Public Good Inputs, Housing Quality, and Locational Stratification

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Our paper contributes to the literature on economic segregation by showing that heterogeneity in preferences for housing quality as a result of the location-specific provision of public goods can generate locational stratification. We develop a general equilibrium framework of endogenous sorting in which agents are allowed to differ in their incomes, opportunity costs of commuting, and preferences for housing quality. Housing quality is endogenously determined by location-specific public infrastructure that is financed by property taxes. We characterize the configuration of the competitive spatial equilibrium. We find that complete integration arises only under a set of parameters of measure zero such that the ratio of the opportunity costs of commuting facing different types of agents is in a specific homothetic relation to the ratio of the marginal valuations of housing quality. By contrast, locational stratification is generic. When the difference between agents' commuting costs is sufficiently large (resp. small) compared to the difference in their respective preferences for housing quality, the equilibrium is segregated with all rich residing in the central city (resp. suburb) as observed in Asia and Europe (resp. U.S. metropolitan areas).

Keywords: economic segregation, endogenous sorting, housing quality, competitive spatial equilibrium
JEL classification: D50, H41, R20

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經濟論文叢刊 (*Taiwan Economic Review*), 41:2 (2013), 195–217。
國立台灣大學經濟學系出版

1 Introduction

Locational stratification has become a major concern of policymakers over the past 5 or 6 decades. In 2000, most of the 30 largest Metropolitan Statistical Areas (MSAs) in the United States were highly segregated (see Peng and Wang (2005) using the dissimilarity index). As claimed in Glazer and Moynihan (1963), "The point about the melting pot...is that it did not happen." Perlmann (1988) and Steinberg (1989) show that a multicultural society and pluralistic cities in the United States are evidence of the persistent existence of social and economic differences among ethnic groups. Jargowsky (1996) finds that in the 1980s there has been a large increase in economic segregation accompanied by a small decline in racial segregation. In this paper, we argue that differences in housing quality as a result of location-specific provision of public goods can be a plausible force behind the locational segregation.

Since the pivotal contribution by Benabou (1996a), economists have highlighted human capital and non-human wealth as the key forces driving locational segregation (e.g., see the income-sorting model by de Bartolome and Ross (2003) and the skill-sorting model by Hanushek and Yilmaz (2007)). Also related to this literature, accessibility to capital markets matters because of its effect on human capital investment (Benabou, 1996b). Beyond this conventional wisdom, Peng and Wang (2005) argue that preference heterogeneity for location-specific public goods can lead to locational stratification. More recently, Bayer et al. (2004) have empirically documented that preference heterogeneity for housing and neighborhood characteristics is crucial for stratified urban housing markets as observed. In this paper, we fully characterize the channels through which heterogeneous preferences for the housing quality can become a source of economic segregation. Our analysis will shed light on possible equilibrium outcomes that correspond to some of the outcomes observed in MSAs in the U.S. Asia, and Europe.

More specifically, we construct a general equilibrium framework of endogenous sorting with two locations/regions and two types of agents. One of the regions can be thought of as the central business district where all jobs are located. Agents differ in their incomes, opportunity costs of commuting, as well as preferences for housing quality. Each region has a fixed plot of land and each is administered by a local government which sets the location-specific property tax rate. Housing quality is endogenously determined by

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location-specific infrastructure established by each local government, and financed by its property tax revenue. Via 'sorting by foot' in the spirit of Tiebout (1956), different types of agent may cluster together and economic stratification may arise as an outcome of the multi-class competitive spatial equilibrium (cf. see the original work by Hartwick et al. (1976) for the concept of such an equilibrium). We extend the insight on endogenous sorting to the study of economic segregation with housing quality depending upon the location-specific provision of public goods. In particular, we are able to establish necessary and sufficient conditions for the local economy to become segregated – when there are less rich than poor, locational segregation features agglomeration with all rich residing in the same location.

Since the pivotal work by Sweeney (1974), there have been numerous studies on housing quality and urban configuration, using static or dynamic frameworks (e.g., see Arnott et al. (1983); Arnott et al. (1999); Lin et al. (2004); Peng and Wang (2009), to name but a few). Nonetheless, a common feature of these previous studies is that households are homogeneous. As a result, one cannot analyze the relationship between housing quality and the agglomeration of high-income, high-skill labor. The heterogeneity of household plays a key role in our paper, driving the endogenous sorting outcomes in equilibrium.

The main findings of our paper are as follows. We show that the equilibrium configuration crucially depends on the heterogeneity of preferences for housing quality versus that of the opportunity costs of commuting. When the difference between agents' commuting costs is sufficiently large compared to the difference in their respective preferences for housing quality, the equilibrium is segregated with all rich residing in the central city. By contrast, when the difference in agents' preferences for housing quality is sufficiently large, the equilibrium is segregated with all rich in the suburb. While the former captures the pattern observed in Asia and Europe, the latter is often seen in the U.S. metropolitan areas. Complete integration arises only under a set of parameters of measure zero when the ratio of commuting costs between the two types is in a specific homothetic relation to the ratio of the marginal valuations of housing quality. Thus, locational stratification is a result of preference heterogeneity and a positive neighborhood externality in the form of housing quality financed by local property taxes. Where the rich reside, however, will crucially depend on their opportunity cost of commuting relative to their preferences for housing quality.

The remainder of this paper is organized as follows. Section 2 sets up

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the basic structure of the model. Section 3 solves the short-run equilibrium for any fixed distribution of workers' type. Section 4 solves the equilibria of Pattern I and Pattern II where high-skilled agents agglomerate in the CBD and suburb, respectively. Finally, some concluding remarks are offered in Section 5.

