ &\quad + p_n(q_n - q_{n+1}) - p_{n-1}(q_n - q_{n+1}) + \delta p_{n-2}(q_{n-1} - q_n)\} \\ &= \hat{R}(q_{n-1}, q_{n+1}), \end{aligned}$$

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where the last equality is derived from the result that $p_{n-1}(q_n - q_{n-1}) + p_n(q_n - q_{n+1}) - p_{n-1}(q_n - q_{n+1}) + \delta p_{n-2}(q_{n-1} - q_n) = 0$. The reason is that by solving the above equation, we obtain

$$q_{n+1} = \frac{q_n(p_n - \delta p_{n-2}) - q_{n-1}(p_{n-1} - \delta p_{n-2})}{p_n - p_{n-1}},$$

which is the definition of q_{n+1}. □

Lemma B.2 Suppose $n \geq 1$. For $q < q_n$, we have $\hat{R}(q_{n-1}, q) > \hat{R}(q_{n-2}, q)$. For $q > q_n$, we have $\hat{R}(q_{n-1}, q) < \hat{R}(q_{n-2}, q)$.

proof. Define

$$\begin{aligned} h_n(q) &= \hat{R}(q_{n-1}, q) - \hat{R}(q_{n-2}, q) \\ &= P(q_{n-1})(q_{n-1} - q) + \delta R(q_{n-1}) \\ &\quad - [P(q_{n-2})(q_{n-2} - q) + \delta R(q_{n-2})]. \end{aligned}$$

Thus,

$$\frac{\partial h_n(q)}{\partial q} = -P(q_{n-1}) + P(q_{n-2}) = -p_{n-1} + p_{n-2} < 0.$$

Since $h_n(q_n) = 0$, we obtain the desired result. □

Lemma B.3 q_n is decreasing in n and there exists $N < \infty$ such that $q_N \leq 0$.

proof. Note first that

$$\begin{aligned} h_n(q_{n-1}) &= \hat{R}(q_{n-1}, q_{n-1}) - \hat{R}(q_{n-2}, q_{n-1}) \\ &= \delta R(q_{n-1}) - R(q_{n-1}) \\ &= -(1 - \delta) R(q_{n-1}). \end{aligned}$$

Secondly,

$$\begin{aligned} q_{n-1} - q_n &= q_{n-1} - \frac{q_{n-1}(p_{n-1} - \delta p_{n-3}) - q_{n-2}(p_{n-2} - \delta p_{n-3})}{p_{n-1} - p_{n-2}} \\ &= \frac{-p_{n-2}q_{n-1} + \delta p_{n-3}q_{n-1} + p_{n-2}q_{n-2} - \delta p_{n-3}q_{n-2}}{p_{n-1} - p_{n-2}} \\ &= \frac{p_{n-2}(q_{n-2} - q_{n-1}) - \delta p_{n-3}(q_{n-2} - q_{n-1})}{p_{n-1} - p_{n-2}} \end{aligned}$$

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$$= \frac{p_{n-2}(q_{n-2} - q_{n-1}) + \delta p_{n-3}(q_{n-3} - q_{n-2}) - \delta p_{n-3}(q_{n-3} - q_{n-1})}{p_{n-1} - p_{n-2}}$$
$$= \frac{R(q_{n-1}) - \delta R(q_{n-1})}{p_{n-1} - p_{n-2}}$$
$$= \frac{-h_n(q_{n-1})}{p_{n-1} - p_{n-2}}$$
$$= \frac{h_n(q_{n-1})}{h_n'(q)}.$$

Then for $n = 2$ we have

$$\begin{aligned} q_1 - q_2 &= \frac{h_2(q_1)}{h_1'(q)} \\ &= \frac{-(1-\delta)R(q_1)}{p_0 - p_1} \\ &= \frac{-(1-\delta)f(1)(1-q_1)}{(1-\delta)(f(1) - f(q_1))} \\ &= -\frac{f(1)(1-q_1)}{f(1) - f(q_1)} \\ &= \frac{f(1)(1-q_1)}{f(q_1) - f(1)} > \frac{f(1)(1-q_1)}{f(q_1)} > \frac{f(1)(1-q_1)}{f(\hat{q})} \equiv \epsilon_0 > 0. \end{aligned}$$

Next, we want to show that for $n > 2$,

$$q_{n-1} - q_n > \epsilon_0.$$

From Lemma B.2 we have

$$\begin{aligned} R(q_{n-1}) &= \hat{R}(q_{n-2}, q_{n-1}) \geq \hat{R}(q_{n-3}, q_{n-1}) \geq \cdots \geq \hat{R}(1, q_{n-1}) \\ &= f(1)(1 - q_{n-1}). \end{aligned}$$

