Estimating Taiwan's Monthly GDP in an Exact Kalman Filter Framework: A Research Note

Yu-Lieh Huang*

In this paper, we describe a framework that nests a wide range of interpolation/distribution setups but relaxes the co-integration condition of temporal disaggregation that has been reported in the literature. Our goal is to evaluate alternative interpolation/distribution models and then generate the monthly deseasonalized real gross domestic product of Taiwan. Our empirical results show that the monthly estimates, incorporated in the information obtained from the industrial production index, are highly consistent with quarterly figures. These estimates should be invaluable to researchers and practitioners for short-run policy analysis in that they signal any emerging economic problems.

Keywords: interpolation, Kalman filter, temporal disaggregation, state-space model.
JEL classification: C51, E31, E47

1 Introduction

A fundamental problem often encountered by macroeconomic researchers lies in the interpolation or distribution of economic time series. For interpolation, an attempt must be made, for example, to estimate the missing values of a stock variable at a higher frequency when the observed values of the series are only reported for lower frequencies. As for distribution, the primary

*Department of Quantitative Finance, National Tsing Hua University. I am indebted to the associate editor and two anonymous referees whose comments lead to a much improved version of the paper. All remaining errors are mine.

經濟論文叢刊 (*Taiwan Economic Review*), 38:1 (2010), 147–160。
國立台灣大學經濟學系出版

focus of this paper, the problem centers on the estimation of intraperiod values for a flow variable subjected to the constraint that their sums, or averages, are equal to the aggregates over a lower frequency of observations. These two processes, usually called "temporal disaggregation techniques", play an important role in the estimation of short-term economic indicators. In fact, a large share of the Euro area quarterly GDP is actually estimated by disaggregation techniques; see, e.g., Eurostat (1999).

Broadly speaking, methods for disaggregation can be classified into two approaches: (1) one that involves the use of observed related series at the desired higher frequency and (2) one that only relies on pure time series dynamic models and that does not make use of information obtained from other related series. The former has been discussed by a number of authors, one of the first of whom was Friedman (1962). Subsequent contributions have been made by Chow and Lin (1971), Fernández (1981), Litterman (1983) and De Alba (1988), to name a few. The second approach, explored by Boot et al. (1967) and Wei and Stram (1990), depends on the dynamic structure of the time series to be disaggregated. Although these approaches both have the potential to be applied to a wide variety of cases, they mainly rely on undesired and/or arbitrary assumptions. To explain, the former does not accommodate the possibility of some underlying dynamic structure of time series (e.g., ARIMA process). It also assumes that there is a co-integration relation between the nonstationary related series and the unobserved disaggregated series *à priori*. Note that such a presumption may cause inaccurate forecasts; see, e.g., Reinsel and Ahn (1992) and Lin and Tsay (1996). As for the second approach, it only extracts signals from the presumed dynamic pattern of the series but in such a way that no other high frequency related information can be added.

In light of the above shortcomings, this paper evaluates an alternative method, namely the state-space approach, first introduced by Harvey (1989) and later developed by Harvey and Koopman (1997). Given the state-space representation discussed in this paper, several features emerge here. First, by assuming that the disaggregated series follows an ARIMAX process, the proposed representation is able to describe the dynamic structure of disaggregated time series, which is in contrast to the models in the first approach. Second, compared to the specification in the existing approaches, the proposed model allows for high frequency related series without imposing the assumption of a co-integration restriction. Note that mis-imposing co-integration constraints between series is unsound and may result in inac-

curate forecasts, cf. Lin and Tsay (1996). Third, for the proposed model, the exact Kalman filtering and smoothing algorithms recently introduced by Koopman (1997) and Koopman and Durbin (2003) are used to evaluate the likelihood function and to estimate real GDP at monthly intervals. Some of the major advantages of using the exact Kalman filter are that it computes the optimal estimates of the latent monthly GDP and provides a way to calculate exact finite-sample forecasts. It also avoids the potential "divergence" problem that may arise from the algorithm proposed by Harvey and Phillips (1979) and Kim and Nelson (1999).[1]