2 The Model

Consider an economy with two regions, $j \in J \equiv \{1, 2\}$, where region 1 is the CBD and the job place to which all workers commute. Both regions have the same land endowment. There are two types of agents, $i \in M \equiv \{H, L\}$, with total mass $N > 0$. Type-H agents (of mass H) have a higher income and a stronger preference for housing quality than type-L agents (of mass L), where we restrict $0 < H < L$ and $H + L = N$. By denoting a type-i agent's income and preference for housing quality as Y^i and γ^i, respectively, $i \in M$, we thus have $Y^H > Y^L > 0$ and $\gamma^H > \gamma^L > 0$. Throughout this paper, the agents' type is indexed by a superscript and their location is indexed by a subscript. The (endogenous) population for type-i agents living in j is denoted by i_j, $i \in M$, $j \in J$. Following Thisse and Wildasin (1992) and Berliant et al. (2006), each agent is assumed to consume one unit of land. In the absence of vacant land, each region is occupied by residents of mass $N/2$. Therefore, the following population identities must hold true:

$$H_j + L_j = \frac{N}{2}, \quad \forall j \in J. \tag{1}$$

More specifically, the population identities can be summarized as follows.

Population	Region 1	Region 2	Total
Type-H	H_1	H_2	H
Type-L	L_1	L_2	L
Total	$\frac{N}{2}$	$\frac{N}{2}$	N

Denote $T^i > 0$ as the commuting cost between the CBD and the suburb for each type-i agent. Defining indicators $I_1 = 0$ and $I_2 = 1$, the commuting cost for each type-i agent residing in j becomes $T^i \cdot I_j$, where $i \in M$ and $j \in J$. Since type-H agents may be thought to have a high valuation of commuting time, it is reasonable to assume that their commuting is more costly, i.e., $T^H > T^L$. Although every agent needs to travel to

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the CBD to work, the local public goods are not the privilege of the CBD residents. Residents in each region are levied a property tax with a tax rate $\tau_j \geq 0$, $j \in J$. By using property tax revenue, each local government can establish location-specific infrastructure as well as LPGs that in turn improve the quality of houses in each region. In addition to the required sewer, road and utilities, such infrastructure includes ameliorating the living environment, building parks and museums, and hosting festivities for the residents.

Upon acquiring one unit of land in his/her region, the type-i agent in region j can enjoy the consumption of a numeraire good (c_j^i) and housing quality (h_j^i), $i \in M$, $j \in J$. To capture neighborhood externalities, we follow the Romer (1986) – Lucas (1988) convention, allowing the average consumption of housing quality in j ($\bar{h}_j$) to affect the utility facing each agent residing in j, where $\bar{h}_j = (H_j h_j^H + L_j h_j^L / N/2)$, which is higher when there are more type-H agents as neighbors. Specifically, we consider quasi-linear preference as proposed by Bergstrom and Cornes (1983): it is linear in consumption for the numeraire good and housing, and the utility sources from housing are given by $\gamma^i U(h_j^i, \bar{h}_j) = \gamma^i (h_j^i)^\alpha \bar{h}_j^{1-\alpha}$, $(1/2) < \alpha < 1$, where it is assumed that own housing quality carries more weight than the average housing quality in the region j (i.e., $\alpha > (1/2)$). The positive externality term $\bar{h}_j$ can be viewed as community housing quality in region j, $j \in J$. While agents can improve the quality of their own homes through individual investment in housing (h_j^i), they are small so that they do not consider the influence of individual choices on the community quality. Notably, while Lin et al. (2004) consider a similar type of Romer-Lucas externalities via home production, our paper models such externalities from the preference side.

Denote p_j as the price of each unit of housing quality in region j, $j \in J$. Each agent will exhaust her income for the consumption of the numeraire good, tax-included housing expenditure and commuting. The optimization problem for a type-i agent residing in region j can be written as:

$$V_j^i = \max_{c,h} \; c + \gamma^i U\left(h, \bar{h}_j\right),$$

$$\text{s.t. } c + p_j\left(1 + \tau_j\right) h + T^i \cdot I_j = Y^i. \tag{2}$$

Under quasi-linear preference, we can substitute the budget constraint (2) into the utility function to eliminate c and derive the first-order condition for h, as follows:

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$$h_j^i = \left[\frac{\alpha\gamma^i}{p_j\left(1+\tau_j\right)}\right]^{\frac{1}{1-\alpha}} \bar{h}_j, \tag{3}$$

where there is no income effect on the individual demand for housing quality. We can then plug (3) into (2) to obtain:

$$c_j^i = Y^i - T^i \cdot I_j - \left[p_j\left(1+\tau_j\right)\right]^{-\frac{\alpha}{1-\alpha}} \left(\alpha\gamma^i\right)^{\frac{1}{1-\alpha}} \bar{h}_j, \tag{4}$$

$i \in M$, $j \in J$. While individual demand for housing quality is increasing in the average housing quality in the region, the individual consumption demand for the numeraire good is decreasing in it. Here, the consumption of the numeraire good absorbs the entirety of the income effect, captured by gross income net of the commuting cost.

3 Market Equilibrium

The aggregate demand for housing quality in region j can then be written as:

$$H_j h_j^H + L_j h_j^L = \left[\frac{\alpha}{p_j(1+\tau_j)}\right]^{\frac{1}{1-\alpha}} \Omega_j \bar{h}_j, \tag{5}$$

where

$$\Omega_j \equiv H_j\left(\gamma^H\right)^{\frac{1}{1-\alpha}} + L_j\left(\gamma^L\right)^{\frac{1}{1-\alpha}}, \tag{6}$$

representing the aggregate preference for housing quality in region j, $j \in J$. Most notably, the composition of residents in each region (type-H versus type-L) will affect both the average housing quality and aggregate preference for housing quality. To pin down the equilibrium price p_j, we combine (5) and the definition of $\bar{h}_j$ to derive:

$$p_j\left(1+\tau_j\right) = \alpha\left(\frac{N}{2}\right)^{\alpha-1} \Omega_j^{1-\alpha}, \quad \forall j \in J. \tag{7}$$

The equilibrium average housing quality in each region is determined by residents' demand for housing quality and the tax revenue they contribute as a source of the regional supply of housing quality. Denoting $G_j > 0$ as the local public good provided by the local government in j (measured

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in the same unit as the composite good), which is financed by property tax revenue, we have:

$$G_j = \tau_j \left(H_j h_j^H + L_j h_j^L\right) p_j, \quad \forall j \in J. \tag{8}$$