So we have

$$\begin{aligned} q_{n-1} - q_n &= \frac{h_n(q_{n-1})}{h_n'(q)} \\ &= \frac{(1-\delta)R(q_{n-1})}{p_{n-1} - p_{n-2}} > \frac{(1-\delta)f(1)(1-q_{n-1})}{p_{n-1} - p_{n-2}} > \epsilon_0, \end{aligned}$$

where the last inequality is due to the fact that

$$p_{n-1} - p_{n-2} < p_{n-1} \leq f(\hat{q}).$$

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Thus, we have $q_1 - q_n > (n-1)\epsilon_0$, i.e., $q_n < q_1 - (n-1)\epsilon_0$. Hence, when $n > 1 + (q_1/\epsilon_0)$, $q_n \leq 0$. □

Proof of theorem: To prove Theorem 2, we need to check that $P(\cdot)$, $t(\cdot)$, and $R(\cdot)$ as defined above satisfy equation (1) to (3).

First, equation (3) is satisfied since for any $q \in (q_{n+1}, q_n]$, we have

$$\begin{aligned} f(q) - P(q) &= f(q) - [(1-\delta)f(q) - \delta p_{n-1}] \\ &= \delta(f(q) - P(q_{n-1})) = \delta(f(q) - P(t(q))). \end{aligned}$$

Secondly, equation (1) is satisfied since for any $q \in (q_{n+1}, q_n]$, we have

$$\begin{aligned} R(q) &= \max_{y\in[q,1]} \{P(y)(y-q) + \delta R(y)\} \\ &= P(t(q))(t(q)-q) + \delta R(t(q)) \\ &= P(q_{n-1})(q_{n-1}-q) + \delta R(q_{n-1}) \\ &= p_{n-1}(q_{n-1}-q) + \delta R(q_{n-1}). \end{aligned}$$

Thirdly, we check that equation (3) is satisfied. Let

$$t'(x) = \min \{\arg \max_{y\in[0,1]} \hat{R}(y,x) \equiv P(y)(y-x) + \delta R(y)\}.$$

We will claim that $t'(x) = 1$ for $x \in (q_1, 1]$ and $t'(x) = q_{n-1}$ for $x \in (q_{n+1}, q_n]$.

First, $t'(x) = 1$ for $x \in (q_1, 1]$ is obtained from Lemma 5. Furthermore, since

$$\begin{aligned} \frac{\partial \hat{R}(y,x)}{\partial y} &= P(y) + (y-x)P'(y) + \delta R'(y) \\ &= P(y) + (y-x)P'(y) + \delta\left(\frac{\partial[(q_{n-2}-y)p_{n-2} + \delta R(q_{n-2})]}{\partial y}\right) \\ &= p_{n-1} + (y-x)(1-\delta)f'(y) - \delta p_{n-2} \\ &= (1-\delta)f(q_{n-1}) + (y-x)(1-\delta)f'(y) > 0, \end{aligned}$$

for all $y \in (q_n, q_{n-1})$ and $y \in [0, q_N)$, $t'(q) \in \{q_{-1}, q_0, \cdots, q_N\}$. Note that we assume $f'(y) \geq 0$ except at $y = \hat{q}$.

Suppose $q \in (q_{n+1}, q_n]$ and $n = 1, \cdots, N$. Lemma B.3 implies $q_N < \cdots < q_{n+1} < q < q_n < \cdots < q_{-1}$. Applying Lemma B.2 recursively, we have $\hat{R}(q_N, q) < \cdots < \hat{R}(q_n, q) < \hat{R}(q_{n-1}, q)$ and $\hat{R}(q_{-1}, q) < \cdots < \hat{R}(q_{n-2}, q) \leq \hat{R}(q_{n-1}, q)$. Thus, we obtain $t'(q) = q_{n-1} = t(q)$ for $n = 1, \cdots, N$. □

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Appendix C

Proof of Proposition 2.
(i) $n < n_0$, is obtained directly from the formula for p_n. For $n \geq n_0$, we have

$$\begin{aligned}\frac{\partial p_n}{\partial k} &= \frac{\partial\left(\delta^{(n-n_0+1)}k\right)}{\partial k} \\ &= \delta^{(n-n_0+1)} - k\delta^{(n-n_0+1)}(\ln\delta)\frac{\partial n_0(k)}{\partial k} \\ &= \delta^{(n-n_0+1)}\left(k - (\ln\delta)\frac{\partial n_0(k)}{\partial k}\right) \\ &= \begin{cases}\delta^{(n-n_0+1)}k > 0, & \text{or,} \\ -\infty < 0,\end{cases}\end{aligned}$$

where $\partial n_0(k)/\partial k = 0$ at points where $n_0(k)$ is differentiable and $\partial n_0(k)/\partial k = \infty$ at points where $n_0(k)$ jumps.
(ii) By a simple calculation, we obtain

$$p_n - p_{n-1} = \begin{cases}\delta^{n-1}(1-\delta)(\delta(\bar{v}-\underline{v})), & n < n_0, \\ \delta^{n-1}(1-\delta)\left(\delta(\bar{v}-\underline{v}) - \delta^{(n_0+1)}k\right), & n \geq n_0.\end{cases}$$