In our empirical study, we apply the proposed model to Taiwan's seasonally adjusted, quarterly real GDP data for the period of 1961:I to 2006:IV with 184 observations. We use the sample period of 1961:I to 2006:II for estimation and remaining data for out-of-sample forecasts. We first report the smoothing estimates and the associated confidence intervals for the most up-to-date information vis-à-vis the state of Taiwan's economy. We then compare the estimated results with those obtained from conventional models using in-sample fitness and out-of-sample forecast ability as measures of performance. Other than transforming quarterly real GDP figures into monthly GDP series, a disaggregation from annually to quarterly estimates is also performed, which can be used to directly compare with the published quarterly real GDP. Our in-sample estimates indicate that the proposed approach generates a relatively smoother path, which is consistent with the underlying characteristics of GDP series. Also, the resulting out-of-sample forecasts show that the proposed model provides the most accurate values. These suggest that the proposed approach may serve as an alternative for the temporal disaggregation of Taiwan's real GDP.

This paper is organized as follows: Section 2 introduces the state-space methodology and demonstrates how it can be applied to a series that can be modelled using nonstationary ARIMAX processes. Section 3 presents the empirical results of Taiwan's real GDP. Section 4 presents our conclusions.

[1] The usefulness of the Kalman filter may be nullified by the phenomenon known as "divergence", as shown in Kalman and Bucy (1961) and Fitzgerald (1971). That is, any inaccuracy in the model or in the computational operations may result in greater errors in the estimates than what the theory predicts.

2 State-Space Model with Exact Kalman Filter

To illustrate our basic concept, let $\ddot{y}_\tau$ be the seasonally-adjusted, quarterly real GDP at time τ and $\ddot{\mathbf{y}}_\tau = (00\ddot{y}_\tau)'$ be a 3×1 vector of observations. Then we stack the observations $\ddot{\mathbf{y}}_\tau$ in a one-column vector to obtain $\mathbf{y} = (\ddot{\mathbf{y}}_1'\ddot{\mathbf{y}}_2' \cdots \ddot{\mathbf{y}}_T')'$, where T is the number of quarterly real GDP. Also let denote the t^{th} element of $\mathbf{y}$. We assume that the unobserved monthly GDP, y_t^*, satisfies the monthly-to-quarterly sum-up constraint:

$$y_t = \sum_{i=0}^{2} y_{t-i}^*, \quad t = 3, 6, 9, \cdots, 3T. \tag{1}$$

We further assume that the difference of y_t^* follows an ARMAX(p, q) process,

$$\Psi(B)z_t^* = x_t'\beta + \Phi(B)\varepsilon_t, \quad t = 1, 2, 3, \cdots, 3T, \tag{2}$$

where $z_t^* = y_t^* - y_{t-1}^*$, ε_t is a martingale difference sequence with mean zero and variance σ_ε^2, $\Psi(B) = 1 - \psi_1 B - \cdots - \psi_p B^p$ and $\Phi(B) = 1 + \varphi_1 B + \cdots + \varphi_q B^q$ are finite-order polynomials of the back-shift operator B such that they have no common factors and their roots are all outside the unit circle. The related monthly indicators x_t are observable and weakly stationary. As can be seen in (1) and (2), the proposed model is capable of exhibiting the dynamic patterns of monthly GDP, and it allows for related exogenous variables. Moreover, in our model, it is only assumed that there exists a linear relationship between z_t^* and x_t which are both weakly stationary. Hence, any co-integration restriction between y_t^* and its related variables is not required. This framework is in sharp contrast with Chow and Lin (1971), and De Alba (1988) where the unobserved variable y_t^* is assumed to have a full co-integration relation with the related series *á priori*.

It is now easy to show that the proposed model in (1) and (2) can be expressed as a state-space model with the following

$$\begin{aligned} y_t &= h_t'\gamma_t, \\ \gamma_{t+1} &= \mu_t + F\gamma_t + R\varepsilon_{t+1}, \end{aligned} \tag{3}$$

for $t = 1, 2, 3, \cdots, 3T$, where a $(r+2)$-dimensional vector $h_t = (10 \cdots 021)'$ if $t = 3, 6, 9, \cdots, 3T$ and $h_t = 0$, otherwise;

$$\gamma_t = \begin{bmatrix} z_t^* \\ \sum_{i=2}^{r} \psi_i z_{t-i+1}^* + \sum_{i=2}^{r} \varphi_{i-1}\varepsilon_{t-i+2} \\ \sum_{i=3}^{r} \psi_i z_{t-i+2}^* + \sum_{i=3}^{r} \varphi_{i-1}\varepsilon_{t-i+3} \\ \vdots \\ \psi_r z_{t-1}^* + \varphi_{r-1}\varepsilon_t \\ y_{t-1}^* \\ y_{t-2}^* \end{bmatrix}_{(r+2)\times 1},$$