Each local government's development technology for housing quality is assumed to take a simple form: $S(G_j) = g \cdot (G_j)^{1/2}$, where g measures technological efficiency for local public good to enhance housing quality. That is, the more local public goods devoted to it, the higher the regional supply of housing quality will be. Since the local public good affects equilibrium housing quality and hence the equilibrium level of utility achieved by households, our model resembles the setup by Greenhut and Mai (1980) in which public facility with endogenously determined location is consumed by households residing at the particular location. Market clearing for housing quality requires:

$$H_j h_j^H + L_j h_j^L = S\left(G_j\right), \quad \forall j \in J, \tag{9}$$

which, together with (8) and the definition of $\bar{h}_j$, yields,

$$G_j = \tau_j p_j S\left(G_j\right), \tag{10}$$

$$\bar{h}_j = S\left(G_j\right)\left(\frac{N}{2}\right)^{-1}. \tag{11}$$

Substituting (11) and (7) into (3), (4) and (10), we obtain:

$$\begin{aligned} h_j^i &= \left(\gamma^i\right)^{\frac{1}{1-\alpha}} \Omega_j^{-1} S(G_j), \\ c_j^i &= Y^i - T^i \cdot I_j - \alpha \left(\gamma^i\right)^{\frac{1}{1-\alpha}} \left(\frac{N}{2}\right)^{\alpha-1} \Omega_j^{-\alpha} S\left(G_j\right), \\ G_j &= \alpha \frac{\tau_j}{1+\tau_j} \left(\frac{N}{2}\right)^{\alpha-1} \Omega_j^{1-\alpha} S\left(G_j\right), \end{aligned} \tag{12}$$

where $i \in M$ and $j \in J$. From (10) and the housing development technology, we have:

$$\tau_j p_j = \frac{G_j}{S\left(G_j\right)} = \frac{\left(G_j\right)^{\frac{1}{2}}}{g}, \quad j \in J. \tag{13}$$

That is, the tax rate for each unit of housing quality can be written as $\tau_j p_j$ from the residents' viewpoint, or $G_j / S(G_j)$ from the supply-side of local

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government. In equilibrium, they should be equalized. (13) together with (7) yield the provision of the local public good G_j as well as the regional supply of housing quality $S(G_j)$ (for $j \in J$):

$$G_j = \left[\alpha g \frac{\tau_j}{1+\tau_j} \left(\frac{N}{2} \right)^{\alpha-1} \Omega_j^{1-\alpha} \right]^2, \tag{14}$$

$$S\left(G_j\right) = \alpha g^2 \frac{\tau_j}{1+\tau_j} \left(\frac{N}{2} \right)^{\alpha-1} \Omega_j^{1-\alpha}. \tag{15}$$

Thus, both the provision of the local public goods and the regional supply of housing quality are increasing in the property tax rate (τ_j) and the aggregate preference for housing quality in region j (Ω_j). Moreover, the more efficient the production of housing quality is (a larger g), or the weaker the neighborhood externality is (a larger α), the larger the sizes of both the provision of the local public goods and the regional supply of housing quality.

We can further substitute (15) into (11) and (12) to obtain:

$$\bar{h}_j = \alpha g^2 \frac{\tau_j}{1+\tau_j} \left(\frac{N}{2} \right)^{\alpha-2} \Omega_j^{1-\alpha}, \tag{16}$$

$$h_j^i = \left(\gamma^i\right)^{\frac{1}{1-\alpha}} \alpha g^2 \frac{\tau_j}{1+\tau_j} \left(\frac{N}{2} \right)^{\alpha-1} \Omega_j^{-\alpha}, \tag{17}$$

$$c_j^i = Y^i - T^i \cdot I_j - \alpha^2 g^2 \left(\gamma^i\right)^{\frac{1}{1-\alpha}} \left(\frac{N}{2} \right)^{2\alpha-2} \Omega_j^{1-2\alpha} \frac{\tau_j}{1+\tau_j}, \tag{18}$$

$i \in M$, $j \in J$. Intuitively, an increase in the aggregate preference for housing quality leads to higher aggregate as well as average demand for housing quality (recall that the population in each region is fixed at $N/2$). As a result of rising aggregate demand, the price must go up, thereby discouraging individual demand for housing quality so long as the positive neighborhood effect is not too strong. By substitution, individual demand for consumption must rise.

We are now prepared to characterize individual locational choices. Denote $f(\tau_j) \equiv (\tau_j/1+\tau_j)$, $k_j \equiv (\Omega_j)^{1-2\alpha}$, $j \in J$, and $\psi^i \equiv \alpha(1-\alpha)g^2(\gamma^i)^{(1/1-\alpha)}(N/2)^{2(\alpha-1)} > 0$, $i \in M$. Substituting (16), (17) and (18) into the utility functions, we have:

$$V_j^i = Y^i - T^i \cdot I_j + \psi^i k_j f\left(\tau_j\right), \quad i \in M, j \in J. \tag{19}$$

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Since $\alpha > 1/2$, k_j is decreasing with Ω_j, $j \in J$. That is, k_j is an inverse index of the aggregate preference for housing quality in j, $j \in J$. When τ_j increases, a higher property tax is levied on the agents, but they can enjoy better housing quality (since there are more public good inputs used for improving the housing environment). Due to the positive neighborhood externalities in the Romer-Lucas convention ($\alpha < 1$), the free-rider problem arises, as one may be riding on positive spillovers from the neighbors in the absence of any public interventions. However, in our model economy, a property tax is imposed and its revenue serves as the sole source for financing the local public good to enhance housing quality. In this case, the housing quality enjoyment effect dominates the property tax burden effect and, as a result, the indirect utility from living in region j is increasing in τ_j, for all $\tau_j \in R_+$, $j \in J$. Therefore, the property tax in this model is used as a centralized device in collecting revenue for the local public good provision, which forces individual investment in housing. As a result, it mitigates the free-rider problem and subsequently raises residents' indirect utility. The reader is reminded, however, that this property need not hold for general utility functional forms with more general setups of location-specific supplies of housing quality or with more general methods of finance of the public good.