Therefore, $p_n - p_{n-1}$ is non-increasing in k. Since $p_n - p_{n-1} > 0$, we must have $\delta(\bar{v}-\underline{v}) - \delta^{(n_0+1)}k > 0$. Hence, $p_n - p_{n-1}$ is decreasing in n.
(iii) Since q_1 satisfies

$$(\hat{q} - q_1)f(\hat{q}) = (1-q_1)f(1),$$

solving the above equation, we obtain

$$q_1 = \frac{\bar{v}\hat{q} - \underline{v} - (1-\hat{q})k}{\bar{v}-\underline{v}},$$

which is a decreasing function of k.
(iv) From the proof of Lemma B.3, we know that

$$\begin{aligned}q_{n-1} - q_n &= \frac{h_n(q_{n-1})}{h_n'(q)} \\ &= \frac{\delta R(q_{n-1})}{p_{n-1} - p_{n-2}}.\end{aligned}$$

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Since $R(q_{n-1})$ is increasing and $p_{n-1} - p_{n-2}$ is decreasing in n, $q_{n-1} - q_n$ is increasing in n. In addition, from (ii), $p_{n-1} - p_{n-2}$ is non-increasing in k. If we can show $R(q_{n-1})$ is non-decreasing in k, then the proof is complete.

We prove that $R(q_n)$ is non-decreasing in k for $n \geq 1$ by induction.

First,

$$\begin{aligned} R(q_1) &= (1 - q_1) f(1) \\ &= \left(1 - \frac{\bar{v}\hat{q} - \underline{v} - (1 - \hat{q})k}{\bar{v} - \underline{v}}\right)(\underline{v} + k) \\ &= \frac{(1 - \hat{q})(\bar{v} + k)(\underline{v} + k)}{\bar{v} - \underline{v}}, \end{aligned}$$

which is increasing in k.

Secondly, suppose that $R(q_{n-1})$ is non-decreasing in k.

$$\begin{aligned} R(q_n) &= p_{n-1}(q_{n-1} - q_n) + \delta R(q_{n-1}) \\ &= p_{n-1} \frac{\delta R(q_{n-1})}{p_{n-1} - p_{n-2}} + \delta R(q_{n-1}), \end{aligned}$$

which is non-decreasing in k since p_{n-1}, $R(q_{n-1})$ is non-decreasing and $p_{n-1} - p_{n-2}$ is non-increasing in k. □

Proof of Proposition 3.

Since q_1 satisfies: $(\hat{q} - q_1) f(\hat{q}) = (1 - q_1) f(1)$, a simple calculation shows that

$$q_1 = \frac{\bar{v}\hat{q} - \underline{v} - (1 - \hat{q}^2)k}{\bar{v} - \underline{v} - (1 - \hat{q})}.$$

In addition, we have

$$\frac{\partial q_1}{\partial k} = \frac{-\bar{v}(1 - \hat{q})}{(\bar{v} - \underline{v} - (1 - \hat{q}))^2} < 0.$$

Then

$$\begin{aligned} p_1 &= (1 - \delta) f(q_1) + \delta p_0 \\ &= (1 - \delta^2)\bar{v} + \delta \underline{v} + \left[(1 - \delta) q_1 + \delta(1 - \delta)\hat{q} + \delta^2\right] k. \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{\partial p_1}{\partial k} &= (1 - \delta)\left(q_1 + k \frac{\partial q_1}{\partial k}\right) \\ &= \frac{\bar{v} - \underline{v} - k(1 - \hat{q})(1 + \bar{v})}{(\bar{v} - \underline{v} - (1 - \hat{q}))^2} \leq 0, \end{aligned}$$

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if, and only if, $k \geq \bar{v} - \underline{v}/(1-\hat{q})(1+\bar{v})$. □

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具網路外部性之耐久財獨佔

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本文分析當耐久財具備非遞延網路外部性時, 獨佔廠商的定價行爲。當網路外部性存在時, 消費者持有此耐久財的願付價格嚴格大於 0, 因此, 交易會在有限的期間內結束。獨佔廠商在跨期價格歧視與加速交易進程之間做取捨, 而非遞延網路外部性讓後者的好處增加, 因此均衡的價格必定遞減。我們證明存在唯一的穩定均衡。均衡的結果滿足 Coase conjecture。而且, 當網路外部性夠大時, 廠商會選擇一個很低的價格立即結清市場。本文也提供一些可以明確的計算出均衡價格與狀態 (state) 的例子。比較靜態的結果顯示: 網路外部性越大, 市場越快達到飽和。

關鍵詞: 耐久財獨佔, 網路外部性, 科斯猜想
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