$r = \max(p, q+1)$, $\psi_i = 0$ for $i > p$ and $\varphi_i = 0$ for $i > q$. The terms F and R are fixed matrixes such that

$$F = \begin{bmatrix} \psi_1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \psi_2 & 0 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \psi_{r-1} & 0 & 0 & \cdots & 1 & 0 & 0 \\ \psi_r & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}_{(r+2)\times(r+2)}, \quad R = \begin{bmatrix} 1 \\ \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_{r-1} \\ 0 \\ 0 \end{bmatrix}_{(r+2)\times 1},$$

and $\mu_t = (x_{t+1}'\beta 0 \cdots 0)'$ is a $(r+2) \times 1$ vector. We will apply model (3) to Taiwan's real GDP in our empirical study.

Because $\{y_t\}$ in (3) is nonstationary, the density of the observations does not exist, and so the likelihood is not defined in the usual sense. We therefore follow De Jong (1991) and compute the likelihood evaluation by using a diffuse initial state in the state-space model.[2] Instead of estimating y_0^* and y_{-1}^* in the initial state vector (i.e., γ_1), we treat these elements as diffuse random elements. One simple approximate technique to handle diffuse random elements is to initiate the Kalman filter using a very large covariance matrix; see, for example, Harvey and Phillips (1979) and Kim and Nelson (1999). While this device is computationally convenient for approximating exploratory work, it is theoretically unsatisfactory since it might result in large rounding errors and to suffer from the potential "divergence" problem, cf. Kalman and Bucy (1961) and Fitzgerald (1971). We thus follow Koopman (1997) and develop an exact initial treatment. A detailed description of this algorithm is given in a separated technical report and is available upon request.

[2] A state is said to be diffuse if its covariance matrix is arbitrarily large. Diffuse initial states arise in the context of model nonstationarity.

From the recursions of the exact Kalman filter algorithm, we obtain the filtered series $\gamma_{t|t} = \mathrm{E}(\gamma_t \mid \Omega^t)$ and all the quantities that are necessary for the evaluation of the diffuse log-likelihood function discussed in Koopman (1997), where Ω^t is the collection of all the observed variables up to time t. Then, using a numerical-search method, we can find the approximate quasi-maximum likelihood estimates (QMLE): $\hat{\theta} = (\hat{\beta}, \hat{\psi}_1, \cdots, \hat{\psi}_p, \hat{\varphi}_1, \cdots, \hat{\varphi}_q, \hat{\sigma}_\varepsilon)'$. Our program is written in GAUSS which employs the Broyden-Fletcher-Goldfarb-Shanno (BFGS) search algorithm. Plugging $\hat{\theta}$ into the smoothing recursions proposed by Koopman and Durbin (2003), we obtain the estimated smoothed series $\gamma_{t|T} = \mathrm{E}(\gamma_t \mid \Omega^T)$, which are the optimal forecasts of γ_t based on all the information in the sample. We then construct the seasonally-adjusted monthly GDP using the smoothed series $\gamma_{t|T}$.

3 Empirical Study

To demonstrate the applicability of the state-space approach, we apply model (3) with a diffuse initial state to Taiwan's real GDP from 1961:I–2006:II for in-sample fitness. Based on economic intuition and the quality of the data, we select a constant term and the change in the industrial production index (IPI) as the observed related monthly indicators x_t. To check whether the difference of IPI contains a unit root, we first apply the Augmented Dickey-Fuller (ADF) test. The resulting ADF statistic is -11.75383 which exceeds the 1% critical value -3.4423 and hence strongly rejects the unit-root hypothesis. Then, we estimate an array of the proposed models with $0 \leq, p, q, \leq 4$. The parameters in these models are estimated using the algorithm described in the previous section. This algorithm is initialized using a broad range of random initial values. The covariance matrix of $\hat{\theta}$ is $-H(\hat{\theta})^{-1}$, where $H(\hat{\theta})$ is the Hessian matrix of the log-likelihood function evaluated at the QMLE $\hat{\theta}$. Among all the models considered, the Schwartz Bayesian information criterion (SIC) selects the model with $p = 1$ and $q = 2$. The estimated results are summarized in Table 1. As the table shows, all parameter estimates are statistically significant at the 5% level. From Table 1, we find that the difference of IPI has a significant impact on $z_t^*(\hat{\beta}_1 = 1.6227)$. This result may justify our selection for the high frequency related series in (2).