The equilibrium locations of type-i agents are determined by the difference in the utility levels achieved by residing in the two regions,

$$V_1^i - V_2^i = T^i + \psi^i \left[k_1 f(\tau_1) - k_2 f(\tau_2)\right]. \tag{20}$$

That is, letting $\Phi^i(\tau_1, \tau_2) \equiv \psi^i[k_2 f(\tau_2) - k_1 f(\tau_1)]$,

$$V_1^i \gtreqless V_2^i \text{ if and only if } T^i \gtreqless \Phi^i(\tau_1, \tau_2), \quad i \in M.$$

4 Equilibrium Characterization

Denote $x_1 \equiv H_1/H$ (to be endogenously determined). The equilibrium distribution of agents can be summarized as follows:

$$H_1 = x_1 H,$$
$$H_2 = (1 - x_1) H,$$
$$L_1 = \frac{N}{2} - x_1 H,$$

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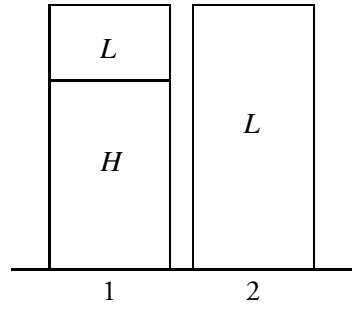


Figure 1: Distribution of Agents' Type in Pattern I

$$L_2 = \frac{N}{2} - (1 - x_1)\, H.$$

Under the assumption of $H < L$, generically there are two equilibrium patterns: in Pattern I, all type-H agents reside in region 1 ($x_1 = 1$); in Pattern II, all type-H agents reside in region 2 ($x_1 = 0$). Of course, there is also a completely mixed equilibrium with some type-H agents in region 1 and some in region 2 ($0 < x_1 < 1$). When there is a dispersion for both types of agents, (20) shows that $T^H/\psi^H = T^L/\psi^L$ must be satisfied. Therefore, only with a set of parameters of measure zero can such a mixed equilibrium arise.

It is noticed that Ω_j, and thus G_j, $\tau_j p_j$, and $S(G_j)$ all depend on the distribution of agents, and therefore, a superscript I or II is added for these variables for clarity when we characterize the two equilibrium patterns to which we now turn.

4.1 Pattern I: $x_1 = 1$

In Pattern I, $x_1 = 1$ indicates that all type-H agents reside in region 1. Thus, we have:

$$H_1 = H, \qquad H_2 = 0,$$
$$L_1 = \frac{N}{2} - H, \qquad L_2 = \frac{N}{2},$$

as depicted in Figure 1.

In equilibrium, it must be the case that type-H agents achieve higher indirect utility by residing in region 1 and that type-L agents are indifferent to their residential locations. In Pattern I, the aggregate preference

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variables become: $\Omega_1^I = H(\gamma^H)^{1/(1-\alpha)} + ((N/2) - H)(\gamma^L)^{1/(1-\alpha)}$ and $\Omega_2^I = (N/2)(\gamma^L)^{1/(1-\alpha)}$, where $\Omega_1^I > \Omega_2^I$. For any given $(\tau_1^I, \tau_2^I) \in \mathbb{R}_+^2$, type-$H$ agents' locational choice implies $V_1^H \geq V_2^H$, and thus:[1]

$$T^H \geq \Phi^H\left(\tau_1^I, \tau_2^I; \Omega_1^I, \Omega_2^I\right) \equiv \psi^H\left[k_2^I f\left(\tau_2^I\right) - k_1^I f\left(\tau_1^I\right)\right], \tag{21}$$

while locational no-arbitrage facing type-L agents requires $V_1^L = V_2^L$; that is,

$$T^L = \Phi^L\left(\tau_1^I, \tau_2^I; \Omega_1^I, \Omega_2^I\right) \equiv \psi^L\left[k_2^I f\left(\tau_2^I\right) - k_1^I f\left(\tau_1^I\right)\right]. \tag{22}$$

Let $(\tau_1^{I*}, \tau_2^{I*}) \equiv \{(\tau_1^I, \tau_2^I) \in \mathbb{R}_+^2 | T^L = \Phi^L(\tau_1^I, \tau_2^I; \Omega_1^I, \Omega_2^I)\}$. For the existence of such a property tax pair $(\tau_1^{I*}, \tau_2^{I*}) \in \mathbb{R}_+^2$ to support the Pattern I equilibrium, it must be the case that $k_2^I f(\tau_2^{I*}) > k_1^I f(\tau_1^{I*})$, which implies:[2]

$$\frac{f\left(\tau_2^{I*}\right)}{f\left(\tau_1^{I*}\right)} > \frac{k_1^I}{k_2^I} = \left(\frac{\Omega_1^I}{\Omega_2^I}\right)^{1-2\alpha}. \tag{23}$$

That is, only when the ratio of the property tax rate in region 2 to that in region 1 is larger than the ratio of the aggregate preference for housing quality among the two regions, can an equilibrium of Pattern I be constituted. Define $k_0^i \equiv T^i/\psi^i$, $i \in M$. We can rewrite (21) and (22) conveniently as:

$$f\left(\tau_2^{I*}\right) \leq \frac{k_0^H}{k_2^I} + \frac{k_1^I}{k_2^I} f\left(\tau_1^{I*}\right), \quad \text{and} \tag{24}$$

$$f\left(\tau_2^{I*}\right) = \frac{k_0^L}{k_2^I} + \frac{k_1^I}{k_2^I} f\left(\tau_1^{I*}\right), \tag{25}$$

where the latter equality can be expressed in terms of a transformed property tax ratio given as follows:

$$\frac{f\left(\tau_2^{I*}\right)}{f\left(\tau_1^{I*}\right)} = \frac{k_0^L}{k_2^I f\left(\tau_1^{I*}\right)} + \frac{k_1^I}{k_2^I}. \tag{26}$$

[1]Although the distribution pattern of H-type agents under $V_1^i = V_2^i$ can be arbitrary, no i-type agent would have incentive to deviate from his/her equilibrium location *given* the equilibrium pattern of agents' distribution.

[2]Recall that $k_j \equiv (\Omega_j)^{1-2\alpha}$ is the inverse index for the aggregate preference for housing quality in region j, $j \in J$.

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Consider,

Condition I. $(T^H/T^L) \geq (\gamma^H/\gamma^L)^{(1/1-\alpha)}$.

Condition I requires that the ratio of agents' commuting costs be sufficiently large compared to the ratio of their respective preferences for housing quality. Intuitively, we note that the property tax augmented price of housing quality is higher in the region where type-H agents agglomerate (as shown in (7)) and the equilibrium is generically partially segregated (as shown in the beginning of this section). Thus, in order for type-H agents to agglomerate to the CBD, their valuation of commuting time must be sufficiently higher than that of L-type agents. Moreover, the higher the type-H agents' relative preference for housing quality is, the greater their relative valuation of commuting time will be required for supporting the Pattern I equilibrium. We can then establish:

Proposition 1. (Pattern I Equilibrium) Under Condition I, the market equilibrium exists in which all type-H agents reside in region 1 whereas both regions are populated by type-L agents (Pattern I).

proof. Substituting (25) into the inequality (24) and manipulating, one obtains: $k_0^H \geq k_0^L$, which holds under Condition I. By plugging (26) into the inequality (23), we can easily see that this inequality holds because $f(\tau_1^{I*}) \geq 0$, $k_0^L > 0$, and $k_2^I > 0$. □

To illustrate the Pattern I equilibrium, we depict the relationship between the equilibrium property tax rates $(\tau_1^{I*}, \tau_2^{I*})$ in Figure 2. In the equilibria of Pattern I, property tax rates fall on the line $f(\tau_2^{I*}) = (k_0^L/k_2^I) + (k_1^I/k_2^I) f(\tau_1^{I*})$. Since equilibrium property tax rates satisfy $f(\tau_2^{I*}) \leq (k_0^H/k_2^I) + (k_1^I/k_2^I) f(\tau_1^{I*})$ and $f(\tau_2^{I*}) > k_1^I/k_2^I) f(\tau_1^{I*})$, for all $\tau_1^{I*} \in \mathbb{R}_+$, there exists an equilibrium of Pattern I for all non-negative property tax rates.

Notice from (26) that when $f(\tau_1^{I*}) \geq (k_0^L/k_2^I - k_1^I)$, $\tau_2^{I*} \leq \tau_1^{I*}$ and that when $f(\tau_1^{I*}) < (k_0^L/k_2^I - k_1^I)$, $\tau_2^{I*} > \tau_1^{I*}$. More specifically, since $(k_1^I/k_2^I) < 1$, whether $(f(\tau_2^{I*})/f(\tau_1^{I*}))$ may exceed one crucially depends on the first term on the right-hand side of (26), $(k_0^L/k_2^I f(\tau_1^{I*}))$. This term corresponds to an *incentive adjustment effect*, which is decreasing in τ_1^{I*}. Intuitively, the smaller τ_1^{I*} is, the larger the ratio of τ_2^{I*}/τ_1^{I*} that is needed to mitigate the free-rider problem so that type-L workers have no incentive to migrate from region 2 to region 1.

Using (14) and (26), we further compute the ratio of local public goods provided in the two regions:

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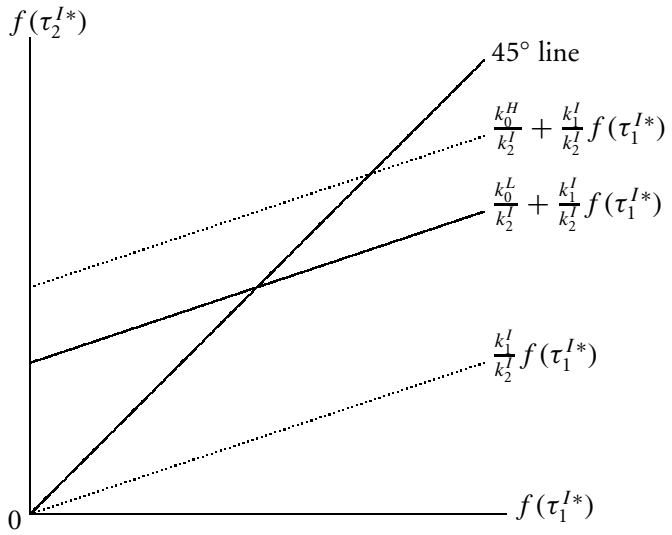


Figure 2: The Relationship between Transformed Property Tax Rates in the Equilibrium of Pattern I

$$\frac{G_2^{I*}}{G_1^{I*}} = \left(\frac{f(\tau_2^{I*})}{f(\tau_1^{I*})}\right)^2 \left(\frac{\Omega_2^I}{\Omega_1^I}\right)^{2(1-\alpha)}$$
$$= \left(\frac{k_0^L}{k_2^I f(\tau_1^{I*})} + \frac{k_1^I}{k_2^I}\right)^2 \left(\frac{\Omega_2^I}{\Omega_1^I}\right)^{2(1-\alpha)}. \quad (27)$$

The second item on the right-hand side of (27), $(\Omega_2^I/\Omega_1^I) < 1$, independent of τ_1^{I*}, represents an *aggregate preference effect*. Since region 1 residents have a stronger aggregate preference for housing quality, they have higher willingness-to-pay for the local public goods. This effect tends to support $(G_2^{I*}/G_1^{I*}) < 1$. Nonetheless, as illustrated above, the first term, $(k_0^L/k_2^I f(\tau_1^{I*}))+(k_1^I/k_2^I)$, could be greater or less than one. When $f(\tau_1^{I*}) \geq (k_0^L/k_2^I - k_1^I)$, the first term is less than one and as a result, we have $(G_2^{I*}/G_1^{I*}) < 1$ unambiguously. Intuitively, when the incentive adjustment effect is small, it is the aggregate preference effect that dominates, so the willingness-to-pay becomes the crucial factor determining the provision of local public goods. In this case, although residing in the CBD results in the enjoyment of a higher housing quality and saves on the commuting cost, the residents must pay a higher property tax than those residing in the suburb.