We now conduct some diagnostic checks on the estimated model, including the Ljung and Box (1978) Q test and the LM test of Engle (1982) on the ARCH effect. The importance of diagnostic checking has usually

Table 1: Quasi-maximum likelihood estimates of the proposed state-space model

Estimate	Estimate	Standard error	t-statistic
$\hat{\beta}_0$	17.1668	4.5819	3.7466*
$\hat{\beta}_1$	1.6227	0.4231	3.8352*
$\hat{\psi}_1$	0.9462	0.0143	66.1678*
$\hat{\varphi}_1$	−2,011.1024	200.1948	−10.0457*
$\hat{\varphi}_2$	2,146.5556	211.0155	10.1725*
$\hat{\sigma}_\varepsilon$	1.6308	0.1730	9.4265*
Log-Likelihood=−3667.960			SIC = 7,377.5089

Note: t-statistics with an asterisk are significant at the 5% level.

been ignored in the literature, one reason being that innovations are not automatically available from the implementation of existing methods. However, the smoothed mean of the residual of the proposed approach, i.e., $\mathrm{E}(\varepsilon_t \mid \Omega^T)$, is easy to calculate and is a by-product of the algorithm. The resulting statistics for the residuals are $Q(12) = 18.147$, $Q(24) = 26.135$ and ARCH(4) = 1.190. These statistics are all insignificant at the 5% level under the $\chi^2(12)$, $\chi^2(24)$ and $\chi^2(4)$ distributions. Hence, there appears to be no serial correlation or conditional heteroskedasticity in these residuals.[3]

In Figure 1, we plot the published quarterly GDP and the estimated monthly real GDP in the panel on the left and that on the right, respectively. The shaded areas designate the recessionary periods identified by the CEPD; the label "P" ("T") denotes the peaks (troughs). It is important to note that both of the series in Figure 1 share a similar dynamic pattern. In Table 2, we summarize the business cycle turning points, identified by following the Bry and Boschan (1971) approach and using the estimated monthly real GDP, IPI, Manufacturing Sales and Nonagricultural Employment (Dataset I).[4] For the purposes of comparison, we also check the dates

[3] We also apply the ADF test to check if z_t^* contains a unit root. The resulting ADF statistic is −5.8128 which exceeds the 1% critical value and hence strongly rejects the unit-root hypothesis.

[4] Bry and Boschan (1971) provide a nonparametric, intuitive and easily im-plementable algorithm to determine peaks and troughs in time series. For a detailed description, the reader is referred to Bry and Boschan's paper or Huang et al. (1998).

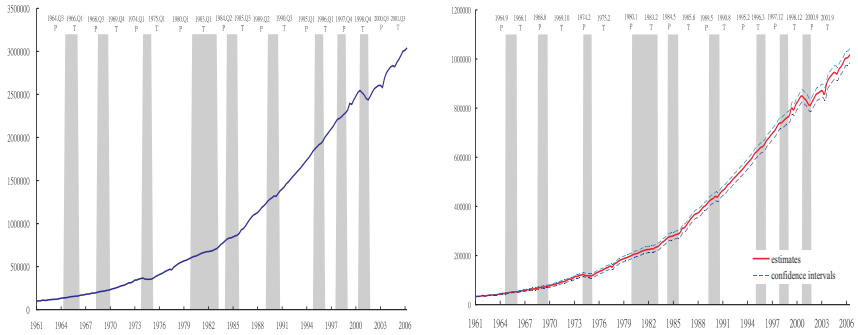


Figure 1: Published quarterly GDP (left) and estimated monthly GDP (right)

of the turning points by using the quarterly GDP, IPI, Manufacturing Sales and Nonagricultural Employment (Dataset II). It is equally interesting to find that both of the turning-point dates match very closely, except for only the trough in 1998. This result shows that the estimated monthly GDP and the published quarterly GDP share common underlying information, and thus the turning points identified respectively are not likely to deviate from each other.[5] Together with the previous result, our overall findings suggest that the estimated monthly GDP data are highly consistent with the quarterly data.