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In the case where $f(\tau_1^{I*}) < (k_0^L/k_2^I - k_1^I)$, the incentive adjustment effect is sufficiently large and we must have $\tau_2^{I*} > \tau_1^{I*}$ in order to mitigate the free-rider problem for type-L workers to feel indifferent regarding residing in either region. That is, the case of a reversed taxation is present in the region with type-H residents that imposes a lower property tax rate. When $f(\tau_1^{I*}) \geq (k_0^L/k_2^I)((\Omega_1^I/\Omega_2^I)^{1-\alpha} - (k_1^I/k_2^I))^{-1}$, even when the local governments set $\tau_2^{I*} > \tau_1^{I*}$, the aggregate preference effect still dominates and, as a result, region 1 still provides more local public goods than region 2 (i.e., $G_1^{I*} > G_2^{I*}$). When $f(\tau_1^{I*}) < (k_0^L/k_2^I)((\Omega_1^I/\Omega_2^I)^{1-\alpha} - (k_1^I/k_2^I))^{-1}$, in addition to reversed taxation, there exists a reversed provision of local public goods (i.e., $G_1^{I*} < G_2^{I*}$) as a consequence of a very strong incentive adjustment effect. In this case, not only does the region with type-H residents impose a lower property tax rate, but it also provides fewer local public goods.

Since k_0^L is increasing in T^L and decreasing in g, an increase in T^L or a decrease in g shifts the type-L agents' locational no-arbitrage condition (25) upward, which is depicted in Figure 2. As the result of a stronger incentive adjustment effect, the required property tax rate in region 2 rises for any given property tax rate in region 1. Similar results hold for a reduction in γ^H, as less efficient public good provisions and less willingness-to-pay by type-H agents in region 1 should have similar positive effects on $(\tau_2^{I*}/\tau_1^{I*})$ (although in this case the type-L agents' locational no-arbitrage condition is rotating counterclockwise, rather than shifting upward). In response to either a larger T^L, a smaller g, or a lower γ^H, it is more likely that a reversion in the taxation and in the provision of local public goods may arise.

4.2 Pattern II: $x_1 = 0$

We now turn to the Pattern II equilibrium with all type-H agents residing in region 2 (i.e., $x_1 = 0$). Since $H < L$, we have:

$$H_1 = 0, \qquad H_2 = H,$$
$$L_1 = \frac{N}{2}, \qquad L_2 = \frac{N}{2} - H,$$

as depicted in Figure 3.

In equilibrium, type-H agents achieve higher indirect utility by residing in region 2, whereas type-L agents are indifferent to residing in either region. In this case, $\Omega_1^{II} = (N/2)(\gamma^L)^{1/(1-\alpha)}$ and $\Omega_2^{II} = H(\gamma^H)^{1/(1-\alpha)} +$

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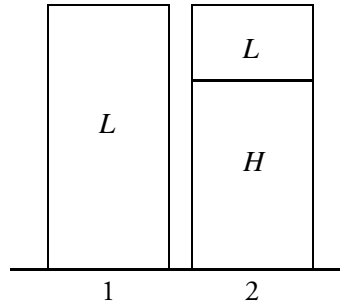


Figure 3: Distribution of Agents' Type in Pattern II

$((N/2) - H)(\gamma^L)^{1/(1-\alpha)}$, where $\Omega_1^{II} < \Omega_2^{II}$. For any given $(\tau_1^{II}, \tau_2^{II}) \in \mathbb{R}^2_+$, this equilibrium pattern requires that $V_1^H \leq V_2^H$ and $V_1^L = V_2^L$, or,

$$T^H \leq \Phi^H\left(\tau_1^{II}, \tau_2^{II}; \Omega_1^{II}, \Omega_2^{II}\right) \equiv \psi^H\left[k_2^{II} f\left(\tau_2^{II}\right) - k_1^{II} f\left(\tau_1^{II}\right)\right], \quad (28)$$

$$T^L = \Phi^L\left(\tau_1^{II}, \tau_2^{II}; \Omega_1^{II}, \Omega_2^{II}\right) \equiv \psi^L\left[k_2^{II} f\left(\tau_2^{II}\right) - k_1^{II} f\left(\tau_1^{II}\right)\right]. \quad (29)$$

Again, $k_2^{II}\ f(\tau_2^{II*}) > k_1^{II}\ f(\tau_1^{II*})$ is required which implies:

$$\frac{f\left(\tau_2^{II*}\right)}{f\left(\tau_1^{II*}\right)} > \frac{k_1^{II}}{k_2^{II}}. \quad (30)$$

Define $(\tau_1^{II*}, \tau_2^{II*}) \equiv \{(\tau_1^{II}, \tau_2^{II}) \in \mathbb{R}^2_+ | T^L = \Phi^L(\tau_1^{II}, \tau_2^{II}; \Omega_1^{II}, \Omega_2^{II})\}$. We follow similar steps to those in the previous subsection, rewriting (28) and (29) as:

$$f\left(\tau_2^{II*}\right) \geq \frac{k_0^H}{k_2^{II}} + \frac{k_1^{II}}{k_2^{II}} f\left(\tau_1^{II*}\right), \quad \text{and} \quad (31)$$

$$f\left(\tau_2^{II*}\right) = \frac{k_0^L}{k_2^{II}} + \frac{k_1^{II}}{k_2^{II}} f\left(\tau_1^{II*}\right), \quad (32)$$

and further express the latter equality below:

$$\frac{f\left(\tau_2^{II*}\right)}{f\left(\tau_1^{II*}\right)} = \frac{k_0^L}{k_2^{II} f\left(\tau_1^{II*}\right)} + \frac{k_1^{II}}{k_2^{II}}. \quad (33)$$

Consider now,

Condition II. $(T^H/T^L) \leq (\gamma^H/\gamma^L)^{(1/1-\alpha)}$.

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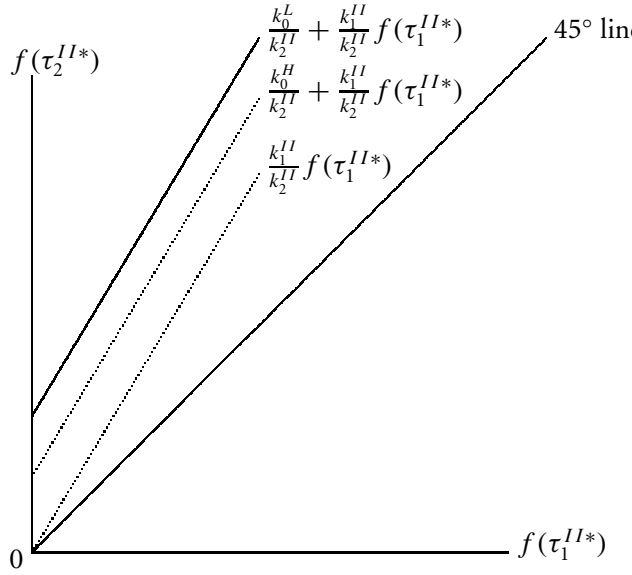


Figure 4: The Relationship between Transformed Property Tax Rates in the Equilibrium of Pattern II

Condition II requires that the difference between agents' commuting costs be sufficiently small compared to the difference in their respective preferences for housing quality. We then have:

Proposition 2. (Pattern II Equilibrium) Under Condition II, the market equilibrium exists in which all type-H agents reside in region 2 whereas both regions are populated by type-L agents (Pattern II).

proof. Plugging (32) into the inequality (31) yields: $k_0^H \leq k_0^L$, which holds under Condition II. Substituting (33) into the inequality (30) shows that the inequality holds as long as $f(\tau_1^{II*}) \geq 0$, $k_0^L > 0$, and $k_2^{II} > 0$. □

The relationship between equilibrium property tax rates $(\tau_1^{II*}, \tau_2^{II*})$ is depicted in Figure 4. In contrast to the Pattern I equilibrium, the Pattern II equilibrium always features $\tau_1^{II*} > \tau_2^{II*}$ and the type-L agents' locational no-arbitrage condition (32) always satisfies $f(\tau_2^{II*}) \geq (k_0^H/k_2^{II}) + (k_1^{II}/k_2^{II}) f(\tau_1^{II*})$ and $f(\tau_2^{II*}) > (k_1^{II}/k_2^{II}) f(\tau_1^{II*})$. In this case, we can observe:

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$$\frac{G_2^{II*}}{G_1^{II*}} = \left(\frac{f\left(\tau_2^{II}\right)}{f\left(\tau_1^{II}\right)}\right)^2 \left(\frac{\Omega_2^{II}}{\Omega_1^{II}}\right)^{2(1-\alpha)} > 1,$$

because $(f(\tau_2^{II*})/f(\tau_1^{II*})) > 1$ and $(\Omega_2^{II}/\Omega_1^{II}) > 1$. That is, under Pattern II, both the incentive adjustment and the aggregate preference effects support a greater provision of local public goods in region 2 where type-H agents reside (i.e., $G_1^{II*} < G_2^{II*}$). That is, in region 2 where type-H agents reside, both the property tax rate and the provision of local public goods are always higher regardless of the level of τ_1^{II*}.

Under Pattern II, the effects of T^L and g on the property tax rates are still transmitted through the incentive adjustment effect (via k_0^L), so the results are the same as under Pattern I regardless of where type-H agents reside. On the contrary, the response to changes in γ^H crucially depends on type-H agents' residential locations. With higher γ^H, type-H agents are more willing to pay for the local public good in region 2, thereby leading to a higher property tax rate in region 2 for a given property tax rate in region 1.

4.3 Equilibrium Configuration

We are prepared to summarize the equilibrium configuration by means of the following figure:

When the difference between agents' commuting costs is sufficiently large compared to the difference in their respective preferences for housing quality (i.e., Condition I is met), the market equilibrium is of Pattern I in which all type-H agents reside in region 1. On the contrary, when the difference between agents' commuting costs is sufficiently small (i.e., Condition II holds), the market equilibrium is of Pattern II with all type-H agents residing in region 2. Figure 5 displays the partition of the parameter space into the two equilibrium patterns. It is clear that the partition curve is above the 45° line, indicating that Pattern II is more likely to arise than Pattern I.

In Pattern I, all high type agents agglomerate in the CBD, which corresponds to the distribution patterns in Asian and European metropolitan areas. In Pattern II, all high type agents agglomerate in the suburb, which captures the distribution patterns observed in the U.S. metropolitan areas. Therefore, under comparable measures of the valuation of commuting time, the commuting costs may be thought of as being similar in the above economies. Our model suggests that the observed distribution patterns in

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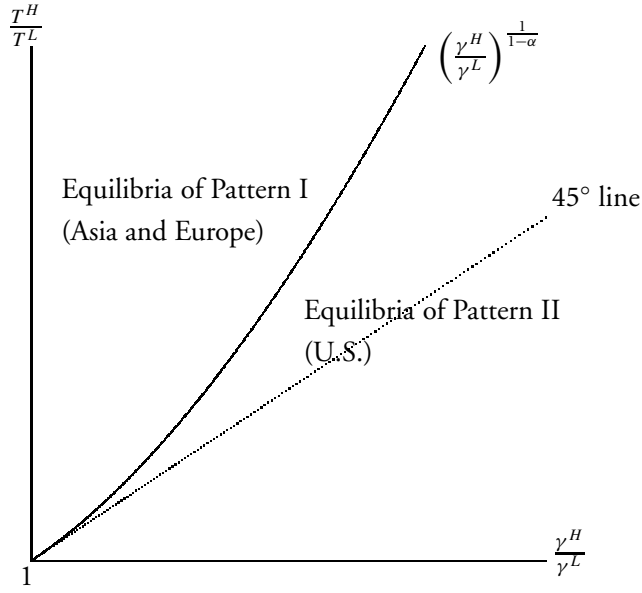


Figure 5: Equilibrium Configurations for Different Commuting Costs and Various Preferences for Housing Quality

different cities may arise from a stronger preference for housing quality in the U.S. compared to Asia and Europe.

Finally, when the ratio of commuting costs between the two types is in a specific homothetic relation to the ratio of the marginal valuations of housing quality (precisely, $(T^H/T^L) = (\gamma^H/\gamma^L)^{(1/1-\alpha)}$), a mixed equilibrium arises where both the rich and the poor reside in both regions. A special case of such a mixed equilibrium is the fully integrated symmetric equilibrium in which each region contains the same proportion of the rich (i.e., $x_1 = x_2 = 1/2$).