To examine the data more carefully, we report the estimated monthly disaggregations of Taiwan's real GDP with the approaches of Chow and Lin (1971), Fernández (1981), Litterman (1983), Boot et al. (1967) and the proposed state-space model. Due to space constraints, we only report the estimated results for the 2001:01–2006:06 period, but a detailed table of the results is available upon request. For easy comparison we plot these estimated monthly GDPs for the period of 2001:01–2006:06 in Figure 2, where the solid (dash-dot, dash-dot-dot, dot, dash) line denotes the estimated monthly GDP of the state-space model (Chow and Lin, Fernández, Litterman, Boot et al.). As shown in Figure 2, the temporal disaggregated GDPs estimated by Chow and Lin (1971), Fernández (1981) and Litterman (1983) are more volatile than those obtained by Boot et al. (1967) and the state-space approach. Although the actual monthly GDP cannot be

[5] We thank the referee for providing this constructive comment.

Table 2: Estimated business cycle turning points

Peaks		Troughs	
Dataset I	Dataset II	Dataset I	Dataset II
1980:07	1980:07	1982:07	1982:07
1984:05	1984:05	1985:09	1985:09
1987:07	1987:07	1988:04	1988:04
1989:04	1989:04	1991:02	1991:02
1991:10	1991:10	1993:12	1993:12
1995:01	1995:01	1996:08	1996:08
1998:01	1998:01	1998:12	1998:11
2000:08	2000:08	2001:08	2001:08
2002:05	2002:05	2003:04	2003:04
2004:05	2004:05	2004:11	2004:11
2005:12	2005:12	—	—

Note: Dataset I (II) includes estimated monthly real GDP (quarterly real GDP), IPI, Manufacturing Sales and Nonagricultural Employment



Figure 2: The estimated monthly GDPs of different approaches

observed, we honestly believe that a reasonable GDP trend should exhibit smoother path as those estimated in Boot et al. (1967) and the proposed state-space model.

Table 3: The out-of-sample forecasts of Taiwan's real GDP

Date	Published GDP	State-Space	Chow & Lin	Fernández	Litterman	Boot et al.
2006:07		1,021,715	861,423	530,329	514,287	971,155
2006:08		1,025,820	870,995	536,717	520,469	998,407
2006:09		1,029,942	869,792	535,914	519,692	1,004,317
2006:III	3109695	3,077,477	2,602,210	1,602,959	1,554,448	2,973,879
2006:10		1,034,082	861,915	530,658	514,605	997,998
2006:11		1,038,239	867,932	534,673	518,491	1,008,086
2006:12		1,042,411	860,602	529,782	513,757	1,006,507
2006:IV	3137611	3,114,732	2,590,449	1,595,112	1,546,853	3,012,591
MSE		39,515.16	746,275.80	2,156,282.85	2,224,703.12	184,596.82

Note: MSE stands for mean square error. The predicted value of 2006:III (2006:IV) is the sum of forecasts from 2006:07 to 2006:09 (2006:10 to 2006:12).

Table 3 contains the 1-step to 6-step ahead out-of-sample forecasts (2006:07–2006:12) for the monthly real GDP using five approaches discussed above. The associated quarterly forecasts and the actual quarterly real GDP are also given. In this table, the forecasts of monthly GDP from 2006:07 to 2006:12 for the proposed state-space model are 1,021,715.23, 1,025,819.70, 1,029,942.43, 1,034,082.42, 1,038,238.75 and 1042410.54, respectively. These show that the predicted quarterly GDP for the 2006:III and 2006:IV period are 3,077,477.36 and 3,114,731.71; the associated mean square error (MSE) is 39,515.16. As compared with the MSEs from other approaches, the proposed state-space model possess the smallest MSE which is only about half the MSE of Chow and Lin.[6] This forecasting result is decisive and apparent to conclude that state-space model achieved better prediction power and thereby proves to serve as a better disaggregation model.

Other than transforming quarterly real GDP into monthly GDP, a disaggregation from annually to quarterly figures is also conducted. To do this, we take seasonally-adjusted, annually real GDP (quarterly IPI) as $\ddot{y}_\tau$ (x_t) and re-construct a state-space model such that the quarterly-to-annually sum-up constraint is satisfied. Figure 3 shows both the published quarterly GDP and the disaggregated state-space quarterly GDPs for the period of 1991:I–2006:IV. In this figure, the solid line denotes the published quarterly GDP and the dash line denotes the estimated quarterly GDP of the state-space

[6] One year out-of-sample forecasts were also performed reaching similar conclusion.

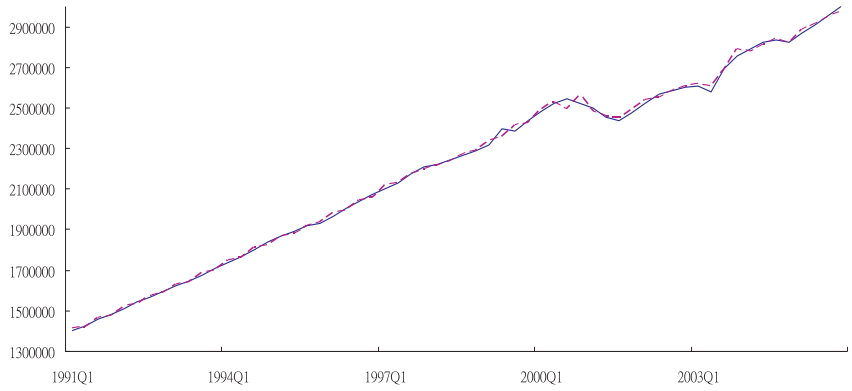


Figure 3: The published (solid line) and estimated (dash line) quarterly GDPs

model. As can be seen, the estimates from the state-space model do not deviate much from the published quarterly GDP. To compare the estimated results with those obtained from conventional models, we, again, use the MSE as a measure of performance for the period from 1961:I–2005:IV. The resulting MSE of the state-space model is 2.0425E+09, while the MSEs of Chow and Lin (1971), Fernández (1981), Litterman (1983) and Boot et al. (1967) are 3.1537E+09, 3.1234E+09, 3.0851E+09 and 2.8712E+09, respectively. Among all the models considered, the state-space model has the smallest MSE.

4 Conclusions

The aim of the present study is the temporal disaggregation of Taiwan's real GDP that is available only at the quarterly frequency of observations; the resulting monthly estimates incorporated in the information contained in the monthly IPI are calculated via the state-space approach. The state-space representation proposed here has several interesting features. First, it allows for the use of dynamic models for the disaggregation of a time series. Second, it admits related high-frequency series and relaxes the assumption of a co-integration relation. Third, it nests the traditional linear disaggregation techniques, such as those of Chow and Lin (1971), Fernández (1981) and Litterman (1983), within more general dynamic specifications; see Proietti

(2006) for detailed comparisons of these techniques. Fourth, it computes the optimal estimates of the latent monthly GDP and provides exact finite-sample forecasts. Thus, the method proposed here adds more flexibility to others previously reported in the literature.

The application of the proposed model to Taiwan's real GDP demonstrates that it is a valuable analytical tool with which to calculate the monthly disaggregation of real GDP. In particular, our empirical results show that the estimated monthly GDP shares a very similar dynamic smooth pattern during the period of analysis. They also show that the turning-point periods of the monthly GDP closely match those based on the quarterly GDP and other related series. Moreover, the proposed approach may provide a more accurate out-of-sample forecasts than those predicted from the other models. The proposed model may therefore serve as an alternative for the temporal disaggregation of time series.

References

Boot, J. C. G., Feibes, W., and Lisman, J. H. C. Lisman (1967), "Further methods of derivation of quarterly figures from annual data", *Applied Statistics*, 16, 65–75.

Bry, G. and Boschan, C. (1971), "Cyclical analysis of time series: Selected procedures and computer programs", NBE Technical Paper, no.20.

Chow, G. C. and Lin, A. L. (1971), "Best linear unbiased interpolation, distribution and extrapolation of time series by related series", *Review of Economics and Statistics*, 53, 372–375.

De Alba, E. (1988), "Disaggregation and forecasting: A Bayesian analysis", *Journal of Business and Economic Statistics*, 6, 197–206.