5 Conclusions

We have developed a general-equilibrium endogenous-sorting model to analyze the pattern of the multi-class competitive spatial equilibrium. We have established that the equilibrium configuration crucially depends on the heterogeneity of preferences for housing quality versus that of the opportunity

costs of commuting. When the difference between agents' commuting costs is sufficiently large compared to the difference in their respective preferences for housing quality, the equilibrium is segregated with all rich residing in the central city. When the reverse is true, the equilibrium is segregated with all rich in the suburb. While in the latter case the suburb with all the rich residing there features a higher property tax and more local public goods provision, in the former case there is no such clear-cut comparison. Under comparable measures of the valuation of commuting time, our results have suggested that the observed distribution patterns in many cities in the U.S. where more rich reside in suburbs may arise from a stronger preference for housing quality by the Americans compared to Asians and Europeans. Moreover, while locational stratification is a result of preference heterogeneity and a positive neighborhood externality in the form of housing quality financed by local property taxes, where the rich reside will crucially depend on their opportunity costs of commuting relative to their preferences for housing quality.

There are several ways to extend the paper, among which we highlight but three. The first, and perhaps the most important, is to generalize the location-specific supply of housing quality. In particular, it would be interesting to allow individual resources to be devoted to enhancing housing quality à la Peng and Wang (2009). This is likely to yield richer equilibrium configuration outcomes due to non-monotonic effects of property taxation. The second is to extend the model economy to a dynamic setting. In particular, it would be interesting to allow the stock of housing quality to be determined by the existing capacity of local public infrastructure as well as the flow input of investment. This may be done in an overlapping-generations setup à la Chen et al. (2009). An intriguing issue is whether locational stratification is more likely to be the stable equilibrium outcome. If it is, one may better understand why economic segregation is often observed and persists in many different societies.

Finally, a possible venue for future research toward a deeper understanding of sorting and segregation is to allow the number of types of agents to exceed the number of regions as analyzed by de Bartolome and Ross (2003) and Hanushek and Yilmaz (2007). Notably, when the number of types of agents equals the number of regions as in our benchmark economy, we have shown that one of the regions features generically homogeneous population in equilibrium. This result may not be what has been found in the empirical literature, as Hardman and Ioannides (2004) document, using the

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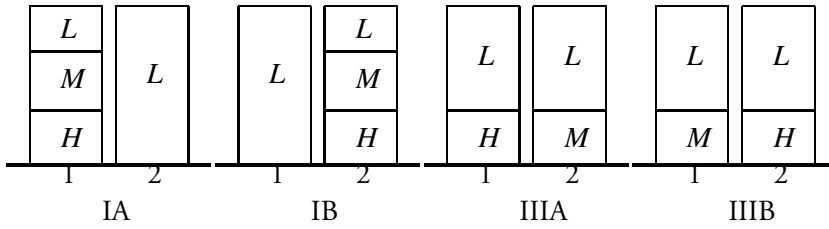


Figure 6a: With Three Types of Agents and Two Regions, When $H + M < L$, There Are Four Equilibrium Patterns

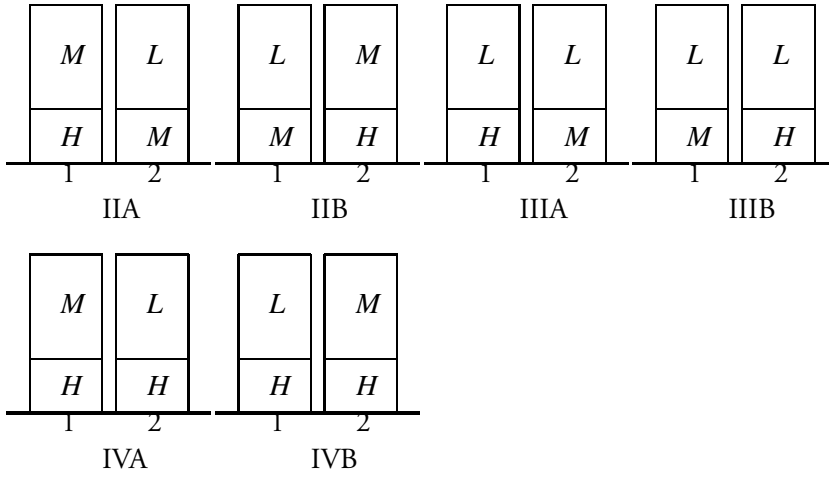


Figure 6b: With Three Types of Agents and Two Regions, When $H + M > L$, There Are Six Equilibrium Patterns

American Housing Survey data, that the heterogeneity within a community typically exceeds the heterogeneity across communities. To rectify this problem, one may consider the most stylized case where there are 3 types and 2 regions, with population H_j, M_j, L_j in region j, where $H_1 + H_2 = H$, $M_1 + M_2 = M$, $L_1 + L_2 = L$ and $H_j + M_j + L_j = N/2$, $j \in J$ and where $H < M < L$. When $H + M < L$, the equilibrium features generically Patterns IA, IB, IIIA, and IIIB (see Figure 6a). When $H + M > L$, the equilibrium features generically Patterns IIA, IIB, IIIA, IIIB, IVA, and IVB (see Figure 6b). Therefore, we have richer patterns of equilibria than in our benchmark model. In particular, while Patterns IA and IB are similar to

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our benchmark Patterns I and II, in other equilibrium patterns all regions feature heterogeneous population as observed in reality.

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投稿日期: 2012年5月29日, 接受日期: 2012年9月24日

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公共財投入、居住品質和區域分隔

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本文對經濟分隔相關文獻的貢獻在於強調對居住品質的偏好差異可以解釋區域分隔現象的產生。本文建構一內生分類 (endogenous sorting) 之一般均衡模型, 兼以考量家戶在所得、通勤時間成本乃至於對居住品質偏好上的差異。居住品質受地方公共財投入內生影響, 而地方公共財投入的規模又由所徵得的財產稅收入決定。本文找出完全競爭空間均衡的特徵條件, 並進一步發現完全整合居住型態的出現需要參數滿足一測度爲零的特定條件。換言之, 一般而言均衡滿足區域分隔型態。當家戶在通勤成本上的差異遠大於其在居住品質偏好上的差異, 均衡時所有富人居住在市中心, 符合亞洲和歐洲的都會區分佈型態。反之則均衡時所有富人居住在市郊, 符合美洲都會區的分佈型態。

關鍵詞: 經濟分隔, 內生分類, 居住品質, 完全競爭空間均衡
JEL 分類代號: D50, H41, R20

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