De Jong, P. (1991), "The diffuse Kalman filter", *Annals of Statistics*, 19, 1073–1083.

Engle, R. F. (1982), "Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation", *Econometrica*, 50, 987–1008.

Eurostat (1999), *Handbook on Quarterly National Accounts*, Luxembourg: European Communities.

Fernández, R. (1981), "A methodological note on the estimation of time series", *Review of Economics and Statistics*, 63, 471–478.

Fitzgerald, R. J. (1971), "Divergence of the Kalman filter", *IEEE Transactions on Automatic Control*, AC-16, 736–747.

Friedman, M. (1962), "The interpolation of time series by related series", *Journal of the American Statistical Association*, 57, 729–757.

Harvey, A. C. (1989), *Forecasting, Structural Time Series and the Kalman Filter*, Cambridge: Cambridge University Press.

Harvey, A. C. and Koopman, S. J. (1997), "Multivariate structural time series models", in B. Hanzon C. Heij, J. M. Shumacher and C. Praagman (eds.), *System Dynamics in Economic and Financial Models*, 269–298, Chichester: John Wiley and Sons.

Harvey, A. C. and Phillips, G. D. A. (1979), "The estimation of regression models with autoregressive-moving average disturbances", *Biometrika*, 66, 49–58.

Huang, Y. L., Kuan, C. M., and Lin, K. (1998), "Identifying the turning points of business cycles and forecasting economic growth rates in Taiwan", *Taiwan Economic Review*, 26, 431–457.

Kalman, R. and Bucy, R. (1961), "New results in linear filtering and prediction theory", *Transactions of the American Society of Mechanical Engineers: Journal of Basic Engineering*, 83, 95–108.

Kim, C.-J. and Nelson, C. R. (1999), *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*, Cambridge: MIT Press.

Koopman, S. J. (1997), "Exact initial Kalman filtering and smoothing for non-stationary time series models", *Journal of the American Statistical Association*, 92, 1630–1638.

Koopman, S. J. and Durbin, J. (2003), "Filtering and smoothing of state vector for diffuse state-space models", *Journal of Time Series Analysis*, 24, 85–98.

Lin, J. L. and Tsay, R. S. (1996), "Co-integration constraint and forecasting: An empirical examination", *Journal of Applied Econometrics*, 11, 519–538.

Litterman, R. B. (1983), "A random walk Markov model for the distribution of time series", *Journal of Business and Economic Statistics*, 1, 169–173.

Ljung, G. M. and Box, G. E. P. (1978), "On a measure of lack of fit in time series models", *Biometrika*, 65, 297–303.

Proietti, T. (2006), "Temporal disaggregation by state-space methods: Dynamic regression methods revisited", *Econometrics Journal*, 9, 357–372.

Reinsel, G. C. and Ahn, S. K. (1992), "Vector autoregressive models with unit roots and reduced rank structure: Estimation, likelihood ratio test, and forecasting", *Journal of Time Series Analysis*, 13, 353–375.

Wei, W. W. S. and Stram, D. O. (1990), "Disaggregation of time series models", *Journal of the Royal Statistical Society Series B*, 52, 453–467.

投稿日期: 2008年7月7日, 接受日期: 2009年4月22日

Estimating Taiwan's Monthly GDP in an Exact Kalman Filter Framework: A Research Note

黃裕烈
國立清華大學計量財務金融學系

國內生產毛值 (Gross Domestic Product, GDP) 是建構景氣基準循環時一個最主要的參考依據。由於 GDP 數列本身爲季資料, 因此進行總體模型分析時必需要將季資料轉換成月資料以符合計算上的需求。本研究主要目的是依據狀態空間 (state-space) 計量模型, 嘗試重新建構適當的 GDP 月資料指標以供政府經濟決策之參考。從計量經濟的角度來看, 此一模型的優點在於其能包含文獻上常用的其它模型, 並且放寬即有共整合的假設, 以建構出適當的 GDP 月資料指標。若從實證結果來看, 本研究所估計出的 GDP 月指標與 GDP 季資料有許多相似的特徵。故我們相信, 利用此模型所建構出的月指標確實可以提供政府短期經濟決策時之參考。

關鍵詞: interpolation, Kalman filter, temporal disaggregation, state-space model
JEL 分類代號: C51, E31, E47